# The Interception Program of Fast Moving Object in Space Using Radar <br> Neetu Agrawal ,**M.P. Singh <br> J.K Institute of Applied Physics , (Electronics Department) <br> Allahabad University ,Allahabad, India <br> neetu24@gmail.com, 9415368870 


#### Abstract

At time objects are seen in space moving towards earth with a great velocity. To detect its position, estimate velocity, trace its trajectory and striking point are problem to be studied. Two planes vertical and horizontal are assumed. The imaginary origin point in the vertical plane has been worked out. A trajectory equation of second order nonlinear has been developed in Cartesian coordinates. The object's position in the trajectory are calculated in terms of radar parameters after time $t \in(0, T)$.


## 1. Introduction:

The path of moving object is called trajectory. Object moves on trajectory, and the curve of trajectory is nonlinear. This has been worked [1] and the trajectory equation given in the paper is
$y=-0.0131 x^{2}+6.5786 x-694.886$
Hence the estimation of distance traveled by the object will be same as the length of the trajectory curve. First the velocity is calculated then for the estimation of the exact time at which the object reaches at a predefined point will be the present problem. For this purpose the length of the curve from the detection point to the predefined point is required to be calculated. The time for reaching the object to predefined point is given by Time $=$ (Length of trajectory from detecting time to predefined)/ (velocity of the object)

$$
\begin{equation*}
=\mathrm{s} / \mathrm{v} \tag{2}
\end{equation*}
$$

Many times it is required to hit the target before it strikes the ground. So to calculate the striking point of the target on the ground as well as time of strike can also be calculated. This provides early warning for the safety of the place and its nearby assets before to get it divested, if interception failed to achieve. For all these purposes the length of trajectory curve is important.

## 2.Measurement of Trajectory Length :

The length of the curve is computed by dividing the curve into small linear parts. The small length can be expressed as $\Delta s=\Delta \bar{x}+\Delta \bar{y}$ is shown in Fig 1. Equation has two variables $\Delta \bar{x}$ which is constant, and $\Delta \bar{y}$ which is varying and is needed to be computed. While calculating the distance segment, x assumes value $x=\Delta x, 2 \Delta x, 3 \Delta x$. $\qquad$ $1000 \Delta x$. When the values of x are substituted in the trajectory equation, we get the


Figure 1: Length of the curve
values of $y=y_{1}, y_{2}, \ldots \ldots . \ldots \ldots . y_{1000}$.The undergiven equations estimates the lengths of segments of trajectory. The summation of these segments, measures the length of trajectory, these are shown as below
$\Delta s_{1}=\sqrt{\Delta x^{2}+y_{1}^{2}}$
$\Delta s_{2}=\sqrt{\Delta x^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$\Delta s_{3}=\sqrt{\Delta x^{2}+\left(y_{3}-y_{2}\right)^{2}}$
-
-

$$
\Delta s_{n}=\sqrt{\Delta x^{2}+\left(y_{n}-y_{n-1}\right)^{2}}
$$

When all piece length are summed, we get the length of the curve:

$$
\begin{equation*}
S=\sum_{i=0}^{1000} \Delta s_{i} \tag{4}
\end{equation*}
$$

The algorithm given below performs the computation of the estimation of length.

```
Begin
        Read \Deltax, y(0)=0
        for i=1 to 1000 in steps of 1 do
        Y(i)=-0.01*\Delta x* \Deltax+6.578* \Delta x-6896.88
    s(i)=Sqrt(\Deltax* 噰[y(i)-y(i-1)]*[y(i)-y(i-1)]
        x= \Delta x+ \Deltax;
                print \Delta s(i);
            stop
        for i=1 to 1000 in steps of 1 do;
        s(i)=s(i+1)+s(i)
    stop
        s=s(i)
    stop
end
```


## 3. Estimation of time Instances:

The entire exercise is time based. It is therefore important to compute the time instances for the movement of object. This is calculated as given below
$T=\frac{s}{v}=\frac{\sum_{i=0}^{1000} \Delta s}{v}$
where $s=$ length of the trajectory from the detected point to the striking point.
$\mathrm{v}=\mathrm{velocity}$ of the object at the instance of detection. Velocity has been measured in [2] $\mathrm{v}=110 \mathrm{~km}$
Let the increment time of computation $\Delta t$, then it is given as
$\Delta t=\frac{T}{1000}=\frac{\sum_{i=0}^{1000} \Delta s}{1000 v}$
The time instants, at which the value of $y(i)$ are determined are given as:
$t_{0}=0$
$t_{1}=\Delta t$
$t_{2}=\Delta t+t_{1}$
$t_{3}=\Delta t+t_{2}$
$t_{n}=\Delta t+t_{(n-1)}$
All these values are given in Table 1. These values are computed using "C" language and algorithm as given below
The first detecting point enable us to calculate the value of x , referring fig 2 as given below.
$x_{1}=d_{1} \cos \alpha \cos \omega=220.69$
The striking point is found out from the trajectory equation putting value $y=0$. The decrement value is then computed by dividing the values into 1000 parts. This is given as

$$
\begin{aligned}
\Delta x & =\left(x_{i}-x_{R}\right) / 1000 \\
& =(220.69-110.34) / 1000 \\
& =0.11
\end{aligned}
$$

When above values are substituted in trajectory equation table 1 is constructed.

Table 1: Values of $x, y$ and $t$

| Sr <br> no. | Time Instances | Increment in $\mathbf{x}$ | The value of <br> $\mathbf{y}$ |
| :--- | :--- | :--- | :--- |
| 1 | $1.34976 \mathrm{e}-05$ | 220.698965 | 117.485285 |
| 2 | $2.69952 \mathrm{e}-05$ | 220.629604 | 117.430903 |
| 3 | $4.04928 \mathrm{e}-05$ | 220.560243 | 117.376396 |
| 4 | $5.399039 \mathrm{e}-05$ | 220.490883 | 117.321762 |
| . |  |  |  |
| . |  |  |  |
| . |  | 151.477979 | 0.363567 |
| 999 | 0.013484 | 151.40862 | 0.183111 |
| 1000 | 0.013498 |  |  |

4. Object position with respect to ground station in terms of time:
The study conducted so far describes the position of moving object in terms of the coordinates in the imaginary plane, and time. This information can be only useful if the information is transformed into time, and distance, angle of azimuth and elevation from the ground station. Referring figure 2. The estimation of these values will be done as discussed below.


Figure 2. Object position at different instances
Consider that an object has been detected and now we desire to know the position, after time T , in terms of azimuth, elevation and distance $\theta_{T}, \alpha_{T}$ and $d_{T}$.
We know from the triangle $\mathrm{Qx}_{1} \mathrm{X}_{2}$ and formula of trigonometry that

$$
\begin{align*}
& \frac{d_{2} \cos \alpha_{2}}{\sin \omega_{1}}=\frac{x_{1} x_{2}}{\sin \left(\theta_{2}-\theta_{1}\right)}  \tag{8}\\
& \sin \omega_{1}=\frac{d_{2} \cos \alpha_{2} \sin \left(\theta_{2}-\theta_{1}\right)}{x_{1} x_{2}}  \tag{9}\\
& =\frac{d_{2} \cos \alpha_{2} \sin \left(\theta_{2}-\theta_{1}\right)}{\sqrt{d_{2}^{2} \cos ^{2} \alpha_{2}+d_{1}^{2} \cos ^{2} \alpha_{1}-2 d_{1} d_{2} \cos \alpha_{1} \cos \alpha_{2} \cos \left(\theta_{2}-\theta_{1}\right)}}
\end{align*}
$$

Where
$x_{1} x_{2}=$
$\sqrt{d_{2}^{2} \cos ^{2} \alpha_{2}+d_{1}^{2} \cos ^{2} \alpha_{1}-2 d_{1} d_{2} \cos \alpha_{1} \cos \alpha_{2} \cos \left(\theta_{2}-\theta_{1}\right)}$
The distance (range) of the target from the ground station is computed considering triangle $\mathrm{Qx}_{1} \mathrm{x}_{2}$ as given below.

$$
\begin{align*}
& \cos w_{1}=\frac{G_{1}^{2}+\left(x_{1}-x_{T}\right)^{2}-\left(Q x_{T}\right)^{2}}{2 G_{1}\left(x_{1}-x_{T}\right)}  \tag{10}\\
& Q x_{T}=\sqrt{G_{1}^{2}+\left(x_{1}-x_{T}\right)^{2}-2 G_{1}\left(x_{1}-x_{T}\right) \cos \omega_{1}} \\
& d_{T}^{2}=Q x_{T}^{2}+y_{T}^{2}  \tag{11}\\
& d_{T}=\sqrt{Q x_{T}^{2}+y_{T}^{2}} \tag{12}
\end{align*}
$$

The angle of elevation of the target at time T will be

$$
\begin{equation*}
\tan \alpha_{T}=\frac{y_{T}}{Q x_{T}} \tag{13}
\end{equation*}
$$

The angle of azimuth is computed considering triangle Q $\mathrm{X}_{\mathrm{T}} \mathrm{X}_{1}$
We will have relation
$\frac{x_{1}-x_{T}}{\sin \theta_{\varsigma}}=\frac{Q x_{T}}{\sin \omega_{1}}$
$\sin \theta_{\varsigma}=\frac{\left(x_{1}-x_{T}\right) \sin \omega_{1}}{Q x_{T}}$
$\theta_{\varsigma}=\sin ^{-1} \frac{\left(x_{1}-x_{T}\right) \sin \omega_{1}}{Q x_{T}}$
$\theta_{T}=\theta \varsigma+\theta_{1}$
Where $\theta_{1}$ is azimuth angle at the time of detection.
Thus the equation (12), (13), (17) measures the position of the object after time T in terms of distance, elevation and azimuth.

## 5. Conclusion:

The Paper discusses the technique to measure the length of the curve of the trajectory. The trajectory and its length have been studied with respect to time and the equation for all the parameters of the object are developed. Program to compare, all the parameters of object has been developed in language C. From this table, the position of the object after time ' T ' determined desired by rotation $\theta_{T}, \alpha_{T}$ and $d_{T}$. These values can be computed and planning can be made to intercept the object.

## 6. References :

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