Anomalous Power Radiation from Material and Metamaterial Resonances

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1. Introduction

A number of recent papers have investigated the radiation field of antennas embedded in metamaterials and/or radiation from resonating modes inside material/metamaterial discontinuity layers [1]. It has been shown that the radiated power can be enhanced significantly at such resonances, which is a source of curiosity for electromagnetic community. In this paper we have investigated the problem using a practical antenna geometry that can be fed easily using common microwave waveguides, coaxial cables through baluns, or other transmission lines as practical antenna feeds. As such, the antenna input must be impedance matched to the feeding transmission line, and the radiated power must be accounted for through the source input power together with the resistive losses of the antennas and those inside the materials enclosing them. In other words, the causality condition must be satisfied and the energy conservation law must prevail, meaning, the radiated power must be equal to the difference of the source power and the resulting power losses in the entire structure. It is shown that the enhanced radiated power is not a physical reality, but it is a consequence of the way the problem is solved mathematically, which fails at resonances, and in the close proximity of the mode resonances. Thus, the solutions give unrealistic radiated power results, in effect violation the conservation of energy law.

2. The Antenna Geometry and its Field Problem

For simplicity, we select a dipole antenna as the radiating source. However, a conventional cylindrical dipole antenna geometry does not coincide with any orthogonal coordinate surfaces, and thus, its electromagnetic problem cannot be solved by exact analytical techniques. The problem becomes even more complex when the dipole is embedded inside multi-layer material coatings. For this reason the dipole geometry here is modified slightly, so that its outer surface coincides with a conducting sphere. In this way, the electromagnetic fields in concentric rings of materials surrounding the conducting sphere, and the space outside, can be expressed in terms of the spherical wave functions, i.e. spherical modes, satisfying proper orthogonality conditions. Thus, the unknown mode coefficients can be determined using the conventional boundary value problem solution techniques, involving the application of orthogonality conditions and the excitation source. The latter entails a circumferential slot on the sphere surface, which is equivalent to a gap voltage feed source in conventional dipole antennas, Fig. 1. Note that such a spherical dipole is a practical antenna and can be fabricated easily and fed using a coaxial transmission line. For a complete solution, all permitted spherical modes of the discrete spectrum must be included in the series. However, as in other practical problems the series involving the solution modes converge rapidly and the unknown mode coefficients can be determined readily. For spherical antennas, it can be shown that the number of terms contributing realistically to the solution are those having mode indices n exceeding a few integers beyond the sphere size ka, where k is the propagation constant in the space enclosing the sphere and a is the sphere radius.

For practical antennas the dipole length is usually about half wavelength, but its length can be allowed to increase, if more directive gain is required. This range of dipole length corresponds to a sphere size of about ka = 1.0, which will be considered in the present study. Also, the geometry of Fig.

1, shows an asymmetric azimuthal slot to feed the dipole, i.e. a gap voltage feed. This is considered as the general case of the spherical antenna. For a symmetric excitation the feeding slot can be moved to the equatorial plane of the antenna by setting θ_o = 90 degrees, without affecting the solution equations. It will influence only the excitation term.

As shown in Fig. 1, the spherical dipole is coated with two concentric layers of homogeneous materials of thicknesses d_1 and d_2. Their permittivity and permeability can be selected arbitrarily to study the problem of resonances and the radiated power anomalies. However, the case of spherical antennas coated with a single layer of homogeneous material has been investigated in [2]. To address the problem involving metamaterials, we consider here a double layer coating, so that layered resonances with varying coating thicknesses can be studied. In the geometry of Fig. 1, therefore, one layer may be a single or double negative metamaterial, and the other a double positive natural material. In this manner, layer resonances with diminishing thicknesses can be obtained and studied. This is done in this paper, and a few results are presented and discussed.

In traditional method of solving electromagnetic problems using separation of variables, here in terms of the spherical wave functions, and after truncating the solution to a few terms beyond the integer value of ka, the wave equation can be cast into a matrix equation like,

$$[\mathbf{Z}]\ [\mathbf{C}] = [\mathbf{F}] \tag{1}$$

where the $n \times n$ square matrix **Z**, is known as the impedance matrix involving spherical Bessel functions, $n \times 1$ matrix **C** contains the unknown coefficients of the solution modes, and **F** is the $n \times 1$ excitation matrix. A solution of (1) yields the unknown coefficients **C**, and thus the field solutions in the entire space outside the spherical dipole antenna. Equation (1) may be solved by any method, already known for the solution of simultaneous equations. However, it is clear that the results will depend on the excitation matrix **F** multiplied by the inverse of the matrix **Z**, or using the steps of common matrix inversion, division by the determinant of the matrix **Z**. In other words, in solving for the unknown coefficients **C** the excitation matrix will be divided by the determinant of the matrix **Z**. It is obvious then that if this determinant goes to zero, the magnitudes of the solved coefficients **C** increase dramatically. In practice, the determinant of the matrix **Z** need not be exactly zero to cause difficulty in solving for the coefficients **C**. As this determinant approaches zero, the condition-number of the matrix equation deteriorates and the solution errors become severe, rendering its solution useless. This problem was previously investigated for single layer coated spheres, where it was shown how the zeros, or quasi zeros of the matrix **Z**, can cause excessive power radiations [2].

The problem of finding the solution to equation (1) at the zeros of matrix **Z**, can be overcome by noting that setting the determinant of **Z** equal to zero makes it the dispersion equation (or transcendental characteristic equation) of the structure in Fig. 1, and its solutions are the resonant modes inside the coating layers. At these resonances the coefficients of the resonant modes far exceed those of the non-resonant ones and the matrix equation (1) reduces to a single term representing the resonant mode, with its unknown coefficient. Thus, at mode resonances there is no need to solve equation (1). One only needs to represent the radiated field by the single resonant mode term and determine its unknown coefficient directly from the difference between the input and reflected powers. That is, the input power of the source and the mismatch conditions control the radiated power, as it should be, which remains finite as long as the input power is finite. An immediate consequence of this approach of solution is the realization that, at mode resonances the antenna radiation patterns and directivity, as well as all other parameters, will be decided by the resonating mode. Thus, all antenna characteristics can be determined readily and easily from the field expressions of the resonating mode. These will be illustrated next.

To demonstrate the problems of radiated power computation, Fig. 2 is included. The first layer is a thin double negative material, but the magnitude of both its permittivity and permeability varies along the left horizontal axis. The relative permittivity of the second layer is fixed at 16, but its thickness is varied along the right horizontal axis. This form of parameter selection allows the detection of mode resonances with both permittivity and thickness variations. The vertical axis is the computed radiated power. The results show basic smooth undulations of the computed power as a function of the second layer thickness, which correspond to the weak resonating modes in the second coating [2]. Superimposed on the wavy results are very sharp resonances that move towards diminishing permittivity of the first layer and the thickness of the second layer. These sharp resonances correspond to resonating modes inside the dual layer coating, where the solution of the matrix equation has failed. To show that at these resonances the resonating modes carry the bulk of the input power (assuming perfect match at the antenna port), the radiated power of different spherical modes are computed and shown in Fig 3, for different resonance conditions. In Fig. 3a, mode n =1 is near resonant. It carries practically all the radiated power. Mode n = 3 has some small contribution, but other modes are nearly zero. Note that the finite contribution of the other modes is because of the fact that we could not determine exactly the parameters for the absolute resonance of the first mode. Fig. 3b shows the results near the resonance of the n = 3 mode. It is obvious that the contribution of the other modes to the radiated power is negligible. For each one of the resonant modes, and for a wide range of coating parameter variations, the radiated power patterns are computed and shown in Fig 4. Except for the last case of n = 5 mode, the results are independent of the coating parameter, an expected result. In the last case, the resonance was not detected accurately and the results are different for different cases. More results and details of the solution technique will be presented in the conference.

3. References

[1] N. Engheta and R. W. Ziolkowski, " Metamaterials physics and Engineering explorations" IEEE Press, John Wiley & Sons Inc , New Jersey, 2006

[2] L. Shafai and R. K. Chugh, "Resonance effects in slotted spherical antennas coated with homogeneous materials," *Can. J. Phys*., vol. 51, pp. 2341-2346, 1973.

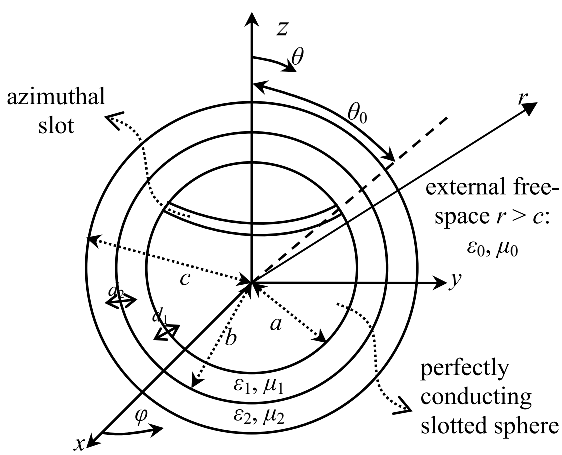

Fig. 1: Geometry of a spherical dipole with a narrow azimuthal excitation slot, and coated with two layers of homogeneous materials.

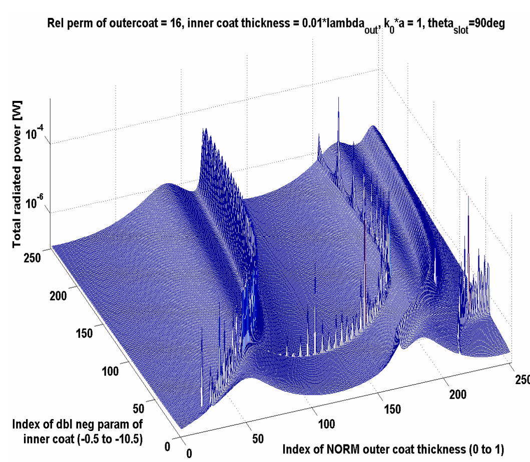

Fig. 2: Total radiated power as a function of the **double negative parameter** ($\varepsilon_1 = \mu_1 < 0$) of the inner layer and thickness of the outer layer, for ε_{r2} = 16. The outer coat thickness d_2 normalized to its dielectric wavelength $\lambda_2 = 1/[f\sqrt{(\mu_2\varepsilon_2)}]$ ranges from 0 to 1. Inner coat thickness $d_1 = 0.01\lambda_2$.

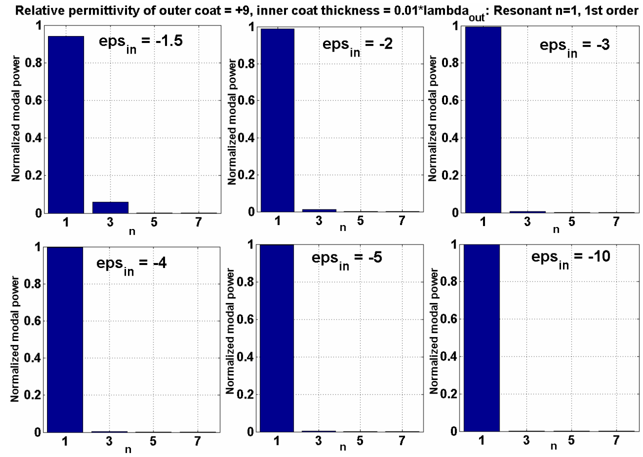

Fig. 3a Modal powers for various n modes, when $\boldsymbol{n}$ **= 1 mode is resonant**, with $\varepsilon_{r2} = +9$, $d_1 = 0.01\lambda_2$, $\varepsilon_1 < 0$.

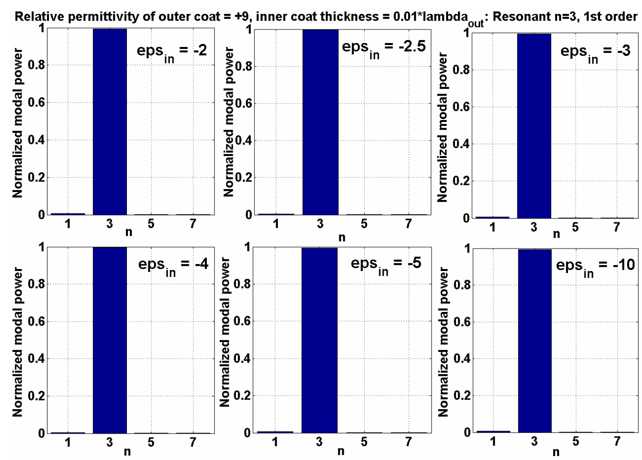

Fig. 3b Modal powers for various n modes, when $\boldsymbol{n}$ **= 3 mode is resonant**, with $\varepsilon_{r2} = +9$, $d_1 = 0.01\lambda_2$, $\varepsilon_1 < 0$.

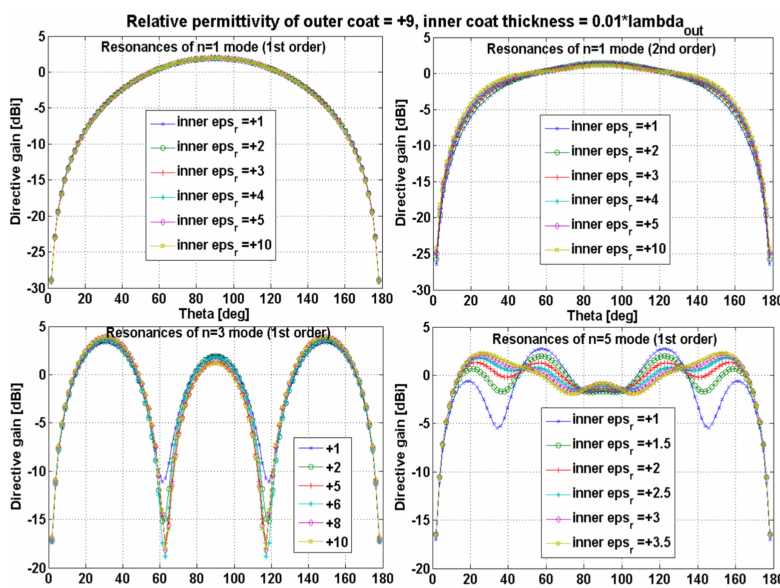

Fig. 4. Directive gain patterns for selected points along resonant modes, $d_1 = 0.01\lambda_2$ and $\varepsilon_1 > 0$. Each cluster of plots corresponds to a certain ε_2, each subplot pertains to a certain n-mode.