

# Analysis of Electromagnetic Wave Propagation along Rough Surface by Using Discrete Ray Tracing Method

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## 1. Introduction

The electromagnetic wave scattering by rough surfaces has attracted researchers' attention from technical view point of radar cross section in relation to remote sensing technology [1], [2]. Recently, a rapid progress has been made in the area of sensor networks to gather physical data and to control natural environments. Sensors are usually distributed on terrestrial surfaces such as deserts, vegetable fields, hills and forests and so on. These surfaces are considered to be random rough surfaces, and thus it is important to investigate propagation characteristics along random rough surfaces to promote sensor networks. [3], [4].

As far as 1D rough surfaces are concerned, analyses based on ray tracing method (RTM) [3] and FVTD method [4] have been reported so far. However, the former method requires much computation time to search many reflection and diffraction rays, and the latter needs much computer memory to deal with relatively large area of rough surfaces compared with the wave length.

In this paper, we propose discrete ray tracing method (DRTM) to numerically analyze electromagnetic wave propagation along 1D rough surface as shown in Fig. 1 as well as along 2D rough surface as shown in Fig. 2. In the present DRTM analyses, we have made some assumptions in order to simplify ray searching algorithm to save computation time. Numerical calculations are carried out for 1D and 2D random rough surfaces, and electric field distributions are shown to check the effectiveness of the proposed DRTM.

## 2. Rough Surface Discretization and DRTM

The generation of random rough surfaces are numerically performed by the direct DFT (discrete Fourier transform) method or convolution method [4]. Parameters used for rough surface generation are correlation length  $cl$ , root mean square or deviation of height  $dv$  together with the type of rough surface spectrum. In this paper we consider only the Gaussian type of spectrum.

The first stage of DRTM is to discretize a rough surface in terms of straight lines for 1D or rectangular plates for 2D so that it could be approximated by piece wise linear lines or piece wise planar plates. This approximation procedure should be performed so that computer memory could be as small as possible. We propose the following discretization method for rough surfaces. First we divide  $x$ -axis or  $(x, y)$ -plane into  $n_x$  straight lines with length  $D_x$  for 1D or  $(n_x, n_y)$  rectangular plates with area  $(D_x, D_y)$  for 2D, respectively. Then we can discretize any types of rough surfaces in terms of representative points as follows;

$$\begin{aligned} \rho_i &= (x_i, H_1(x_i)) \quad (i = 0, 1, 2, \dots, n_x) \quad (\text{for 1D}) \\ \mathbf{r}_{ij} &= (x_i, y_j, H_2(x_i, y_j)) \quad (i = 0, 1, 2, \dots, n_x \quad j = 0, 1, 2, \dots, n_y) \quad (\text{for 2D}) \end{aligned} \quad (1)$$

where

$$\begin{aligned} x_i &= D_x i \quad (i = 0, 1, 2, \dots, n_x) \quad (\text{for 1D}) \\ (x_i, y_j) &= (D_x i, D_y j) \quad (i = 0, 1, 2, \dots, n_x \quad j = 0, 1, 2, \dots, n_y) \quad (\text{for 2D}) \end{aligned} \quad (2)$$

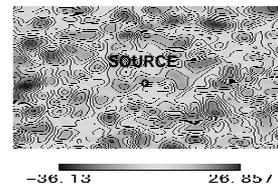
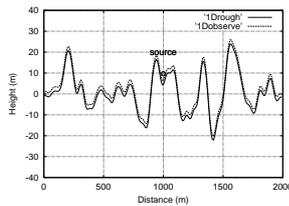


Figure 1: Geometry of 1D rough surface and source.

Figure 2: Geometry of 2D rough surface and source.

and  $H_1(x)$  and  $H_2(x, y)$  are the height functions of 1D and 2D rough surfaces, respectively.

Next we derive the normal vectors of the discretized lines or plates by the following relations:

$$\begin{aligned} \mathbf{n}_i &= (\mathbf{u}_z \times \mathbf{a}_i) / |\mathbf{u}_z \times \mathbf{a}_i| \quad (i = 0, 1, 2, \dots, n_x - 1) \quad (\text{for 1D}) \\ \mathbf{n}_{ij} &= (\mathbf{a}_{ij} \times \mathbf{b}_{ij}) / |\mathbf{a}_{ij} \times \mathbf{b}_{ij}| \quad (i = 0, 1, 2, \dots, n_x - 1 \quad j = 0, 1, 2, \dots, n_y - 1) \quad (\text{for 2D}) \end{aligned} \quad (3)$$

where  $\mathbf{u}_z$  is the unit vector in z-direction, and

$$\begin{aligned} \mathbf{a}_i &= (\boldsymbol{\rho}_{i+1} - \boldsymbol{\rho}_i) \quad (\text{for 1D}) \\ \mathbf{a}_{ij} &= (\mathbf{r}_{i+1j} - \mathbf{r}_{ij}) \quad \mathbf{b}_{ij} = (\mathbf{r}_{ij+1} - \mathbf{r}_{ij}) \quad (\text{for 2D}) \end{aligned} \quad (4)$$

It should be noted that only the position vectors  $\boldsymbol{\rho}_i$  or  $\mathbf{r}_{ij}$  and normal vectors  $\mathbf{n}_i$  or  $\mathbf{n}_{ij}$  are enough to search rays numerically for 1D or 2D discretized rough surfaces, respectively. This fact results in simplifying the ray searching algorithm and also saving computer memories considerably.

The essence of the algorithm of the proposed DRTM could be summarized in the following. We assume that arbitrary two lines of 1D or plates of 2D discretized rough surfaces are in line of sight or in short LOS, if a representative point of one line or plate is in LOS with that of another line or plate. Otherwise, they are not in line of sight or in short NLOS. This assumption enables to simplify greatly the ray searching algorithm, and as a result, it helps saving much computation time. It is worth noting that this algorithm can be readily modified to achieve more accurate rays.

In conventional ray tracing method (RTM), rays are classified into incident, reflection and diffraction rays; in DRTM, however, we divide them into incident, source diffraction and image diffraction rays [5]. The source diffraction is closely related to the incident wave with two types; one is a source diffraction in the illuminated region when the two representative points are in LOS, and the other is a source diffraction in the shadow region when the two representative points are in NLOS. In this DRTM, we employ only the source diffraction rays with the shortest path, and thus these diffraction rays are constructed so that the two representative points in LOS or NLOS may form the shortest path between them.

The image diffraction is closely associated with reflection which can be described in the geometrical optics as the emission from the image of source. We can construct image diffraction rays by connecting different two lines or plates using representative points successively when they are in LOS. It should be noted that the conventional reflection ray is also included in the present image diffraction rays as a special type of ray satisfying the Snell's law or the relationship that the reflection angle equals the incident angle.

### 3. Field Computations

We have discussed the principle of DRTM ray searching algorithm in the preceding section, and based on the far field approximation, we can evaluate electromagnetic fields in terms of the ray parameters such as reflection and diffraction points together with the rough surface parameters such as the position and normal vectors of the lines or plates of discretized rough surfaces. Although detailed discussions are omitted here, the electric field  $\mathbf{E}$  at the receiver is formally expressed in the following diadic and vector form:

$$\mathbf{E} = \sum_{n=1}^N \left[ \prod_{m=1}^{m=M_n^r} (\mathbf{R}_{nm}) \cdot \prod_{k=1}^{k=M_n^d} (\mathbf{D}_{nk}) \cdot \mathbf{E}_0 \right] \frac{e^{-\kappa r_n}}{r_n} \quad (5)$$

where  $\mathbf{E}_0$  is the electric field of the n-th ray at the first reflection or diffraction point, and  $\kappa$  is the wave number in the free space. N is the total number of rays considered,  $M_n^d$  is the number of times of its source diffractions, and  $M_n^r$  is the number of times of its image diffractions. Based on the ray data, the distance of the n-th ray from source to receiver is given by

$$r_n = \sum_{k=0}^{k=M_n^r + M_n^d} r_{nk} \quad (n = 1, 2, \dots, N) \quad (6)$$

where  $r_{nk}$  is the k-th distance from one reflection or diffraction point to the next one.

The diadic function ( $\mathbf{R}$ ) for the image diffraction is given by the ray data and the Fresnel's reflection coefficients for horizontal (h) and vertical (v) polarizations given by Eq.(7) and the complex type of Fresnel function defined by Eq.(8). On the other hand, the diadic function ( $\mathbf{D}$ ) for the source diffraction is given by the ray data and the Fresnel function. Detailed discussions are omitted here for brevity.

$$R^h = \frac{\cos \theta - \sqrt{\epsilon_c - \sin^2 \theta}}{\cos \theta + \sqrt{\epsilon_c - \sin^2 \theta}} \quad R^v = \frac{\epsilon_c \cos \theta - \sqrt{\epsilon_c - \sin^2 \theta}}{\epsilon_c \cos \theta + \sqrt{\epsilon_c - \sin^2 \theta}} \quad (7)$$

$$F(X) = \frac{e^{\frac{\pi}{4}j}}{\sqrt{\pi}} \int_X^{\infty} e^{-ju^2} du \quad (X > 0) \quad (8)$$

## 4. Numerical Examples

Fig. 1 shows an example of 1D rough surface of length 2[Km] with height deviation  $dv=10[m]$  and correlation length  $cl=50[m]$ , and Fig. 2 shows 2D rough surface example of area (1,1)[Km] with height deviation  $dv=5[m]$  and correlation length  $cl=50[m]$ . The dielectric constant and conductivity of the medium constituting rough surface are chosen as  $\epsilon_r=5$  and  $\sigma=0,0023[S/m]$ , and the operating frequency is selected as  $f=1GHz$ . The spectrum type of the random rough surface is assumed to be Gaussian. The source is located at the center of the 1D or 2D rough surface with  $h_s=1[m]$  above it, and the receiver is moved along the surface at  $h_r=0.5[m]$  above it.

First we show some numerical examples for 1D rough surfaces. Fig. 3 shows electric fields computed by the incident ray, zero order ray, in the illuminated region without any reflections or diffractions. Fig. 4 shows fields computed by three types of first order rays, that is, image diffraction ray with once reflection, source diffraction ray in the illuminated region, and that in the shadow region. Fig. 5 shows fields computed by three types of second order rays, that is, image diffraction rays with twice reflections, image diffraction ray experienced once source diffraction, and source diffraction ray experienced once image diffraction. Fig. 6 shows fields computed by four types of third order rays, that is, source diffraction ray experienced twice image diffractions, image diffraction ray with twice reflections after once source diffraction, image diffraction ray after source and image diffractions, and three times reflection rays. From these results it is concluded that the three order of source and image diffractions are enough to evaluate electric fields for computing propagation characteristics along rough surfaces.

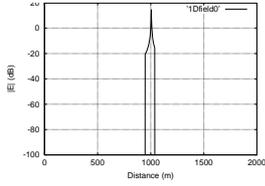


Figure 3: Field computed by zero order ray.

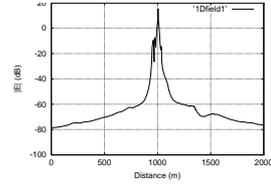


Figure 4: Field computed by first order rays.

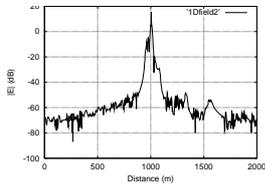


Figure 5: Field computed by second order rays.

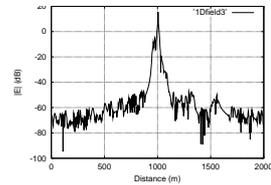


Figure 6: Field computed by third order rays.

Fig. 7 and Fig. 8 show ensemble averaged electric and magnetic fields computed by using 100 samples of generated rough surfaces for E-wave and H-wave, respectively. The height deviation is chosen as  $dv=20[m]$ ,  $10[m]$  and  $5[m]$ , and the field distributions in the free space are also depicted. It is demonstrated that the larger the height deviation  $dv$  becomes, the larger the wave attenuation is increased; characteristics of the propagation loss are similar to those in the urban areas [6].

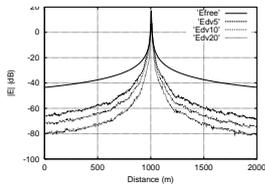


Figure 7: E-wave distributions with  $dv$  as a parameter.

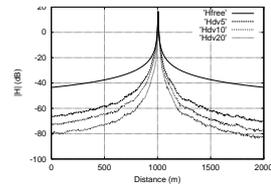


Figure 8: H-wave distribution with  $dv$  as a parameter.

Now, we show some numerical examples for 2D rough surfaces where height deviation and correlation length are chosen as  $dv = 10[m]$  and  $cl = 50[m]$ , respectively. Fig. 9 and Fig. 10 show electric fields computed by incident zero order ray in the illuminated region for vertical and horizontal small dipole antennas, respectively. The operating frequency is  $f = 1GHz$  and the complex dielectric constants are the same as the former 1D cases.

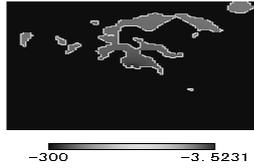


Figure 9: Field computed by zero order ray (v).

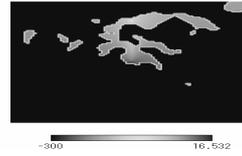


Figure 10: Field computed by zero order ray (h).

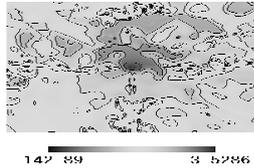


Figure 11: Field computed by first order rays (v).

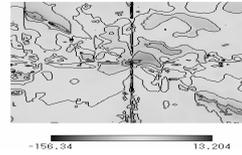


Figure 12: Field computed by first order rays (h).

Fig. 11 and Fig. 12 show electric fields computed by first order of image and source diffraction rays for vertical and horizontal small dipole antennas, respectively.

Fig. 13 and Fig. 14 show electric fields computed by second order image and source diffraction rays for vertical and horizontal small dipole antennas, respectively. It is shown that almost the same characteristics are obtained both for the vertical and horizontal polarizations except for the directivity of the source. It can be concluded that the present DRTM is effective to the analyses of propagation characteristics along 1D and 2D rough surfaces.

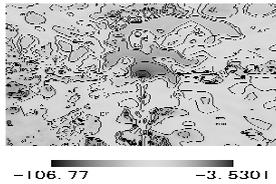


Figure 13: Field computed by second order rays (v).

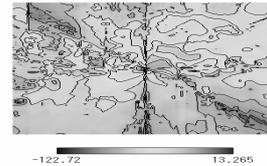


Figure 14: Field computed by second order rays (h).

## 5. Conclusion

In this paper, we have proposed DRTM for numerical analysis of characteristics of electromagnetic propagation along 1D and 2D random rough surfaces. The point of the present method is to discretize not only rough surface but also ray tracing. The former helps saving computer memories and the latter does simplifying ray searching algorithm resulting in saving computation time. Numerical calculations were carried out for 1D and 2D rough surfaces to check the effectiveness of the proposed DRTM.

Improvement of the present DRTM and its application to various electromagnetic problems deserve as future investigations.

## References

- [1] E.I. Thoros, "The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian roughness spectrum," *J. Acoust. Soc. Am.*, vol. 83, no.1, pp.78-92, Jan. 1988.
- [2] K.Y. Yoon, M. Tateiba and K. Uchida, "FVTD Simulation for Random Rough Dielectric Surface Scattering at Low Grazing Angle," *IECE TRANS. ELECTRON.*, vol.E83-C, no.12, pp.1836-1843, June 2000.
- [3] K. Uchida, H. Fujii, M. Nakagawa, J. Honda and T. Morikawa, "Ray Tracing Analysis of Electromagnetic Wave Propagation along Rough Surface," Technical Report of IEEJ, EMT-05-48, pp.105-108, (2005-11).
- [4] K. Uchida, H. Fujii, M. Nakagawa, X.F. Li and H. Maeda, "FVTD Analysis of Electromagnetic Wave Propagation along Rough Surface", *IEICE TRANS. COMMUN.*, vol.J90-B, no.1, pp.48-55, (2007-01).
- [5] K. Uchida and Ch.-K. Lee, "Image and Source Coefficients for a Diffraction Problem", *The Journal of "Electromagnetic Waves and Electronic Systems"*, no.4-5, T.7, pp.84-87, (2002).
- [6] M. Hata, "Empirical Formula for Propagation Loss in Land Mobile Radio Services", *IEEE Trans. Veh. Technol.*, vol. VT-29, no.3, pp.317-325, (1980-08).