# Design of multibeam dielectric lens antennas <br> with the fan beam by multi-objective optimization 

Y. Kuwahara<br>Graduate School of Technologies, Shizuoka University<br>3-5-1 Johoku Naka Hamamatsu 432-8561 Japan<br>tykuwab@ipc.shizuoka.ac.jp

## 1. Introduction

In advanced ITS application, the radar antenna is expected to resolve several objects in the horizontal plane. These applications demand multibeam antennas. Multibeam antennas have to form high gain and low sidelobe radiation patterns at different directions. In case of the anticollision vehicular radar, a vertical sector beam will be required when considering the inclination of the road.

The typical antenna with multibeam performance is the bifocal antenna with dielectric lens [1]. Though the bifocal lens is guaranteed to equalize the aperture phase distribution on the specified design directions in the scanning plane, it is not guaranteed on the transverse plane due to its astigmatism as well as the other directions. Of course, the vertical sector beam cannot be formed. No previous reports have adequately addressed these points, especially vertical sector beam.

We propose an effective method to design multibeam dielectric lens antennas with the fan beam. To optimize the lens antennas balancing the gain (including pattern loss) and sidelobe level of the scanning and transverse patterns taking into account of beamwidth of the vertical pattern, the pareto-genetic algorithm [2] is introduced.

## 2. Proposed Lens Design Using PARATO GA

In our approach, the initial shape of lens is determined so as to yield a single focal lens. Next, the shape of the lens is modified so as to obtain both multibeam and vertical sector beam characteristics. The modifying values are given by the chromosomes. The positions of the feeds are also determined from the chromosomes. The radiation pattern of each structure is calculated and evaluated by the pareto ranking method. The individuals and the calculation procedure are shown in Fig. 1. The definition of lens coordinates considering the multibeam goal is shown in Fig. 2. First of all, $\mathrm{P}_{1}(1)$ is defined. $\phi$ is the angle between the z -axis and the line from the origin to the lens edge. When $\mathrm{D}_{\mathrm{x}}$ is fixed, $\phi$ is a function of $z$. Accordingly, when Feed (1) is placed at the origin, $z_{1}(1)$ in $P_{1}(1)$ is determined so as to maximize the radiation efficiency [3].

$$
\begin{equation*}
P_{1}(1)=\left[0 z_{1}(1)+\triangle z_{1}(1)\right]^{T} \tag{1}
\end{equation*}
$$

$z_{1}(1)$ is the value yielded by the maximization of the radiation efficiency. $\triangle z_{1}(1)$ is the correction value derived from the individuals. Applying $\triangle z_{1}(1)$ can yield superior results in multibeam antenna design. Though the optimized lens shape including $\triangle z_{1}(1)$ may not offer maximum efficiency, it obtains better performance from the view point of the values of multiple objective functions.

The coordinates of the first plane that starts from $\mathrm{P}_{1}(1)$ are determined. The interval, in the x -direction, of each $\mathrm{P}_{1}(\mathrm{n})$ is constant, $\triangle \mathrm{x}_{1}$. The difference, in the z -direction, from the previous point is defined as $\triangle z_{1}(n)$. Accordingly, $P_{1}(n)$ is expressed by the following recursion.

$$
\begin{equation*}
P_{1}(n)=\left[x_{1}(n-1)+\Delta x_{1} \quad z_{1}(n-1)+\Delta z_{1}(n)\right]^{T} \quad(2 \leq n \leq N) \tag{2}
\end{equation*}
$$

$\triangle z_{1}(n)$ are derived from the individuals. Referring to $P_{1}(n)$, the coordinates of the second plane are determined so that a single focal lens can be formed. The criterion of the path length is satisfied when the ray from the feed (1) to $\mathrm{P}_{1}(\mathrm{~N})$ becomes parallel to the z -axis after refraction at the lens edge. Considering this path length and the refraction at the first plane, all $\mathrm{P}_{2}(\mathrm{n})$ can be determined. The result is a single focal lens with modified first plane.

Next, $\mathrm{P}_{2}(\mathrm{n})$ are modified to $\mathrm{P}^{\prime}(\mathrm{n})$ to obtain the desired multibeam characteristics. Similar to $\mathrm{P}_{1}(\mathrm{n})$, $\mathrm{x}_{2}(\mathrm{n})$ in $\mathrm{P}_{2}(\mathrm{n})$ are left and $\mathrm{z}_{2}(\mathrm{n})$ are modified to $\mathrm{z}_{2}^{\prime}(\mathrm{n})$.

$$
\begin{align*}
& \mathrm{P}_{2}^{\prime}(\mathrm{N})=\left[\mathrm{x}_{2}(\mathrm{~N}) \mathrm{z}_{2}(\mathrm{~N})+\triangle \mathrm{z}_{2}(\mathrm{~N})\right]^{\mathrm{T}}  \tag{3}\\
& P_{2}^{\prime}(n)=\left[\begin{array}{ll}
x_{2}(n) & z_{2}(n)+\Delta z_{2}(n)+z_{2}^{\prime}(n+1)-z_{2}(n+1)
\end{array}\right]^{T} \quad(N-1 \geq n \geq 1) \tag{4}
\end{align*}
$$

$\Delta \mathrm{z}_{2}(\mathrm{n})$ are also derived from the individuals. $\Delta z_{2}(n-1)+z_{2}^{\prime}(n)-z_{2}(n)$ represents the accumulation of correction value from the lens edge. (4) can gradually change toward the lens center. By rotating the cross section made by $\mathrm{P}_{1}(\mathrm{n})$ and $\mathrm{P}_{2}(\mathrm{n})$ around z axis, form of lens is achieved.

Feed (2) is placed on the line formed by $\theta(2)=10^{\circ}$. When the lens thickness on $z$-axis is $T$, the intersection point between the $z$-axis and $R(m)$ is defined as $T /$ ref from $P_{1}(1)$ [1]. Feed (3) and feed (4) are placed at $20^{\circ}$ and $30^{\circ}$, respectively. Each feed is directed to $\mathrm{P}_{1}(1)$ to reduce the spillover. The focal lengths have yet to be determined. Because the focal point of xz-plane and yz-plane differs due to astigmatism, GA decides the focal lengths. The guesses of $R(m)$ are calculated from [4]. The focal point on the scanning plane is

$$
\begin{equation*}
\mathrm{R}(\mathrm{~m})=\mathrm{F} \cos ^{2}(\theta(\mathrm{~m})) \quad(m=1 \ldots 4) \tag{5}
\end{equation*}
$$

where F is the focal length shown in Fig. 2. The focal point on the transverse plane is

$$
\begin{equation*}
\mathrm{R}(\mathrm{~m})=\mathrm{F} \quad(m=1 \ldots 4) \tag{6}
\end{equation*}
$$

From (5) and (6), the search range of feed locations is determined. The positions are also derived from the individuals to obtain better performance.

To investigate the beam width of the vertical pattern and the pattern loss of multibeam, the major axis $\mathrm{D}_{\mathrm{x}}$, and the minor axis $\mathrm{D}_{\mathrm{y}}$ of the lens are incorporated into the individual as the chromosome.

To evaluate the modified lens performance, the radiation patterns of the designed shapes $\mathrm{E}_{\mathrm{rm}}(\theta, \varphi)$ are calculated. The patterns from $\mathrm{E}_{\mathrm{r} 1}(\theta, \varphi)$ to $\mathrm{E}_{\mathrm{r} 4}(\theta, \varphi)$ correspond to each feed. Here three objective functions are defined. The first object is the minimum gain where two beams intersect.

$$
\begin{equation*}
\operatorname{Obj} 1=\min \left[\mathrm{E}_{\mathrm{r} 1}\left(\theta_{\text {cross }}(1), 0\right) \quad \mathrm{E}_{\mathrm{r} 2}\left(\theta_{\text {cross }}(2),\right) \quad \mathrm{E}_{\mathrm{r} 3}\left(\theta_{\text {cross }}(3), 0\right)\right] \tag{7}
\end{equation*}
$$

$\theta_{\text {cross }}(\mathrm{m})$ is an angle intersecting $\mathrm{m}^{\text {th }}$ beam with $\mathrm{m}+1^{\text {th }}$ beam.
The second object is the minimum value of the ratio between the gain and the sidelobe level.

$$
\begin{align*}
\operatorname{Obj} 2= & \min \left[\mathrm{E}_{\mathrm{r} 1}(\theta(1), 0) / \operatorname{SLL}\left(\mathrm{E}_{\mathrm{r} 1}(\theta, 0)\right) \mathrm{E}_{\mathrm{r} 2}(\theta(2), 0) / \operatorname{SLL}\left(\mathrm{E}_{\mathrm{r} 2}(\theta, 0)\right)\right. \\
& \mathrm{E}_{\mathrm{r} 3}(\theta(3), 0) / \operatorname{SLL}\left(\mathrm{E}_{\mathrm{r} 3}(\theta, 0)\right) \mathrm{E}_{\mathrm{rt}}(\theta(4), 0) / \operatorname{SLL}\left(\mathrm{E}_{\mathrm{r} 4}(\theta, 0)\right) \\
& \left.\mathrm{E}_{\mathrm{r} 1}(\theta(1), 0) / \operatorname{SLL}\left(\mathrm{E}_{\mathrm{r} 1}(\theta(1), \varphi)\right) \mathrm{E}_{\mathrm{r} 2}(\theta(2), 0) / \operatorname{SLL}\left(\mathrm{E}_{\mathrm{r} 2}(\theta(2), \varphi)\right)\right) \\
& \left.\mathrm{E}_{\mathrm{r} 3}(\theta(3), 0) / \operatorname{SLL}\left(\mathrm{E}_{\mathrm{r} 3}(\theta(3), \varphi)\right) \mathrm{E}_{\mathrm{r} 4}(\theta(4), 0) / \operatorname{SLL}\left(\mathrm{E}_{\mathrm{r} 4}(\theta(4), \varphi)\right)\right] \tag{8}
\end{align*}
$$

$\operatorname{SLL}(\cdot)$ represents the calculated value of the maximum sidelobe level. The first half of (8) represents the multibeam pattern in the horizontal plane. The second half of (8) represents the sector beam pattern in the vertical plane.

The third object is formation of sector beam in the vertical plane. Here, it is assumed to form beamwidth of $20^{\circ}$.

$$
\begin{equation*}
O b j_{3}=\min \left[-\left|\phi_{3 \mathrm{~dB}}(1)-20\right| \quad-\left|\phi_{3 \mathrm{~dB}}(2)-20\right| \quad-\left|\phi_{3 \mathrm{~dB}}(3)-20\right| \quad-\left|\phi_{3 \mathrm{~dB}}(4)-20\right|\right] \tag{9}
\end{equation*}
$$

$\phi_{3 \mathrm{~dB}}(\mathrm{~m})$ is beamwidth of vertical pattern of $\mathrm{m}^{\text {th }}$ beam. (7), (8) and (9) are evaluated for all individuals.

## 3. Numerical Simulation

The validity of the proposed method is demonstrated in this section. The radiation pattern of the feed was $\cos ^{2.8} \theta$ in both horizontal and vertical plane based on numerical simulation results of Vivaldi antenna. The upper limitation of $\mathrm{D}_{\mathrm{x}}$ and $\mathrm{D}_{\mathrm{y}}$ is decided so as to achieve beamewidth of $10^{\circ}$ by the circular aperture under the conditions that radiation efficiency becomes maximum value [3].

The calculation result is shown below. The values of objective functions (7) to (9) at the final generation are shown in Fig. 3. The circles in this figure represent each individual's performance. Individual A to C are pareto-optimal. Individual A is superior to gain, Individual B is superior to sidelobe level, and Individual C is superior to beamwidth of vertical sector beam.

## 4. Conclusion

We have proposed an effective method to design multibeam dielectric lens antennas with the fan beam. To optimize the lens antennas balancing the gain including pattern loss of multibeam and sidelobe level of the scanning and transverse patterns taking into account of beamwidth of the vertical pattern, the pareto-genetic algorithm is introduced. The validities have been confirmed by numerical simulation.

## References

[1] A. L. Peebles, "A dielectric bifocal lens for multibeam antenna applications," IEEE Trans. Antennas Propagat., vol. 36, no. 5, pp. 599-606, May. 1988.
[2] J. Horn, N. Nafpliotis, and D. E. Goldberg, "A niched pareto genetic algorithm for multiobjective optimization," in Proc. 1st IEEE Conf. Computation, pp. 82-87, 1994.
[3] Y. Kuwahara, T. Ishita, Y. Matsuzawa, and Y. Kadowaki, "An X-band phased array antenna with a large elliptical aperture," IEICE Trans. Commun., vol. E76-B, no. 10, pp. 1249-1257, Oct. 1993.
[4] R. M. Brown, "Dielectric bifocal lenses," IRE International Conv. Rec., pp.180-187, Mar. 1956.


Fig. 1 Calculation procedure
Fig. 2 Lens configuration

Table 1 Parameters of simulation

| separation angle between feeds $\triangle \theta$ | $10^{\circ}$ |
| :---: | :---: |
| maximum steering angle $\theta(M)$ | $30^{\circ}$ |
| feed pattern $n \cos \theta^{\wedge} n$ | 2.8 |
| aperture $D_{x}, D_{y}$ | $1 \sim 20 \lambda$ (variable) |
| distance between lens and feed $z_{l}(1)$ | $1 \sim 7 \lambda$ (variable) |
| interval $\Delta x_{l}$ | $0.5 \lambda$ |
| $\triangle z_{l}(n)$ | $-0.05 \sim+0.05 \lambda$ (variable) |
| focal distance $R(m)$ (except of $m=1)$ | $F \cos (\theta(m)) \sim F($ variable) |
| refractive index $R e f$ | 1.6 |
| number of individuals | 40 |
| number of chromosomes | $49\left(D_{x}, D_{y,}, z_{l}(1), \Delta z_{l}(n) \times 21, \Delta z(n) \times 22, R(m) \times 3\right)$ |
| bit number of chromosomes | 8 |
| Number of generation | 100 |
| Mutation probability | $0.2($ start $)$ to $0.1($ end $)$ |
| Crossover probability | 0.01 |



Fig. 3 Three objective functions at final generation.

Table 2 Performances of 3 parato optimal.

| Individual | Obj $_{1}[\mathrm{~dB}]$ | Obj $_{2}[\mathrm{~dB}]$ | Obj $_{3}[\mathrm{Deg}]$ |
| :---: | :---: | :---: | :---: |
| A | 21.7 | 12.7 | -10.1 |
| B | 16.5 | 19.1 | -2.72 |
| C | 9.71 | 18.1 | -0.04 |

