Closed-Form Explicit Formula for the TOA-Distribution of Multipaths between a Mis-Aligned Directional Transceiver & an Omni-Directional Transceiver Enclosed among Scatterers

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Abstract

This work is first in the "geometric modeling" literature to derive a closed-form explicit formula for the time-of-arrival (TOA) distribution of the uplink/downlink multipaths between an omni-directional transceiver and a (possibly *mis-aligned*) directional transceiver that points directional beam towards an *arbitrary* direction. This propagation-model idealizes the scatterers as uniformly distributed spatially over a circular disc, which encloses the omni-directional transceiver and is centered at the directional transceiver. However, the directional beam-width reduces the effective scattering spatial region from a circular disc to a "pie-cut" shape. This proposed model allows the omni-directional transceiver to lie anywhere inside this pie-cut region, depending on the directional beam mis-alignment.

1 Introduction

Directional antennas have been in use in cellular communications for space-division multiple-access and for beamforming. For "geometrical models" involving directional antennas, [1] and [2] analytically derive closed-form formulas for the multipaths' *direction*-of-arrival (DOA) distribution and *time*-of-arrival (TOA) distribution, respectively. There, a directional transmitter is modeled to lie at the center of a circular-disc scatterer region, within which lies a wireless receiver. (The wireless receiver is often enclosed by proximate scatterers, especially in highly clustered propagation environments, e.g., for a low-lying receiver or for indoor.) With the transmitter's directionality, this circular-disc scatterer region becomes effectively "pie-cut" in shape, with the pie-cut's "angular width" defined by the directional transmitter's azimuth-beam-width (which is symbolized by β in Figure 1). However, the derivations in [1] and [2] deal only with the case of the transmit-beam's beam-center aligning *exactly* towards the direction of the receiver. In field operations, however, non-negligible beam-pointing errors would likely exist with the transmit-beam. Hence, this paper relaxes the above limitation in [1] and [2] so as to allow arbitrary mis-alignment in the transmit-beam pointing direction. However, this paper still succeeds in deriving a closed-form TOA-distribution formula for this more complex geometric model. Indeed, this new derivation covers even the case when the receiver lies outside the transmit-beamwidth.

2 The Present "Pie Cut" Geometric Model

The scatterers are uniformly distributed over a circular disc, of radius R and centered at the origin of a two-dimensional Cartesian plane. Located at this origin is a directional transmitter (Tx), with a beamwidth spanning the azimuth range of $[\beta_1, \beta_2]$ where $-\pi/2 \leq \beta_1 < 0 < \beta_2 \leq \pi/2$. The receiver (Rx) is omni-directional and is located (without loss of generality) at the Cartesian coordinates $(D, 0)$, where $D \leq R$. Please refer to Figure 1.

Because the transmitter's azimuth-beam-width is limited, only those scatterers within the above azimuth-angular sector would have any multipath bouncing off them onwards to the receiver. Hence, the scatterers' effective spatial density may be idealized as

$$\begin{aligned} & f_{x,y}(x,y) \qquad (1) \\ = & \begin{cases} \frac{1}{\beta R^2}, & \text{if } \left(x^2+y^2 \leq R^2\right) \;\&\; \left(\arctan\left|\frac{y}{x}\right| \leq \beta_2\right) \;\&\; \left(\arctan\left|\frac{y}{x}\right| \geq \beta_1\right) \;\&\; (x \geq 0); \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

The above corresponds to the shaded "pie-cut" region in Figure 1.

Like all earlier papers that analytically derive closed-form explicit expressions of the TOA-distribution based on geometrical models, these four standard assumptions are made: (a) Each propagation path, from the mobile to the base-station, reflects off exactly one scatterer. (b) Each scatterer acts (independently of other scatterers) as an omni-directional lossless re-transmitter. (c) Negligible complex-phase effects in the receiving-antenna's vector-summation of its arriving multipaths. That is, all arriving multipaths arriving at each receiving-antenna are assumed to be temporally in-phase among themselves. (d) Polarizational effects may be ignored.

3 The Derived Distribution of the Multipaths' TOA

Three sub-cases exist:

(1) When the pie-cut arc (but not necessarily the pie-cut's straight edges) lies entirely outside the elliptical rim (except possibly at one "tangential" point), the ellipse and the pie-cut boundary cross over each other at only two points.

(2) This is when the pie-cut arc lies at least partly inside the elliptical rim.

(3) When the pie-cut region lies completely inside the ellipse, the TOA distribution-density thus equals $f_\tau(\tau) = 0$.

Presented below is the analytically derived TOA-distribution of the arriving multipaths. (The detailed derivation will be presented in a future journal-edition of this work.)

$$f_\tau(\tau) = \begin{cases} \frac{1}{A}\left[h(\tau,\beta_2) - h(\tau,\beta_1)\right], & \text{for } \tau \in \left[\frac{D}{c}, \frac{D}{c}\eta\right] \\ \frac{1}{A}\left\{H(R - r(\tau,\beta_2))\left[h(\tau,\beta_2) - h(\tau,\theta_0(\tau))\right]\right. \\ \left. -\, H(R - r(\tau,\beta_1))\left[h(\tau,\beta_1) + h(\tau,\theta_0(\tau))\right]\right\}, & \text{for } \tau \in \left[\frac{D}{c}\eta, \frac{D}{c}\max\{\rho_1,\rho_2\}\right] \\ 0, & \text{Otherwise} \end{cases} \quad (2)$$

where

$$\begin{aligned} H(\alpha) &= \begin{cases} 0, & \alpha \le 0 \\ 1, & \alpha > 0 \end{cases} \\ \eta &= 2\frac{R}{D} - 1 \\ \rho_1 &= \frac{R}{D} + \underbrace{\sqrt{\left(\frac{R}{D}\right)^2 - 2\left(\frac{R}{D}\cos\beta_1\right) + 1}}_{=\xi_1} \\ \rho_2 &= \frac{R}{D} + \underbrace{\sqrt{\left(\frac{R}{D}\right)^2 - 2\left(\frac{R}{D}\cos\beta_2\right) + 1}}_{=\xi_2} \end{aligned}$$

$H(.)$, in the middle case in (2), switches among three sub-cases under case (2). The derived (2) is plotted in Figures 2 to 4.

For each propagation-path traveling from a directional transmitter to the omni-directional receiver, there would exist a corresponding propagation-path traversing the same spatial route but in the opposite direction, but with the receiver now being directional and the transmitter now being omni-directional. Hence, the same TOA distribution could apply for both the uplink and the downlink.

References

[1] L. Jiang & S. Y. Tan, "Simple Geometrical-Based AOA Model for Mobile Communication Systems," *IEE Electronics Letters*, vol. 40, no. 19, pp. 1203-1205, September 2004.

[2] L. Jiang & S. Y. Tan, "Geometrically-Based Channel Model for Mobile-Communication Systems," *Microwave and Optical Technology Letters*, vol. 45, no. 6, pp. 522-527, June 2005.

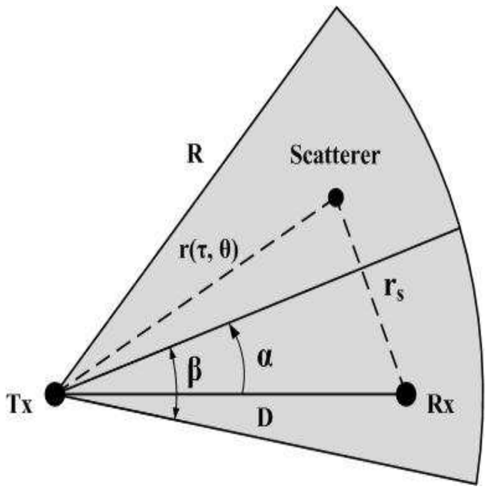

Figure 1: Geometry of pie-cut model relating the transmitter, the scatterers' effective spatial region, and the receiver.

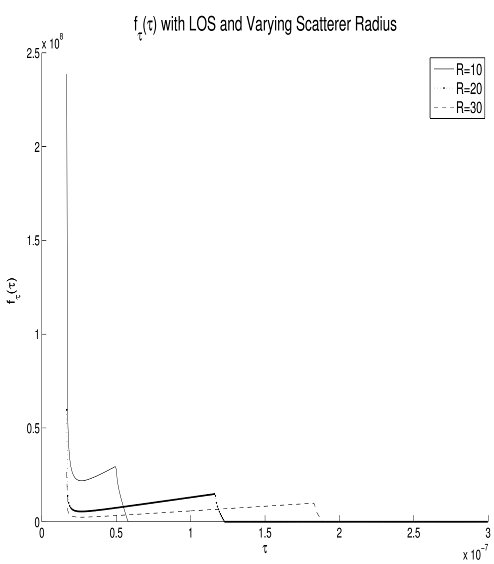

Figure 2: $f_\tau(\tau)$ at BW= $\pi/2$, $D = 5$ meters, and $R = 10, 20, 30$ meters.

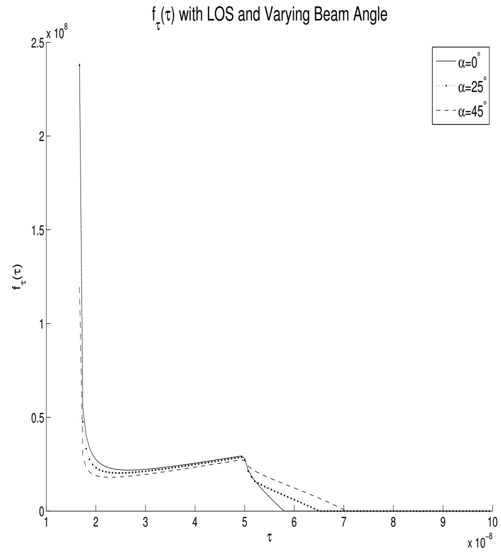

Figure 3: $f_\tau(\tau)$ at $R = 10$ meters, BW= $\pi/2$, $D = 5$ meters, and $\alpha = 0°, 25°, 45°$.

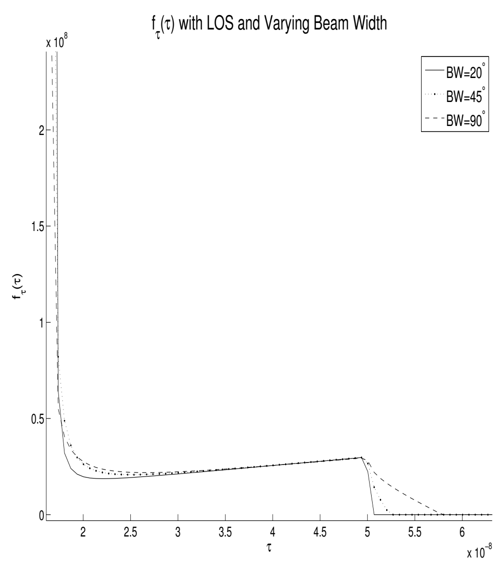

Figure 4: $f_\tau(\tau)$ at $R = 10$ meters, $\alpha = 0$, $D = 5$meters for $BW = 20°, 45°, 90°$.