Trajectory Estimation of Moving Object in Space

Neetu Agrawal
J.K Institute of Applied Physics (Electronics Department)
Allahabad University Allahabad, India 211002
neetu24@gmail.com

ABSTRACT

The paper describes classical technique to estimate the trajectory of moving object in space. A hypothetical vertical plane on which the object moves, trajectory equation has been developed in Cartesian coordinates. The Cartesian values of variables are transformed to distance, elevation and azimuth angle so that reference of the trajectory can be used for other studies.

KEYWORDS
Trajectory Estimation, Regression technique

1. Introduction:

It is an important and interesting exercise to evaluate the velocity [1] and path of the moving object with respect to time. Such information is of great importance to track the object and to carry out the follow up action. Equipment which can propagate fan beam initially is used in scanning process [2] to detect the object. Once the moving object is detected the fan beam is converted to pencil beam and thus scanning process is changed to tracking process of the object[5].

The movement of the object while being at three places 1st, 2nd , 3rd are shown in figure 1 . Let they are attained at time $\Delta t_1, \Delta t_2, \Delta t_3$ .The line diagram shows the geometry formed by the object with the ground position at the above mentioned time. It is clear from the figure 1, that there are two distinct planes, one hypothetical vertical plane on which the trajectory is formed , and other plane where the equipments are installed. The reference line is drawn at the second plane. The object moves in the vertical plane and the equation for the trajectory is to describe the moving path of the object which remains in this plane. The equation variables are x and y. The path is smooth and nonlinear. Obviously such path can be described by second order nonlinear equation[4]. The Figure.1. shows the situation

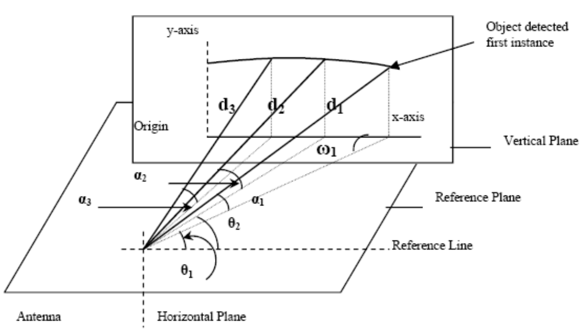

Figure. 1. Object Position at different instances

2. Development of Trajectory Equation of Moving Object :

Trajectory equation can be expressed in xy coordinates provided location of origin and points of xy plane can be expressed in terms of distance of the object, azimuth and elevation angle formed at radar station. .Let the trajectory equation follows second order nonlinear polynomial as given below

$$y = a_2 x^2 + a_1 x + a_0 \qquad (1)$$

If the coefficients of above equation are determined from the initial instants of the fast moving object then the equation assumes very high accuracy for its trajectory. The trajectory points determine the futuristic positions of the object in terms of time and space and hence are important for many studies.

The experiment conducted for the distance and elevation angle are applied for the zero error states in theoretical and measurement of value. There are three coefficient a_1, a_2, a_0 ,these are to be determined for the zero error curve fittings.

The Figure,1 represents the two planes. Horizontal plane and vertical plane. When the object was detected its distance, azimuth angle and elevation are noted d_1, θ_1, α_1 respectively. Five times an experiment is conducted to determine distance, azimuth and elevation angles. These values enable the processors to compute the values of coefficient for deciding the trajectory equation which is assumed to be quadratic as given below

$$y = a_2 x^2 + a_1 x + a_0$$

When the values of (x_i= 0, 1,2,3,4,5) are substituted in the equation (1) for evaluating $\hat{y}_i$ then we will have the estimated path as plotted in the figure 2. The other plot is actual path of the equation(1). The estimated path $\hat{y}_i$ can be expressed as

$$\hat{y}_i = a_2 x_i^2 + a_1 x_i + a_0 \qquad (2)$$

Where i = 1,2,3,4,5

The error in theoretical and measured values for y is expressed as given below

$$y_i - \hat{y}_i = y_i - [a_2 x_i^2 + a_1 x_i + a_0] \qquad (3)$$

The values of x_i is common for both the curves, hence error will be only in y. If total error is calculated, it will be given as :

$$\sum_{i=1}^{5}(y_i - \hat{y}_i) = \sum_{i=1}^{5}[y_i - a_2 x_i^2 + a_1 x_i + a_0] \tag{4}$$

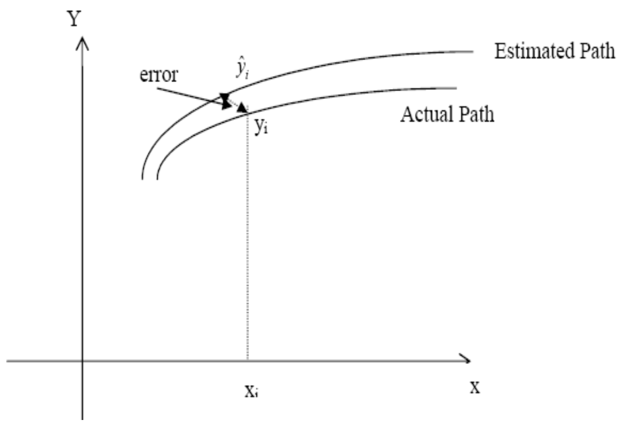

Figure 2 : Estimated and actual path

This error may be positive or negative hence to ensure it to be positive only, we square both side of the equation

$$S = \sum_{i=1}^{5}\left(y_i - \hat{y}_i\right)^2 = \sum_{i=1}^{5}\left[y_i - \left(a_2 x_i^2 + a_1 x_i + a_0\right)\right]^2 \tag{5}$$

If s is minimal, then $\frac{ds}{da_2}$ or $\frac{ds}{da_1}$ or $\frac{ds}{da_0}$ should be equal to zero. If this is applied to above equation we get three simultaneous equations as given below.

$$\begin{aligned}
\sum y_i &= \sum a_2 x_i^2 + \sum a_1 x_i + n a_0 \\
\sum x_i y_i &= a_0 \sum x_i + \sum a_1 x_i^2 + a_2 \sum x_i^3 \\
\sum x_i^2 y_i &= a_2 \sum x_i^4 + \sum a_1 x_i^3 + a_0 \sum x_i^2
\end{aligned} \tag{6}$$

Where n= number of instances selected for detection measurement, which is presently 5.The values of $x_1, x_2 \ldots\ldots\ldots x_5$ are distance in the x axis from the origin and $y_1, y_2 \ldots\ldots\ldots y_5$ height in xy vertical planes. These values are to be transformed using distance measured of the object from the radar installation, angles of azimuth and angle of elevation noted at the instance propagating the signal. Let these are represented by symbols $d_1, d_2 \ldots\ldots\ldots d_5, \theta_1, \theta_2 \ldots\ldots\ldots \theta_5$ and $\alpha_1, \alpha_2 \ldots\ldots\ldots \alpha_5$

Then the values of x y are expressed as

$$\begin{aligned}
x_1 &= d_1 \cos\alpha_1 \cos\omega_1, y_1 = d_1 \sin\alpha_1 \\
x_2 &= d_2 \cos\alpha_2 \cos\omega_2, y_2 = d_2 \sin\alpha_2 \\
x_3 &= d_3 \cos\alpha_3 \cos\omega_3, y_3 = d_3 \sin\alpha_3 \\
x_4 &= d_4 \cos\alpha_4 \cos\omega_4, y_4 = d_4 \sin\alpha_4 \\
x_5 &= d_5 \cos\alpha_5 \cos\omega_5, y_5 = d_5 \sin\alpha_5
\end{aligned} \tag{7}$$

Where angles $\alpha_1, \alpha_2 \ldots\ldots\ldots \alpha_5$ and $\omega_1, \omega_2 \ldots\ldots\ldots \omega_5$ are shown in the figure1 and $\angle\omega$ can be calculated as

$$\sin\omega_1 = \frac{d_2 \cos\alpha_2 \sin(\theta_2 - \theta_1)}{\sqrt{[d_2^2 \cos^2\alpha_2 + d_1^2 \cos^2\alpha_1 - 2d_1 d_2 \cos\alpha_1 \cos\alpha_2 \cos(\theta_2 - \theta_1)]}}$$

$$\cos\omega_1 = \sqrt{1 - \left(\frac{d_2^2 \cos^2\alpha_2 \sin^2(\theta_2 - \theta_1)}{d_2^2 \cos^2\alpha_2 + d_1^2 \cos^2\alpha_1 - 2d_1 d_2 \cos\alpha_1 \cos\alpha_2 \cos(\theta_2 - \theta_1)]}\right)}$$

Now the values of $(\left(x_i, i \in (1,5)\right)$ and $(\left(y_i, i \in (1,5)\right)$ are computed and then, computation of following is carried out

$$\sum_{i=1}^{5} x(i) = \sum_{i=1}^{5} d(i)\cos\alpha(i)\cos\omega(i)$$

$$\sum_{i=1}^{5} y(i) = \sum_{i=1}^{5} d(i)\sin\alpha(i)$$

$$\sum_{i=1}^{5} x^2(i) = \sum_{i=1}^{5}\left(d(i)\cos\alpha(i)\cos\omega(i)\right)^2$$

$$\sum_{i=1}^{5} x^2(i)y(i) = \sum_{i=1}^{5} d^2(i)\cos^2\alpha(i)\cos^2\omega(i)\sin\alpha(i)$$

$$\sum_{i=1}^{5} x^3(i) = \sum_{i=1}^{5}\left(d(i)\cos\alpha(i)\cos\omega(i)\right)^3 \tag{10}$$

$$\sum_{i=1}^{5} x^4(i) = \sum_{i=1}^{5}\left(d(i)\cos\alpha(i)\cos\omega(i)\right)^4$$

$$\sum_{i=1}^{5} x(i)y(i) = \sum_{i=1}^{5} d^2(i)\cos\alpha(i)\cos\omega(i)\sin\alpha(i)$$

If the above mentioned values are calculated in the equation then we can evaluate

$$\begin{aligned}
a_0 &= \frac{1}{n}\left[\sum y_i - a_1 \sum x_i - a_2 \sum x_i^2\right] \\
a_1 &= \frac{\left[\sum x_i \sum y_i - n\sum x_i y_i\right]\left[\sum x_i^2 \sum x_i^2 - \sum x_i^2 n\right] - \left[\sum x_i^2 \sum y_i - n\sum x_i^2 y_i\right]\left[\sum x_i^2 \sum x_i - \sum x_i^3 n\right]}{\left[\left(\sum x_i\right)^2 - n\sum x_i^2\right]^2 - \left[\sum x_i^2 \sum x_i - \sum x_i^3 n\right]^2} \\
a_2 &= \frac{\left[\sum x_i \sum y_i - n\sum x_i y_i\right]\left[\sum x_i \sum x_i^2 - \sum x_i^3 n\right] - \left[\left(\sum x_i\right)^2 - n\sum x_i^2\right]\left[\sum x_i^2 \sum y_i - n\sum x_i^2 y_i\right]}{\left[\sum x_i^2 \sum x_i - n\sum x_i^3\right]^2 - \left[\left(\sum x_i^2\right)^2 - n\sum x_i^2\right]^2}
\end{aligned} \tag{11}$$

Once these values are computed they are substituted to represent the trajectory in equation (1).

3. Computed Result:

The values are measured while conducting the experiments are given in the following Table1

Table 1: Values of Different Parameters

Instances	Distance	Azimuth angle	Elevation angle	Instance of transmitting pulse
1	d_1=250km	$\alpha_1=28^0$	$\theta_1=35^0$	$t_1=0$
2	d_2=240km	$\alpha_2=27.5^0$	$\theta_2=36^0$	$t_2=1$
3	d_3=235km	$\alpha_3=27^0$	$\theta_3=37^0$	$t_3=1.5$
4	d_4=230km	$\alpha_4=26.5^0$	$\theta_4=38^0$	$t_4=2$
5	d_5=225km	$\alpha_5=26^0$	$\theta_5=39^0$	$t_5=2.5$
6	d_6=220km	$\alpha_6=25.5^0$	$\theta_6=40^0$	$t_6=3.0$

The values computed for the Cartesian coordinates are as
x_1=22.73, x_2=196.96, x_3=202.86, x_4=202.45, x_5=200.17
y_1=117.3, y_2=110.81, y_3=106.68, y_4=102.62, y_5=98.6

From the table we have computed the following values

$$\sum_{i=1}^{5} x(i) = 050.90, \qquad \sum_{i=1}^{5} y(i) = 536.3333,$$

$$\sum_{i=1}^{5} x(i)^2 = 221080.556, \qquad \sum_{i=1}^{5} x(i)^3 = 4.65520,$$

$$\sum_{i=1}^{5} x(i)y(i) = 112932.419 , \qquad \sum_{i=1}^{5} x(i)^4 = 9.81139,$$

$$\sum_{i=1}^{5} x^2(i)y(i) = 2.38014 \tag{12}$$

These values will compute the coefficient as given below

$$a_0 = -694.8868, \quad a_1 = 6.5786, \quad a_2 = -0.01313 \tag{13}$$

This leads to equation of trajectory as given below.

$$y = a_2x^2 + a_1x + a_0$$

$$y = -0.0131x^2 + 6.5786x - 694.886 \tag{14}$$

4. Conclusion:

A moving object in space if is required to be studied, then its position with respect to ground station , velocity and the path of movement is required to be found out. While carrying out the study, two hypothetical planes are assumed, one vertical on which the trajectory is formed and other horizontal plane on which ground station is situated. The object has been detected five times and distances, and angles are measured accurately. An origin is determined in the vertical plane; it is the point from where if a perpendicular is drawn it passes through the ground station. The Cartesian coordinates of the object at five points are shown in the figure1. The trajectory equation has been assumed a second order in x and y coordinates and their coefficients are computed using regression technique for minimal error. The equation such obtained describes the path of the trajectory and the position of the object with respect to time can be computed for its future movement.

References :

1. K.S Knudsen etal , “ Moving Object Detection and Trajectory Estimation in the Transform / Spatio Temporal Mixed Domain” IEEE trans 1992 , pp 505-508.
2. Geogory w Donohoc, “ A Combined Analog Digital Technique for Normalizing Video Signals for the Detection of Moving Object” IEEE trans 1992 , pp 437-440.
3. Paulo A.C Marques, “SAR Moving Object Trajectory Estimation: Solving the Blind angle Ambiguity with a Single Sensor.” IEEE trans 2000 , pp 77-80.
4. Simon J Julier A, “ New Extension of the Kalman Filter to Nonlinear System” IEEE trans 2000.
5. Wenqin wang, “An Approach for Multiple Moving Target Detection and Velocity Estimation” IEEE conference 2006 , pp 749-753.