On Approximation for Spectral Expression of Scattered Field from A Thick Half-Plate

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1. Introduction

The two-dimensional (2D) plane-wave scattering by a thick conducting half-plane is one of the fundamental electromagnetic field problems. It includes reflection at the faces of the half-plate as well as multiple diffractions at the two wedges. Although this problem can be solved rigorously in spectral domain by applying Wiener-Hopf (WH) technique to the 2D wave equation [1]- [4], its Fourier inverse transformation cannot be performed analytically because the rigorous WH solutions in the spectral domain include coefficients to be solved by infinite simultaneous equations and factorization of kernel functions [5] [6]. From these reasons, the rigorous WH solutions are too complicated for us to apply them to practical electromagnetic field problem.

When we analyze the complicated problems such as propagation in urban area, the analytical scattered fields should be expressed in a compact form and the results should exhibit versatility, high accuracy and stability. In this context, we attempt to introduce an approximate spectral expression based on the WH solutions for the present problem.

In this paper, we deal with the case of E-wave incidence focusing on the far fields. Before we introduce the approximate spectral expressions, we first rewrite the WH solutions in spectral domain so as to calculate analytically its Fourier inverse transformation, that is, the spectral expression by the two half-planes is expressed by using the Fresnel integral and the spectral expression by the thickness of the half-plate is expressed by using the saddle point method [7]. Then, we investigate the behavior of the WH solutions numerically in spectral domain focusing on the effect of the incident angle and the thickness of the half plane.

As the numerical results, we introduce the approximation for the spectral expression of the scattered field by the thickness of the half-plate and it is expressed in some compact forms by using the principal values at the pole and sampling function. In order to check the accuracy of this approximation, we compare the approximated solutions at the grazing incident angle with the WH ones in spectral domain. It is demonstrated that both solutions are in good agreement.

2. Wiener-Hopf Solutions

Fig.1 shows the geometry of the 2D plane wave scattering by a thick conducting half-plate problem. The thickness of the half-plate is 2b. A plane wave (E-wave) is incident upon the half-plate with angle θ $(0 < \theta < \pi)$ with respect to the x-axis.

Now, we define the total (t), incident (i) and scattered (s) fields are as follows:

$$(\boldsymbol{E}^t, \boldsymbol{H}^t) = (\boldsymbol{E}^i, \boldsymbol{H}^i) + (\boldsymbol{E}^s, \boldsymbol{H}^s) \tag{1}$$

where the leading function of the incident wave (E-wave) is expressed as follows:

$$E_z^i = e^{-j\kappa x\cos\theta - j\kappa y\sin\theta}. \tag{2}$$

The time dependence $e^{j\omega t}$ is assumed and the wave number in the free space is expressed as $\kappa = \omega\sqrt{\epsilon_0\mu_0}$. In order to obtain the spectral function of the scattered field based on the WH technique, we use the

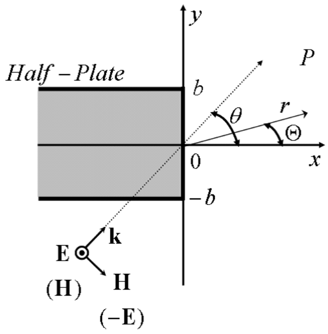


Figure 1: Geometry of the problem.

following Fourier transformation pair defined by

$$F(\zeta) = \int_{-\infty}^{\infty} f(x)e^{j\zeta x}dx, \qquad f(x) = \frac{1}{2\pi}\int_{c} F(\zeta)e^{-j\zeta x}d\zeta. \tag{3}$$

The rigorous solutions of the scattered field in spectral domain can be obtained by applying the WH technique after calculating the complex Fourier transformation of the wave equation. In this study, we decompose the spectral expressions into two parts, regular in the upper or lower half plane in the spectral domain. Neglecting detailed discussions, the final results are summarized as follows [5]:

$$\begin{aligned} E_z^s(\zeta, y) =& \frac{je^{-jk(y-b)}}{2G_c^+(\zeta)} \cdot \frac{U_c^+(\zeta_\theta)G_c^+(\zeta_\theta)}{(\zeta-\zeta_\theta)} + \frac{je^{-jk(y-b)}}{2G_c^+(\zeta)} \sum_{n=1}^{\infty} \frac{U_c^+(-k_{cn})b_{cn}^2 G_c^-(k_{cn})}{(k_{cn}+\zeta_\theta)(\zeta-k_{cn})k_{cn}} \\ &+ \frac{je^{-jk(y-b)}}{2G_s^+(\zeta)} \cdot \frac{U_s^+(\zeta_\theta)G_s^+(\zeta_\theta)}{(\zeta-\zeta_\theta)} + \frac{je^{-jk(y-b)}}{2G_s^+(\zeta)} \sum_{n=1}^{\infty} \frac{U_s^+(-k_{sn})b_{sn}^2 G_s^-(k_{sn})}{(k_{sn}+\zeta_\theta)(\zeta-k_{sn})k_{sn}} \\ =& \Lambda_p(\zeta)\frac{j\sqrt{\kappa-\zeta_\theta}}{(\zeta-\zeta_\theta)\sqrt{\kappa-\zeta}}e^{-jk(y-b)} + \Omega_p(\zeta)\frac{j\sqrt{\kappa}}{\sqrt{\kappa-\zeta}}e^{-jk(y-b)} \qquad for \quad y > b \end{aligned} \tag{4}$$

$$\begin{aligned} E_z^s(\zeta, y) =& \frac{je^{jk(y+b)}}{2G_c^+(\zeta)} \cdot \frac{U_c^+(\zeta_\theta)G_c^+(\zeta_\theta)}{(\zeta-\zeta_\theta)} + \frac{je^{jk(y+b)}}{2G_c^+(\zeta)} \sum_{n=1}^{\infty} \frac{U_c^+(-k_{cn})b_{cn}^2 G_c^-(k_{cn})}{(k_{cn}+\zeta_\theta)(\zeta-k_{cn})k_{cn}} \\ &- \frac{je^{jk(y+b)}}{2G_s^+(\zeta)} \cdot \frac{U_s^+(\zeta_\theta)G_s^+(\zeta_\theta)}{(\zeta-\zeta_\theta)} - \frac{je^{jk(y+b)}}{2G_s^+(\zeta)} \sum_{n=1}^{\infty} \frac{U_s^+(-k_{sn})b_{sn}^2 G_s^-(k_{sn})}{(k_{sn}+\zeta_\theta)(\zeta-k_{sn})k_{sn}} \\ =& \Lambda_m(\zeta)\frac{j\sqrt{\kappa-\zeta_\theta}}{(\zeta-\zeta_\theta)\sqrt{\kappa-\zeta}}e^{jk(y+b)} + \Omega_m(\zeta)\frac{j\sqrt{\kappa}}{\sqrt{\kappa-\zeta}}e^{jk(y+b)} \qquad for \quad y < -b. \end{aligned} \tag{5}$$

In the above expressions, we have omitted the scattered field in the region $-b < y < b$, since the present discussions are restricted only to the far field. It should be noted that the infinite number of unknown coefficients must be determined by the related infinite set of algebraic equations [1].

In order to obtain the scattered field in the space domain in a compact form, we rearrange the spectral expressions in Eqs. (4) and (5) into two parts depending on whether it includes infinite number of unknown coefficients or not. They are defined by use of the functions $\Lambda_{p,m}(\zeta)$ and $\Omega_{p,m}(\zeta)$ as follows:

$$\Lambda_{p,m}(\zeta) = \frac{\sqrt{\kappa-\zeta}G_c^+(\zeta_\theta)U_c^+(\zeta_\theta)}{2\sqrt{\kappa-\zeta_\theta}G_c^+(\zeta)} \pm \frac{\sqrt{\kappa-\zeta}G_s^+(\zeta_\theta)U_s^+(\zeta_\theta)}{2\sqrt{\kappa-\zeta_\theta}G_s^+(\zeta)} \tag{6}$$

$$\Omega_{p,m}(\zeta) = \frac{\sqrt{\kappa-\zeta}}{2\sqrt{\kappa}G_c^+(\zeta)} \sum_{n=1}^{\infty} \frac{U_c^+(-k_{cn})b_{cn}^2 G_c^-(k_{cn})}{(k_{cn}+\zeta_\theta)(\zeta-k_{cn})k_{cn}} \pm \frac{\sqrt{\kappa-\zeta}}{2\sqrt{\kappa}G_s^+(\zeta)} \sum_{n=1}^{\infty} \frac{U_s^+(-k_{sn})b_{sn}^2 G_s^-(k_{sn})}{(k_{sn}+\zeta_\theta)(\zeta-k_{sn})k_{sn}} \tag{7}$$

where $\Lambda_{p,m}(\zeta)$ and $\Omega_{p,m}(\zeta)$ are associated with the spectral functions of the scattered fields at the two half-planes($y = \pm b$, $x < 0$) and at the thickness part of the half-plate ($|y| < b$, $x = 0$), respectively.

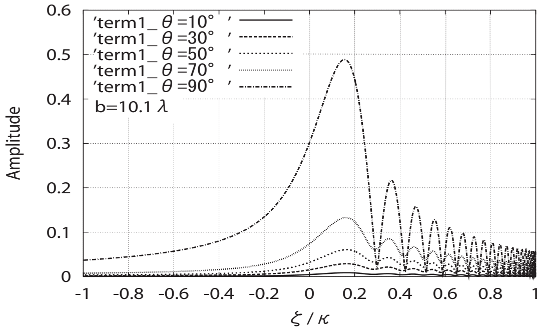


Figure 2: Dependence of first term of $\Omega_{p,m}(\zeta)$ on incident angles (θ <= 90°, b=10.1λ).

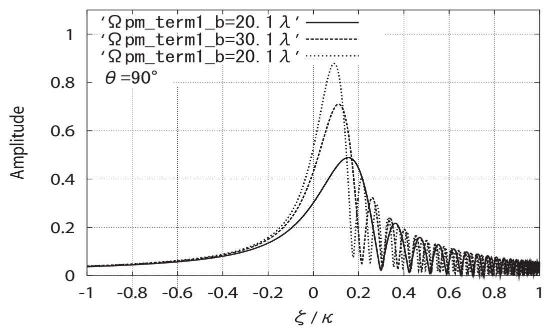


Figure 3: Dependence of first term of $\Omega_{p,m}(\zeta)$ on thickness of the half-plate (θ = 90°, b=10.1λ, 20.1λ, 30.1λ).

3. Approximate spectral expressions

In this section, we attempt to introduce an approximate spectral expression for $\Omega_{p,m}(\zeta)$ function in Eq.(7) by investigating the behavior of the WH solutions numerically. We choose two numerical parameters, the incident angle (θ) and the thickness of the half-plate (b). Here, $\Omega_{p,m}(\zeta)$ function includes two terms, that is, p and m are expressed as addition or subtraction of each term. Therefore, we derive the approximation for each term.

We show the behavior of the first term of $\Omega_{p,m}(\zeta)$ function depending on the incident angles and thickness of the half-plate in the spectral domain at $\zeta[-\kappa : \kappa]$. Fig.2 shows that the behavior of $\Omega_{p,m}(\zeta)$ function depending on the incident angles $\theta \leqq 90°$ for b=10.1λ. From these results in case of $\theta < 90°$, it is found that the overall behavior of $\Omega_{p,m}(\zeta)$ function is same and its amplitude changes depending on incident angle. According to the dependence of the $\Omega_{p,m}(\zeta)$ function on the thickness of the half-plate, when the thickness of the half-plate is increased, the width of the main lobes is narrowed as shown in Figs.3.

On the basis of some numerical results, we derive the approximate expressions for Eq.(7) as follows:

$$\Omega_{p,m}(\zeta) \approx \frac{\cos[K_1 - k_\Theta b]}{2(\zeta - \zeta_\theta)} \pm j\frac{\sin[K_2 + k_\Theta b]}{\sqrt{2}(\zeta - \zeta_\theta)} \qquad for \quad 0.16\kappa < \zeta < \kappa \qquad (K_1, K_2 : Constant). \tag{8}$$

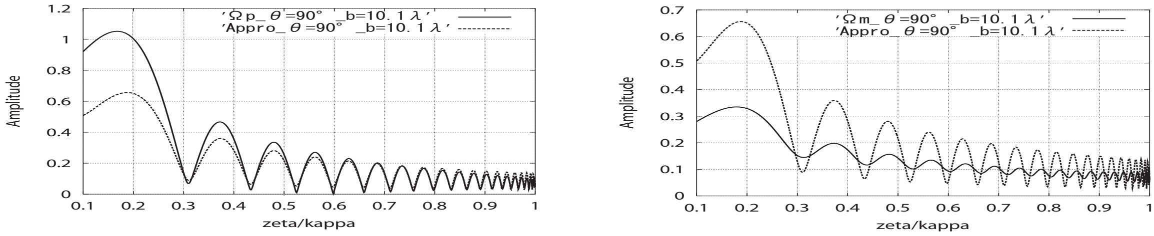
4. Numerical examples

In order to check the accuracy of the present approximate spectral expressions, we compare the approximations at the grazing incident angle with the WH solutions in spectral domain. Figs.4 and 5 show comparison of the approximations with the WH solutions ((a) Ω_p, (b) Ω_m), where we have selected numerical parameters as $\theta = 90°$, $b = 10.1\lambda, 20.1\lambda$ and $K_1 = 0.65$, $K_2 = 0.5$. In these figures, we have limited ζ at $[0.1\kappa\ , \kappa]$.

It is found from these results that the approximations are in good agreement with the WH solutions with respect to phase relation, even if the thickness of the half-plate is changed from $b = 10.1\lambda$ to $b = 20.1\lambda$. It is also shown that the approximation with respect to amplitude relation needs a weight function.

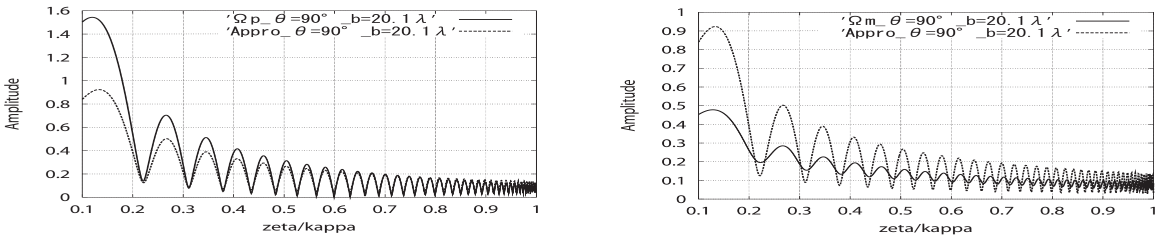
5. Conclusions

In this paper, we have attempted to introduce an approximation for spectral expression of scattered field from a half-plate by investigating the behavior of the rigorous WH solution depending on incident angle and thickness of the half-plate. We have derived the approximate spectral expression in Eq.(8) by using the principal values at the pole and sampling function. According to comparison of the approxima-



(a) Ω_p (b) Ω_m

Figure 4: Comparison of the approximation with the WH solution for $\theta = 90°$, $b = 10.1\lambda$.



(a) Ω_p (b) Ω_m

Figure 5: Comparison of the approximation with the WH solution for $\theta = 90°$, $b = 20.1\lambda$.

tions with the WH solutions, these two solutions are in good agreement with each other except for their amplitudes. Thus, it is found that we need to add a weighting function to the approximate solutions.

We treated here only the grazing incident angle ($\theta = 90°$). We need to consider the oblique incidence case in order to apply the present approximate solutions to a complicated problem such as the propagation in urban area. It deserves as a future problem.

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