Time Domain Analysis of Characteristics of a Microstrip Patch Antenna with Metamaterial Substrate

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1. Intruduction

From the end of the last century, there has been a growing interest in the investigation of artificial materials, which are called as metamaterials [1]- [3]. The material, also named as the left-handed material, is theoretically investigated by Veselago [4]. The metamaterial has a negative refractive index, and a negative refraction is then occurred at the interface between an ordinary dielectric material and the metamaterial. Furthermore, the direction of an energy flow is opposite to the wave vector of a plane electromagnetic wave in the metamaterial. While the material does not exist in nature, the metamaterial may be fabricated by periodic arrays of thin wires and split ring resonators [5].

This paper presents radiation characteristics of a microstrip patch antenna using a metamaterial substrate. A metal patch is located on the metamaterial substrate, and the microstrip line is connected to the patch to feed the antenna. Since the permittivity and the permeability of the metamaterial are dispersive, we express those parameters of the material as the Lorentz model [6]. Using the dispersion model and the constitutive relations of the metamaterial, we can derive partial differential equations for the electric and the magnetic field vectors of the metamaterial. The electromagnetic radiation field from the microstrip patch antenna may be obtained based on the ordinary FDTD method [7] and the ADE-FDTD method [6]. Numerical results are shown that the effectiveness of the microstrip patch antenna with metamaterial substrate.

2. Theory

Figure 1 shows the geometrical configuration of a microstrip patch antenna with a metamaterial substrate. A rectangular radiation patch is located upon the substrate backed by a ground plane and the feed point of the antenna is at the edge of a microstrip line, which is connected to the patch. Assuming $\exp(j\omega t)$ time dependence, the constitutive relations for the metamaterial are expressed as [6]

$$\varepsilon_0\,\varepsilon_r(x,y,z,\omega) = \varepsilon_\infty(x,y,z) + [\varepsilon_s(x,y,z) - \varepsilon_\infty(x,y,z)]\,\frac{\omega_{pe}^2(x,y,z)}{\omega_{pe}^2(x,y,z) + 2j\omega\delta_{pe}(x,y,z) - \omega^2} \tag{1}$$

and

$$\mu_0\,\mu_r(x,y,z,\omega) = \mu_\infty(x,y,z) + [\mu_s(x,y,z) - \mu_\infty(x,y,z)]\,\frac{\omega_{pm}^2(x,y,z)}{\omega_{pm}^2(x,y,z) + 2j\omega\delta_{pm}(x,y,z) - \omega^2}, \tag{2}$$

where the parameters ω_{pe} and ω_{pm} are the plasma frequencies, δ_{pe}, δ_{pm} denote damping coefficients for each material parameter, ε_0 and μ_0 the permittivity and the permeability of free space, and ε_r and μ_r denote the relative permittivity and the relative permeability. Furthermore, $\varepsilon_s = \varepsilon_0\varepsilon_r(\omega = 0)$, $\mu_s = \mu_0\mu_r(\omega = 0)$ and $\varepsilon_\infty = \varepsilon_0\varepsilon_r(\omega = \infty)$, $\mu_\infty = \mu_0\mu_r(\omega = \infty)$, respectively.

From Eqs. (1) and (2), we have

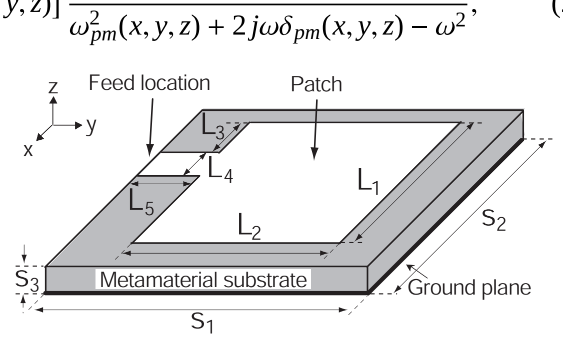

Figure 1: Geometry of the problem.

$$\begin{aligned}\mathbf{D}(x,y,z,\omega) &= \left\{\varepsilon_\infty(x,y,z) + [\varepsilon_s(x,y,z) - \varepsilon_\infty(x,y,z)]\,\frac{\omega_{pe}^2(x,y,z)}{\omega_{pe}^2(x,y,z) + 2j\omega\delta_{pe}(x,y,z) - \omega^2}\right\}\\ &\quad\cdot \mathbf{E}(x,y,z,\omega)\\ &= \frac{1}{\omega_{pe}^2(x,y,z) + 2j\omega\delta_{pe}(x,y,z) - \omega^2}\left\{\varepsilon_\infty(x,y,z)\left[2j\omega\delta_{pe}(x,y,z) - \omega^2\right]\right.\\ &\quad\left. + \varepsilon_s(x,y,z)\omega_{pe}^2(x,y,z)\right\}\mathbf{E}(x,y,z,\omega)\end{aligned}$$

$$\begin{aligned}\left[\omega_{pe}^2(x,y,z) + 2j\omega\delta_{pe}(x,y,z) - \omega^2\right]\mathbf{D}(x,y,z,\omega) &= \left\{\varepsilon_\infty(x,y,z)\left[2j\omega\delta_{pe}(x,y,z) - \omega^2\right]\right.\\ &\quad\left. + \varepsilon_s(x,y,z)\omega_{pe}^2(x,y,z)\right\}\mathbf{E}(x,y,z,\omega) \qquad (3)\end{aligned}$$

and

$$\begin{aligned}\mathbf{B}(x,y,z,\omega) &= \left\{\mu_\infty(x,y,z) + [\mu_s(x,y,z) - \mu_\infty(x,y,z)]\,\frac{\omega_{pm}^2(x,y,z)}{\omega_{pm}^2(x,y,z) + 2j\omega\delta_{pm}(x,y,z) - \omega^2}\right\}\\ &\quad\cdot \mathbf{H}(x,y,z,\omega)\\ &= \frac{1}{\omega_{pm}^2(x,y,z) + 2j\omega\delta_{pm}(x,y,z) - \omega^2}\left\{\mu_\infty(x,y,z)\left[2j\omega\delta_{pm}(x,y,z) - \omega^2\right]\right.\\ &\quad\left. + \mu_s\omega_{pm}^2(x,y,z)\right\}\mathbf{H}(x,y,z,\omega)\end{aligned}$$

$$\begin{aligned}\left[\omega_{pm}^2(x,y,z) + 2j\omega\delta_{pm}(x,y,z) - \omega^2\right]\mathbf{B}(x,y,z,\omega) &= \left\{\mu_\infty(x,y,z)\left[2j\omega\delta_{pm}(x,y,z) - \omega^2\right]\right.\\ &\quad\left. + \mu_s(x,y,z)\omega_{pm}^2(x,y,z)\right\}\mathbf{H}(x,y,z,\omega). \qquad (4)\end{aligned}$$

Using the inverse Fourier transform relation $j\omega \to \partial/\partial t$, Eqs. (3) and (4) may be express in the time domain as

$$\begin{aligned}&\omega_{pe}^2(x,y,z)\mathbf{D}(x,y,z,t) + 2\delta_{pe}(x,y,z)\frac{\partial\mathbf{D}(x,y,z,t)}{\partial t} + \frac{\partial^2\mathbf{D}(x,y,z,t)}{\partial t^2}\\ &\quad= \omega_{pe}^2(x,y,z)\varepsilon_s(x,y,z)\mathbf{E}(x,y,z,t) + 2\delta_{pe}(x,y,z)\varepsilon_\infty(x,y,z)\frac{\partial\mathbf{E}(x,y,z,t)}{\partial t} + \varepsilon_\infty\frac{\partial^2\mathbf{E}(t)}{\partial t^2} \qquad (5)\end{aligned}$$

and

$$\begin{aligned}&\omega_{pm}^2(x,y,z)\mathbf{B}(x,y,z,t) + 2\delta_{pm}(x,y,z)\frac{\partial\mathbf{B}(x,y,z,t)}{\partial t} + \frac{\partial^2\mathbf{B}(x,y,z,t)}{\partial t^2}\\ &\quad= \omega_{pm}^2(x,y,z)\mu_s(x,y,z)\mathbf{H}(x,y,z,t) + 2\delta_{pm}(x,y,z)\mu_\infty(x,y,z)\frac{\partial\mathbf{H}(x,y,z,t)}{\partial t} + \mu_\infty\frac{\partial^2\mathbf{H}(t)}{\partial t^2}. \qquad (6)\end{aligned}$$

Introducing the ADE-FDTD method [6], the positioning of the components of **E**, **H**, **D**, and **B** is done by a standard Yee lattice [7] and $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$. Considering leap-frog algorithm and the constitutive relations of the metamaterial, time locations of each component of **E**, **H**, **D**, and **B** are determined. Thus an FDTD update equation for **E** is expressed as follow:

$$\begin{aligned}\mathbf{E}^n(x,y,z) &= \left\{\left[\omega_{pe}(x,y,z)(\Delta t)^2 + 2\delta_{pe}(x,y,z)\Delta t + 2\right]\mathbf{D}^n(x,y,z) - 4\mathbf{D}^{n-1}(x,y,z)\right.\\ &\quad + \left[\omega_{pe}^2(x,y,z)(\Delta t)^2 - 2\delta_{pe}(x,y,z)\Delta t + 2\right]\mathbf{D}^{n-2}(x,y,z) + 4\varepsilon_\infty(x,y,z)\mathbf{E}^{n-1}(x,y,z)\\ &\quad\left. - \left[\omega_{pe}^2(x,y,z)\varepsilon_s(x,y,z)(\Delta t)^2 - 2\delta_{pe}(x,y,z)\varepsilon_\infty(x,y,z)\Delta t + 2\varepsilon_\infty(x,y,z)\right]\mathbf{E}^{n-2}(x,y,z)\right\}\\ &\quad / \left[\omega_{pe}^2(x,y,z)\varepsilon_s(i,j,k)(\Delta t)^2 + 2\delta_{pe}(x,y,z)\varepsilon_\infty(x,y,z)\Delta t + 2\varepsilon_\infty(x,y,z)\right]. \qquad (7)\end{aligned}$$

Similarly, we have an FDTD update equation for magnetic field is obtained as follow:

$$\begin{aligned}\mathbf{H}^{n+\frac{1}{2}}(x,y,z) = &\Big\{\Big[\omega_{pm}(x,y,z)(\Delta t)^2 + 2\delta_{pm}(x,y,z)\Delta t + 2\Big]\mathbf{B}^{n+\frac{1}{2}}(x,y,z) - 4\mathbf{B}^{n-\frac{1}{2}}(x,y,z) \\ &+ \Big[\omega_{pm}^2(x,y,z)(\Delta t)^2 - 2\delta_{pm}(x,y,z)\Delta t + 2\Big]\mathbf{B}^{n-\frac{3}{2}}(x,y,z) + 4\mu_\infty(x,y,z)\mathbf{H}^{n-\frac{1}{2}}(x,y,z) \\ &- \Big[\omega_{pm}^2(x,y,z)\mu_s(x,y,z)(\Delta t)^2 - 2\delta_{pm}(x,y,z)\mu_\infty(x,y,z)\Delta t + 2\mu_\infty(x,y,z)\Big]\mathbf{H}^{n-\frac{3}{2}}(x,y,z)\Big\} \\ &/ \Big[\omega_{pm}^2(x,y,z)\mu_s(x,y,z)(\Delta t)^2 + 2\delta_{pm}(x,y,z)\mu_\infty(x,y,z)\Delta t + 2\mu_\infty(x,y,z)\Big] \end{aligned} \tag{8}$$

On the other hand, FDTD update equations for **D** and **B** are derived from Maxwell's equations:

$$\frac{\mathbf{D}^n(x,y,z) - \mathbf{D}^{n-1}(x,y,z)}{\Delta t} = \nabla \times \mathbf{H}^{n-\frac{1}{2}}(x,y,z) \tag{9}$$

and

$$\frac{\mathbf{B}^{n+\frac{1}{2}}(x,y,z) - \mathbf{B}^{n-\frac{1}{2}}(x,y,z)}{\Delta t} = -\nabla \times \mathbf{E}^n(x,y,z). \tag{10}$$

Thus the electromagnetic field inside the metamaterial is obtained from these FDTD update equations.

3. Numerical Results

Computer simulations are performed for a microstrip patch antenna having a homogeneous metamaterial substrate. The computation domain that covers free space outside the antenna is divided into $210 \times 230 \times 54$ cells. The cell sizes of $\triangle x$, $\triangle y$, and $\triangle z$ are 0.12967mm, 0.13333mm, and 0.08833mm, and $\Delta t = 0.2029$ps. The perfectly matched layer (PML) [8] is implemented as an absorbing boundary condition to truncate the computational domain. The dimensions of the microstrip patch antenna are $L_1 = 12.45$mm, $L_2 = 16.00$mm, $L_3 = 2.09$mm, $L_4 = 2.46$mm, $L_5 = 4.00$mm, $S_1 = 24.00$mm, $S_2 = 23.34$mm, and $S_3 = 0.795$mm. The material parameters of the metamaterial substrate are as follows: $\varepsilon_s = 3.0\varepsilon_0$, $\mu_s = 3.0\mu_0$, $\varepsilon_\infty = 1.2\varepsilon_0$, $\mu_\infty = 1.2\mu_0$, $\omega_{pe} = \omega_{pm} = \omega_p = 2\pi \times 10.0 \times 10^9$ rad/sec, $\delta_{pe} = \delta_{pm} = \delta_p = 0.1\omega_{pe}$ rad/sec. Furthermore, the input impedance of the antenna is set to be 50 ohms.

The relative permittivity and the relative permeability of the metamaterial are illustrated in Fig. 2. As is seen from this figure that the refractive index of the metamaterial is negative from $f = 6.560$GHz to $f = 9.640$GHz. Figure 3 shows the return loss of the antenna. From the figure, some resonant frequencies are occurred including at the region that the refractive index of the metamaterial is negative. Radiation pattern of the antenna in x-z plane at $f = 3.130$GHz is presented in Fig. 4. Since the radiation pattern is omni-directional, the pattern is due to the elementally mode. Radiation pattern of the antenna in x-z plane at $f = 9.085$GHz is presented in Fig. 5. Note that the refractive index of the metamaterial is negative at the frequency.

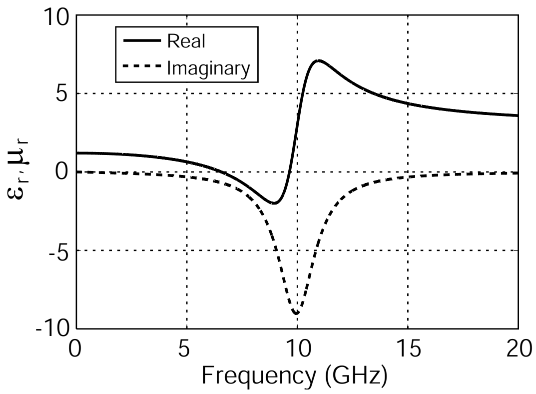

Figure 2: Relative permittivity and permeability of the metamaterial substrate.

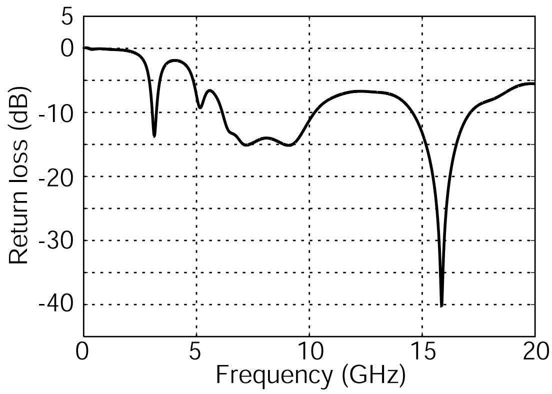

Figure 3: Return loss of the microstrip patch antenna with the metamaterial substrate.

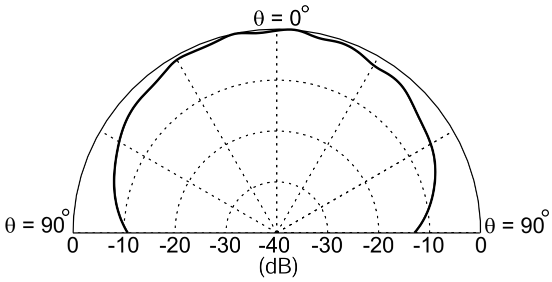

Figure 4: Radiation pattern of the microstrip patch antenna in the *x*-*z* plane at $f = 3.130$GHz.

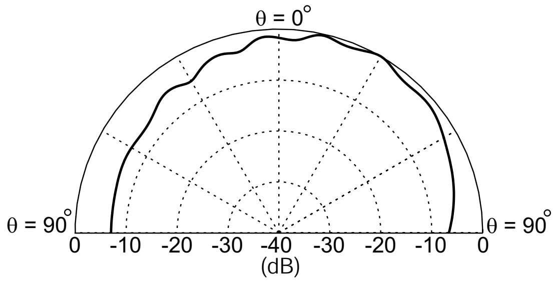

Figure 5: Radiation pattern of the microstrip patch antenna in the *x*-*z* plane at $f = 9.085$GHz.

From Figs. 2-5, the antenna works as an omni-directional antenna even though the refractive index of the substrate is negative. Furthermore, we can see that the use of the metamaterial substrate is effective in the design of a microstrip patch antenna with low return loss.

4. Conclusion

Radiation characteristics of a microstrip patch antenna with a metamaterial substrate have been investigated in time domain. Since the metamaterial is dispersive, the permittivity and the permeability of the material are modeled by the Lorentz equations. The ADE-FDTD method is employed to obtain the electromagnetic field in the metamaterial. Numerical results confirm the effectiveness of the use of the metamaterial in the substrate of a microstrip patch antenna, which shows good radiation characteristics.

Study on the effects of a patch shape and a feeding method on radiation characteristics of a microstrip patch antenna remains a topic for future work.

Acknowledgments

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