Velocity Measurement of Fast Moving Object in Space using FFT

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ABSTRACT
The paper describes FFT technique as a tool to estimate the velocity and other associated parameter along with its trajectory from the assigned ground station on a hypothetical vertical plane where the object moves. At each predefined points of trajectory distances with elevation and azimuth angles are measured from the ground station and then velocity is determined by matching the samples of frequency domain.

KEYWORDS
Velocity Estimation, FFT technique

1. Introduction:

The technique is fastest and reliable for the estimation of the velocity. The method evolves for having conducted experiments during peace time and for the known velocity and their DFT sequences are computed and stored. If the number of samples are assumed to be 6, the FFT technique reduces down the computation time using two twiddle matrix [1] of size (3×3) and (2×2).

The six samples in time domain are used to convert six samples in frequency domain .These DFT values are computed employing FFT. The sample height in the time domain is considered when sampling duration T =1 sec. This will obviously make the computation of twiddle matrix very fast[1]. The sample height in time domain is assumed the height of the object from the ground horizontal plane shown in the Figure 1.

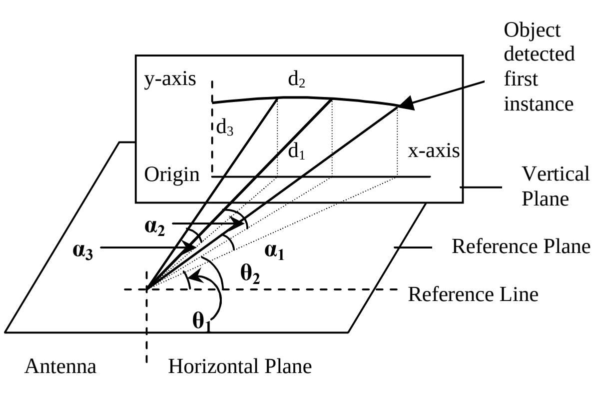

Figure 1: Object Position at Different Instances

These heights are measured from the ground station estimating the distance of the moving object and its angle of elevation. The distance of the object is measured [2] by measuring the time between the transmitted impulse and its received impulse echo. For measuring the time a special hardware is employed and processor computes the distance very accurately. Such equipments are available in American, French and Germany markets[3] .The angle of elevation measures from the vertical position of antenna which is controlled by stepper motor. This is measured accurately by vertical shift carried out with pulses of microwave frequency.

A FFT program is developed to compute DFT of the time domain samples. The following generalized picture depicts the computational algorithm of the FFT program[1]. For any number of samples in time domain ,the input sequence is arranged in array and is given array in next section

2. Pictorial Generalized representation of FFT:

A generalized layout is given in Figure 2 , for the conversion of time sequence to frequency domain sequence using FFT technique for N=P×Q.

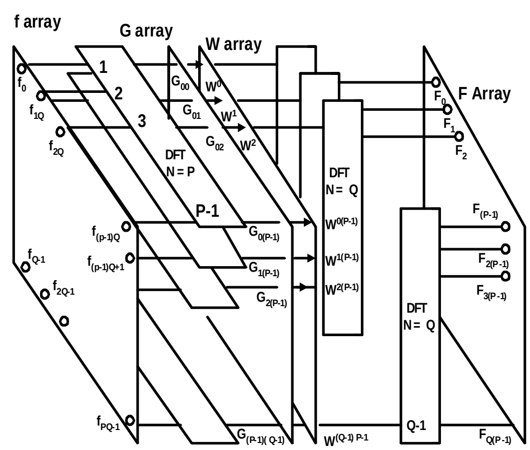

Figure 2 : Generalized Representation of FFT for N=P×Q

The picture enables to write the values of output sequence [F_1, F_2, F_3….F_{PQ}] directly from the figure 2. There is a considerable enhance in speed in processing the DFT applying FFT technique. The figure 2 estimates the number of multiplication and additions carried out in computation of the output. The number of multiplication and addition in FFT program are given by following

equation which display the reduced number of multiplication and addition using FFT.
With FFT
M=N(P+Q-3)+1=13 (1)
A=N(P+Q-2)=18

Without FFT
M= N × N=36
A=N(N-1)=30
The number of samples if chosen are N=P×Q then the input array will be shown as given below.

Input Array – f

$$
\begin{array}{llllll}
\text{0th row} & f_0 & f_Q & f_{2Q} & \dots & f_{(p-1)Q} \\
\text{1st row} & f_1 & f_{Q+1} & f_{2Q+1} & \dots & f_{(p-1)(Q+1)} \\
\text{2nd row} & f_2 & f_{Q+2} & f_{2Q+2} & \dots & f_{(p-1)(Q+2)} \\
\dots\dots & . & & & & \\
\dots\dots & . & & & & \\
(Q-1)\text{ row} & f_{(Q-1)} & f_{(2Q-1)} & f_{(3Q-1)} & \dots & f_{(pQ-1)}
\end{array} \quad (2)
$$

And the output array of the samples in frequency domain are represented in array as given below.

Output Array –F

$$
\begin{array}{llllll}
\text{0th row} & F_0 & F_Q & F_{2Q} & \dots & F_{(Q-1)p} \\
\text{1st row} & F_1 & F_{Q+1} & F_{2Q+1} & \dots & F_{(Q-1)(p+1)} \\
\text{2nd row} & F_2 & F_{Q+2} & F_{2Q+2} & \dots & F_{(Q-1)(p+2)} \\
\dots\dots & . & & & & \\
\dots\dots & . & & & & \\
(Q-1)\text{ row} & F_{(Q-1)} & F_{(2Q-1)} & F_{(3Q-1)} & \dots & F_{(pQ-1)}
\end{array} \quad (3)
$$

In the figure there is another array known as multiplying array which is represented in twiddle form. The matrix so formed is shown as below.

Multiplying matrix –W

$$
\begin{array}{llllll}
\text{0th row} & W^0 & W^0 & W^0 \dots & W0 \\
\text{1st row} & W^0 & W^1 & W^2 & W^{p-1} \\
\text{2nd row} & W^0 & \dots\dots & . & \\
\dots\dots & . & & & \\
\dots\dots & . & & & \\
(Q-1)\text{ row} & W^0 & \dots\dots & & W^{(P-1)(Q-1)}
\end{array} \quad (4)
$$

The inputs are given to matrices for N= Q., twiddle matrices of size [P×P] . The output of these matrices for N=P are shown in the following figure. The values Gij will be output for i =0….p and j=0…..q.

ARRAY – G

$$
\begin{array}{llllll}
0^{th}\text{ row} & G_{0,0} & G_{0,1} & G_{0,2}\dots G_{0,j} & - & G_{0,P-1} \\
1^{st}\text{ row} & G_{1,0} & G_{1,1} & G_{1,2}\dots G_{1,j} & - & G_{1,P-1} \\
\dots & & & & & \\
\dots & & & & & \\
i^{th}\text{ row} & G_{i,0} & G_{i,1} & G_{i,2}\dots G_{i,j} & - & G_{i,P-1} \\
(Q-1)^{th} - & G_{(Q-1),0} & G_{(Q-1),1} & G_{(Q-1),2}\dots G_{(Q-1),j} & \dots & G_{(Q-1)(P-1)}
\end{array}
$$

The matrix for samples N and DFT is equal to P are arranged one above other and they are Q in numbers and having size P×P. The other set of matrix DFT=Q or P in numbers and are vertically arranged near the array of output.

3. Example for DFT Computation using FFT for N=6:

To carry out computation work for the program to express the methodology, N has been put 6 which makes the pictorial representation of figure 2 to the figure 3.

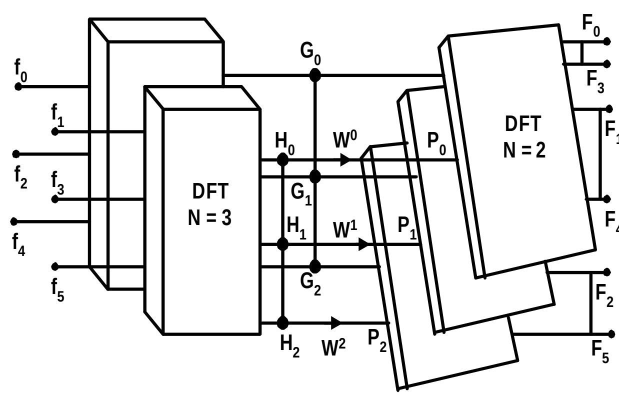

Figure 3: FFT representation for N=6 =3×2

It is clear from the figure3 , that decimation has been done at the input stage that is in time domain. If the entire figure3 is divided into two sets of matrix then the mathematical equation satisfy the picture will be as given below.
Considering RHS i.e DFT N=2

$$
\begin{aligned}
F_0 &= w^0 G_0 + w^0 p_0 \\
F_1 &= w^0 G_1 + w^0 p_1 \\
F_2 &= w^0 G_2 + w^0 p_2 \\
F_3 &= w^0 G_0 + w^0 p_0 \\
F_4 &= w^0 G_1 + w^0 p_1 \\
F_5 &= w^0 G_2 + w^0 p_2
\end{aligned} \quad (6)
$$

Considering LHS for DFT N=3

$$
\begin{aligned}
G_0 &= w^0 f_0 + w^0 f_2 + w^0 f_4 \\
G_1 &= w^0 f_0 + w^2 f_2 + w^4 f_4 \\
G_2 &= w^0 f_0 + w^4 f_2 + w^2 f_2 \\
p_0 &= w^0 f_1 + w^0 f_3 + w^0 f_5 \\
p_1 &= w^1 f_1 + w^3 f_3 + w^5 f_5 \\
p_2 &= w^2 f_1 + w^0 f_3 + w^4 f_5
\end{aligned}
\tag{7}
$$

when these equations are solved by program, when $f_{0,}$ f_1.....f_5 are given , we get the output sequence of DFT.

4. Computed results :

The twiddle matrices are computed and are given below

$$
\begin{bmatrix} DFT \\ N=6 \\ Q=2 \end{bmatrix} = \begin{bmatrix} w^0 & w^0 \\ w^0 & w^3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & e^{-j\pi} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & \cos\pi - j\sin\pi \end{bmatrix}
$$

$$
\begin{bmatrix} DFT \\ N=3 \\ \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5-j.866 & -0.5+j.866 \\ 1 & -0.5+j.866 & -0.5-j.866 \end{bmatrix} \tag{8}
$$

The values of f_0, f_1.....f_5 are computed from the figure 1 are given as:

$$
\begin{aligned}
f_0 &= d_0 \sin\alpha_0 = 117.3 \\
f_1 &= d_1 \sin\alpha_2 = 110.8 \\
f_2 &= d_2 \sin\alpha_2 = 106.68 \\
f_3 &= d_3 \sin\alpha_3 = 102.62 \\
f_4 &= d_4 \sin\alpha_4 = 98.6 \\
f_5 &= d_5 \sin\alpha_5 = 94.7
\end{aligned}
\tag{9}
$$

When equation 6,7,8,9,10 are jointly computed we get the values of DFT for N=6 as

F_0 , $F_{1,....}$ F_5 is given as below

$$
\begin{bmatrix} F_0 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} = \begin{bmatrix} 630.25 \\ 14.8-20.94j \\ 2.66-6.94j \\ 14.91 \\ 14.51+6.94j \\ 26.66+20.94j \end{bmatrix} \tag{10}
$$

The 6 samples in frequency domain of different velocities collected during peace period.

During the peace period planes carry out practice flight in the radar zone too. The pilots are advised to make the flight at a given speed. Let these given speeds are 2950km/h, 3000km/h, 3050km/h, 3100km/h, 3150km/h, 3200km/h. The experiment conducted during this time gives the values of their FFT output as given below in tabular form.

Table 1: FFT output values in the peace time

Velocity	F_0	F_1	F_2	F_3	F_4	F_5
2950km/h	630.00	14.6-20.14j	2.50-6.90j	14.50	14.00+6.50j	26.50+20.50j
3000km/h	630.25	14.8-20.94j	2.66-6.90j	14.91	14.51+6.94j	26.50+20.94j
3050km/h	630.50	14.8-20j	2.70-6.99j	14.50	14.00+6.00j	26.50+20j
3100km/h	630.25	14.8-20.94j	2.66-6.90j	14.91	14.51+6.0j	26.66+20.94j
3150km/h	630.25	14.9-20.94j	2.70-6.90j	14.94	14.61+6.94j	26.70+30.00j
3200km/h	630.75	14.10-20.9j	2.73-.70j	15.00	14.70+7.0j	20.75+25.94j

4.1 Matched digital Filtering – Applied to Velocity Measurement:

The method developed to measure the velocity of the target, is to determine the samples in frequency domain from the samples of the time domain in equal numbers. Farming twiddle matrix of the size equal to the number of samples then making the computation using FFT technique to calculate the samples in frequency domain carries this out.

This set of measured samples in frequency domain of the object is compared with the sets of samples of frequency domain of priori-stored value of the object. The comparison is done and if the threshold value of hamming distance is equal to predefined value, the velocity is declared. The technique is termed as digital filter matching to determine the velocity.

4.2 Digital Filter Matching technique

Matching technique can be done by any method. The proven methods so far are:

(1) Student hypothesis method
(2) Chi square method
(3) Hamming distance method

The last two methods are of widely used. In hamming distance method, distance is measured by using following operational equations

$$
H_{d_i} = \sum f_i \otimes y_i \tag{11}
$$

In this case if y_i is equal to f_i for i=0,1…..5 , then H_{d_i} = 1.

In the present case the operation is carried out 6 times if the H_{d_i} value is close to 6, which is threshold value, It is declared that it is fully matched.

In the above case the following table depicts the hamington distance.

Table 2 – Hamming Distance

Velocity	$H_D F_0$	$H_D F_1$	$H_D F_2$	$H_D F_3$	$H_D F_4$	$H_D F_5$	Total
2950km/h	0	0	0	0	0	0	0
3000km/h	1	1	1	1	1	1	6
3050km/h	0	0	0	0	0	0	0
3100km/h	1	1	1	1	1	0	5
3150km/h	0	0	0	0	0	0	0
3200km/h	0	1	1	1	0	0	3

From the above table it is clear that the velocity of the object is 3000km/h.

5. Conclusion:

FFT technique has been employed to compute samples in time domain. Six samples are measured in time domain and their counterpart samples in frequency domain are computed forming twiddle matrix of size [3×3] and [2×2]. The samples height so obtained is compared from the priori computed samples height of successive velocities. The comparison has been done using hamming distance method. An example has been worked out to demonstrate the method.

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