# FVTD Analysis of Propagation Characteristics in Relation to Random Rough Surface Spectrum


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## 1. Introduction

The role of information and communication technology is increasing in the modern society. Recently, the sensor network among the wireless networks have been attracting attention of research community. In order to construct the wireless network system among many sensors located randomly on rough surfaces, such as desert, hilly terrain, forest and sea surfaces, it is important to theoretically investigate the propagation characteristics of electromagnetic waves under these physical enviroments[1]. It is also important to investigate radiation problem from antenna of sensor device located on the rough surface.

We investigate propagation characteristics of electromagnetic waves along random rough surfaces. Gaussian spectrum is often used for random rough surface generation, but it is inadequate to deal with complicated environments. We consider here three types of spectrum density functions for random rough surfaces, that is, Gaussian, n-th order power law and exponential. Rough surfaces are generated by Fast Fourier Transform (FFT), and electromagnetic fields along rough surfaces are computed numerically by using Finite Volume Time Domain (FVTD) method[2]. The distance characteristics of propagation above rough surfaces are discussed.

## 2. 1D Random Rough Surface Statistic

Let $K$ be a spatial angular frequency, then the spectrum density functions are given as follows:

1. Gaussian Spectrum:

$$W_g(K) = \left(\frac{clh^2}{2\sqrt{\pi}}\right) e^{-\frac{K^2cl^2}{4}} \tag{1}$$

2. N-th Order Power Law Spectrum:

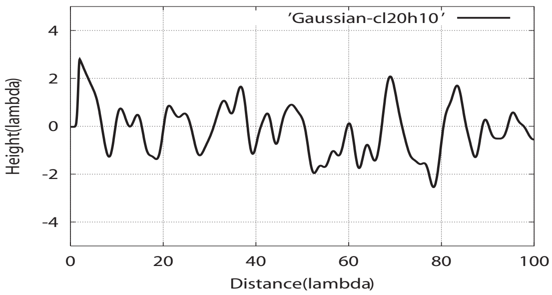
$$W_{pN}(K) = \left(\frac{clh^2}{2\sqrt{\pi}}\right) \left\{1 + \frac{\Gamma^2(N-\frac{1}{2})}{\Gamma^2(N)} \frac{K^2cl^2}{4}\right\}^{-N} \tag{2}$$

3. Exponential Spectrum:

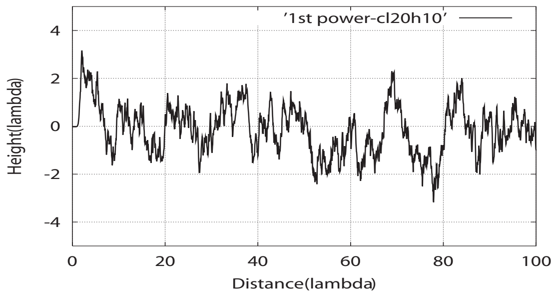
$$W_e(K) = \left(\frac{clh^2}{\pi}\right) \left\{1 + K^2cl^2\right\}^{-1} \tag{3}$$

where $cl$ is the correlation length of the rough surface and $h$ is the variation of its height. Moreover, $\Gamma(x)$ is the $\Gamma$ function for the n-th order power law spectrum.

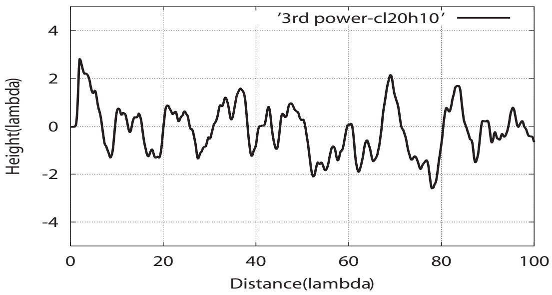
There are two types of random rough surface generation method, that is, direct DFT[3] and convolution. Direct DFT is generation method for small area, and convolution is for large area. In this paper, we generate the random rough surfaces with three types of spectra by using direct DFT method.



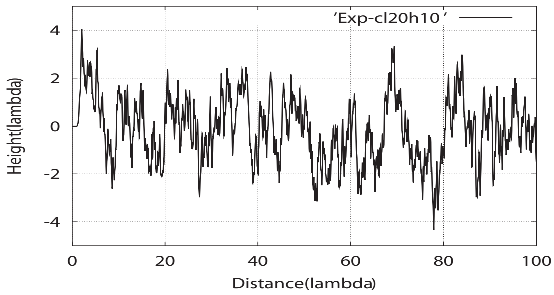

(a)Gaussian spectrum




(b)1st order power law spectrum




(c)3rd order power law spectrum




(d)Exponential spectrum

Figure 1: Examples of random rough surface (cl=2.0$\lambda$, h=$\lambda$).

We show some examples of random rough surfaces in ($a$) through ($d$) of Fig.1. ($a$) is Gaussian type random rough surface. ($b$) is 1st order power law type, ($c$) is 3rd order power law type, and ($d$) is exponential type random rough surface. We select two parameters for random rough surface, $cl = 2.0\lambda$ and $h = \lambda$. N-th order power law type rough surfaces approach the Gaussian distribution as $N \to \infty$. 1st order power law type rough surface is similar to exponential type, but exponential type have a property that peak is higher and sharper than 1st order power law type.

## 3. FVTD Method

The FVTD method has widely been applied to the numerical computation of the electromagnetic fields. However, it requires much computer memory. As a result, when the computational size is much larger than the wavelength, it is difficult to perform the FVTD computations by use of a small computer such as a PC. We use PML for absorbing boundary condition[4]. We treat 1D random rough surfaces for computing electromagnetic fields above rough surface, because we can not deal with a 2D random rough surface due to present computers' ability.

We normalize the magnetic field as follows:

$$\tilde{\boldsymbol{H}} = Z_0 \boldsymbol{H} = \sqrt{\mu_0/\epsilon_0}\boldsymbol{H}. \tag{4}$$

In this paper, we investigate 2D propagation problem along 1D random rough surfaces. Therefore, electromagnetic fields can be decomposed to two different types of independent polarization, E and H waves. The E wave has a polarization described as $\tilde{\boldsymbol{H}} = (\tilde{H}_x, \tilde{H}_y, 0)$ and $\boldsymbol{E} = (0, 0, E_z)$. 2D FVTD equations for E-wave are given as follows[2]:

$$\begin{aligned} \tilde{H}_x^{n'+1}(i,j) &= \tilde{H}_x^{n'}(i,j) - \Lambda_y[E_z^n(i,j+1) - E_z^n(i,j-1)] \\ \tilde{H}_y^{n'+1}(i,j) &= \tilde{H}_y^{n'}(i,j) + \Lambda_x[E_z^n(i+1,j) - E_z^n(i-1,j)] \\ E_z^{n+1}(i,j) &= E_z^n(i,j) \\ &\quad + \Gamma_x[\tilde{H}_y^{n'+1}(i+1,j) - \tilde{H}_y^{n'+1}(i-1,j)] - \Gamma_y[\tilde{H}_x^{n'+1}(i,j+1) - \tilde{H}_x^{n'+1}(i,j-1)] \end{aligned} \tag{5}$$

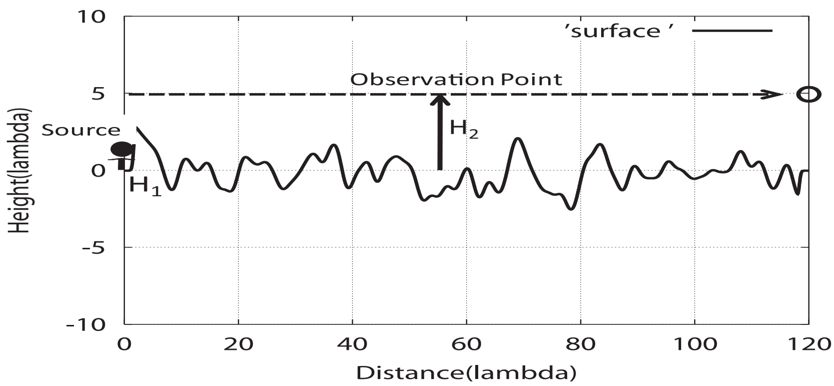



Figure 2: Geometry of the problem.

where the step parameters are defined by

$$\begin{aligned}\Lambda_{x,y,z} &= (c\Delta t)/(2\mu_r\Delta_{x,y,z})\\ \Gamma_{x,y,z} &= (c\Delta t)/(2\epsilon_r\Delta_{x,y,z})\ .\end{aligned} \tag{6}$$

On the other hand, in case of H-wave with polarization described as $\tilde{\boldsymbol{H}} = (0, 0, \tilde{H}_z)$ and $\boldsymbol{E} = (E_x, E_y, 0)$, 2D FVTD equations are given as follows[2]:

$$\begin{aligned}\tilde{H}_z^{n'+1}(i,j) &= \tilde{H}_z^{n'}(i,j)\\ &\quad + \Lambda_y[E_x^n(i,j+1) - E_x^n(i,j+1)] - \Lambda_x[E_y^n(i+1,j) - E_y^n(i-1,j)]\\ E_x^{n+1}(i,j) &= E_x^n(i,j) + \Gamma_y[\tilde{H}_z^{n'+1}(i,j+1) - \tilde{H}_z^{n'+1}(i,j-1)]\\ E_y^{n+1}(i,j) &= E_y^n(i,j) - \Gamma_x[\tilde{H}_z^{n'+1}(i+1,j) - \tilde{H}_z^{n'+1}(i-1,j)].\end{aligned} \tag{7}$$

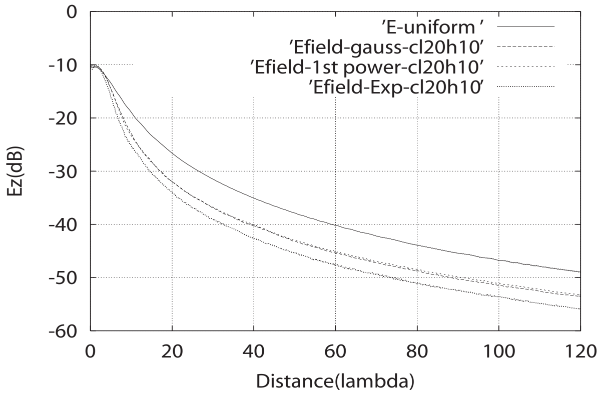
## 4. Numerical Example

In Fig.2, we show the geometry of the problem for computing propagation characteristics. We select the operating frequency and parameters for random rough surfaces as follows:

- Frequency $f = 1.0GHz$ $(\lambda = 0.3[m])$
- Permittivity $\epsilon_r = 5.0$
- Permeability $\mu_r = 1.0$
- Conductivity $\sigma = 0.0023[S/m]$ .

Height of source point is very close to rough surfaces, $H_1 = 0.1\lambda$, and height of observation point is above the rough surfaces, $H_2 = 5.0\lambda$.

We show the field intensity distribution along rough surfaces that are varied depending on the spectra, as shown in Fig.3 and 4. We select the number of sampled rough surfaces as 100. Since the source point is very close to rough surface, field intensities of the observation point above rough surfaces are much attenuated because of the shadowing area. In addition, E waves are more influenced by rough surfaces than H waves. In those figure, it is shown that the attenuation characteristics along rough surfaces are large compared with uniform ground, and it is also shown that they exhibit constant attenuation characteristics. Attenuation characteristic of 2D free space is $r^{-1/2}$. We assume here attenuation characteristics along rough surfaces are $r^{-n/2}$. Using the least squares method, we can obtain order $n$ easily. In Table 1, we show the order $n$ depending on the rough surface spectra, $cl = 2.0\lambda$ and $h = \lambda$. As the result, it is shown that the propagation losses varies with random rough surface spectrum, and it is described by order $n$.



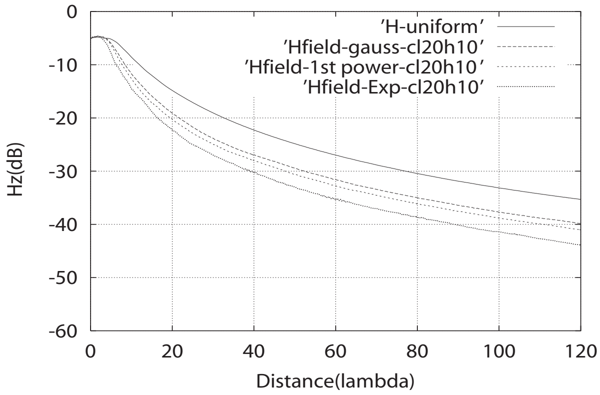




Figure 3: Electric field intensity (cl=2.0$\lambda$, h=$\lambda$). Figure 4: Magnetic field intensity (cl=2.0$\lambda$, h=$\lambda$).

Table 1: Propagation loss in terms of order *n*.

| Spectrum | Correlation length cl ($\lambda$) | Variation of height h ($\lambda$) | E wave | H wave |
|---|---|---|---|---|
| Uniform | 2.0 | 0.0 | *n*=2.78 | *n*=1.87 |
| Gauss | | 1.0 | *n*=3.13 | *n*=2.19 |
| 1st power law | | | *n*=3.11 | *n*=2.27 |
| 2nd power law | | | *n*=3.14 | *n*=2.23 |
| 3rd power law | | | *n*=3.14 | *n*=2.22 |
| Exponential | | | *n*=3.28 | *n*=2.43 |

## 5. Conclusions

In this paper, we investigated propagation characteristics along random rough surfaces by using the FVTD. Based on the Direct DFT method, we generated three types of random rough surfaces, that is, Gaussian, n-th order power law and exponential spectra. We numerically compute electromagnetic field intensities along rough surfaces by using the FVTD. We have found E waves are more influenced by rough surfaces than H waves. We showed the propagation losses along rough surfaces in terms of order *n*.

We need to calculate propagation losses by using random rough surface parameters, correlation length (*cl*) and variation of rough surface's height (*h*). Moreover, we need to treat 2D rough surfaces. We would like to investigate these problems in a near future.

## References

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