# On the resonant transmission width of the narrow gap between two overlapped half-plane conductors 

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## 1. Introduction

The various problems [1-3] of electromagnetic coupling from one region to another through a transmission-cavity-resonance (TCR) structure have been dealt with in the electromagnetic community. The transmission cavity in these problems can take the structures of two types; one type [1] is the one cavity formed by two conducting transverse walls having a small coupling hole (or iris) in them as in the aperture coupled microwave band pass filter. In contrast to this type, the open ends (corresponding to slit or aperture) at both extreme ends of the cavity region, by themselves behave like lossy magnetic walls as in [2] and in the cavity model for the microstrip patch antenna [4], and in the present work, the transmission cavity may be also formed by the partially overlapped conducting planes without necessity of using the transverse conducting walls with a small hole in them.

The transmission resonance problem through the cavity of the former type was first considered by Leviatan [1] and that of the latter type was investigated by Harrington [2]. Some years later, it was reported that the transmission resonance phenomena similar to those in the prior works [1-2] are observed also in the seemingly quite different structure which is composed of two slot perforated parallel conducting planes [3]. Motivated by this report [3], we investigated the reason why the similar transmission resonance phenomena occur for the seemingly different structures in [2] and [3]. For this purpose, we have searched into the structure which shows the transmission resonance phenomena same as that in the previous structure [2] by introducing some suitable modifications [5] in the original structure in [3].

## 2. Theory and discussions

Fig. 1 illustrates the structure which has been searched for - the structure composed of two parallel conducting plates partially overlapped. In Fig. 1, $\vec{E}_{\text {inc }}$ and $\vec{H}_{\text {inc }}\left\{=-\hat{z} H_{0} \exp \left[j k_{0}(x \cos \phi+y \sin \phi)\right], \hat{z}:\right.$ unit vector in the $z$ direction $\}$ denote the electric and magnetic field vectors, respectively, of the incident plane wave with the incidence angle $\phi$, and $\vec{k}_{0}$ means the propagation vector. The medium constants ( $\varepsilon_{0}$ and $\mu_{0}$ ) are assumed to be the same as those of the free space, $l$ denotes the length of the overlapped region, and $d$ is the separation between two conducting planes. Here $d$ is assumed to be much smaller than the free space wavelength $\lambda_{0}$, i.e., $k_{0} d \ll 1$ so that only the TEM mode may propagate and higher order modes may exponentially decay on being generated and leaving the port $a-a^{\prime}$ in the parallel plate waveguide (PPW) region for $0 \leq x \leq d$ and $-l \leq y \leq 0$.

In this case, the equivalent circuit for the structure of Fig. 1 is given as in Fig. 2, which is of the same form as that for the narrow slit problem in the thick conducting screen [2]. In this equivalent circuit, $Y_{r}(=G+j B)$ represents the radiation admittance for both ports $a-a^{\prime}$ and $b-b^{\prime}$, which can be obtained through the use of the method in [5]. $Y_{0}\left(=1 / \eta_{0}\right)$ and $\beta\left(=k_{0}=\omega \sqrt{\mu_{0} \varepsilon_{0}}\right)$ mean, respectively, the characteristic admittance and propagation constant of the parallel plate transmission line between input port $a-a^{\prime}$ and output port $b-b^{\prime}$. $I_{i}$ means the Norton equivalent current source. For $k_{0} d \ll 1, I_{i}$ is approximately $2 H_{0}$ (i.e., $I_{i} \approx\left|\hat{n} \times 2 \vec{H}_{\text {inc }}\right|_{x=0, y=0}=2 H_{0}$; here the unit normal vector $\hat{n}=\hat{x}$ ) because, in the limiting case where $k_{0} d$ approaches near zero, $I_{i}$ converges to $2 H_{0}$ with the port $a-a^{\prime}$ shorted. Note also that for $k_{0} d \ll 1$, the radiation conductance $G\left(=1 /\left(120 \lambda_{0}\right)[\mathrm{S} / \mathrm{m}]\right)$ in Fig. 2 is identical to that for the narrow slit in a flanged PPW [2], as discussed in [5]. Only the difference between two equivalent circuits for the present and previous works [2] is that $B$ in Fig. 2 means the radiation susceptance for the open ends at port $a-a^{\prime}$ or $b-b^{\prime}$ in Fig. 1, whereas $B$ in [2] means the radiation susceptance for the flanged narrow slit. Here it is also worth while to mention that $I_{i} \approx 2 H_{0}$ irrespective of the incidence angle $\phi$. For $k_{0} d \ll 1, G \ll B$ and $|G+j B| \ll Y_{0}$. So in this case, the open ends at port $a-a^{\prime}$ and $b-b^{\prime}$ behave like the lossy magnetic wall similar to that in the cavity model in the microstrip patch antenna theory [4]. Here, 'lossy' means small amount of radiation through the lossy magnetic wall.

By use of the above equivalent circuit representation, the transmission resonance condition is obtained as follows: The total input admittance at port $a-a^{\prime}$ can be found as

$$
\begin{equation*}
Y_{i n}^{a-a^{\prime}}=G+j B+Y_{0} \frac{G+j\left(B+Y_{0} \tan \beta l\right)}{Y_{0}-B \tan \beta l+j G \tan \beta l} . \tag{1}
\end{equation*}
$$

From the usual resonance condition under which the imaginary part of the total input admittance $Y_{i n}^{a-a^{\prime}}$ vanishes, the resonant line length $l[6]$ for $k_{0} d \ll 1$ is found to be

$$
\begin{equation*}
\tan \beta l=2 Y_{0} B /\left(G^{2}+B^{2}-Y_{0}^{2}\right) \cong-2 B / Y_{0} . \tag{2}
\end{equation*}
$$

At this resonance, the total input admittance $Y_{i n}^{a-a^{\prime}}$ becomes purely real as $Y_{i n}^{a-a^{\prime}}=2 G[\mathrm{~S} / \mathrm{m}]$ and the resonant length $l_{\text {res }}$ is approximately given by

$$
\begin{equation*}
l_{\text {res }} \cong\left(n-\frac{2 B}{\pi Y_{0}}\right) \frac{\lambda_{0}}{2} \cong n \frac{\lambda_{0}}{2}, n=1,2,3, \cdots \tag{3}
\end{equation*}
$$

under the assumption that $\tan \beta l$ varies linearly in the vicinity of each zero. This has been the resonance condition for the equivalent circuit. Under this resonance condition, the power $P_{G}^{b-b^{\prime}}$ delivered to the load $G$ at port $b-b^{\prime}$, by use of the well-known circuit theory as in [2], is found to be $P_{G}^{b-b^{\prime}}=H_{0}^{2} / 2 G[\mathrm{~W} / \mathrm{m}]$. This corresponds to the power $P_{t}$ per unit length along the $z$ axis which is transmitted through the gap region and reradiated into the upper half space $(x<0)$.

As mentioned above, the Norton equivalent circuit source for $k_{0} d \ll 1$ is given by $I^{i} \cong 2 H_{0}$ regardless of the incidence angle $\phi$. So the result for $P_{t}$ has a physical significance that the power coupled into the input port and radiated into the upper half space at the resonance is constant
independently of the incident angle $\phi$. Note that the incident power density is given by $P_{i}=(1 / 2) \eta_{0} H_{0}^{2}\left[\mathrm{~W} / \mathrm{m}^{2}\right]$ with $\eta_{0}=\sqrt{\mu_{0} / \varepsilon_{0}}=120 \pi[\Omega]$. So if we define the effective gap height $d_{\text {eff }}$ as the ratio of the transmitted power $P_{t}\left(=P_{G}^{b-b^{\prime}}\right)[\mathrm{W} / \mathrm{m}]$ to the incident power density $P_{i}$, it is found to be $P_{t} / P_{i}=\lambda_{0} / \pi[\mathrm{m}]$. This result means that, at the resonance, the power incident upon the transverse width of $\left(\lambda_{0} / \pi\right)[\mathrm{m}]$ along the direction of the incident electric field vector direction on the constant phase plane, as shown in Fig. 1, is coupled into the input port $a-a^{\prime}$ and radiated into the upper half space independent of the incident angle, if the incident power density $P_{i}$ is kept constant. This result of $\left(\lambda_{0} / \pi\right)[\mathrm{m}]$ is the same as the well-known effective height of the halfwavelength dipole antenna.

For the sake of looking into this physical significance, we investigate the effective height of the present structure through the use of the general definition for the effective height of a linear receiving antenna. The definition is given as $\vec{E}_{i n c} \cdot \vec{h}_{e f f}=v_{o c}$, where $\vec{E}_{\text {inc }}\left[=\eta_{0} H_{0}(-\hat{x} \sin \phi+\hat{y} \cos \phi)=-\hat{\phi} \eta_{0} H_{0}\right]$ means the incident electric field vector of arbitrary incidence angle $\phi$ at the input port $a-a^{\prime}, \vec{h}_{e f f}$ means the effective height vector of the present structure in Fig. 1, and $v_{o c}$ means the open circuit voltage with port $a-a^{\prime}$ open circuited. Since the total admittance at port $a-a^{\prime}$ is $Y_{i n}^{a-a^{\prime}}=2 G[\mathrm{~S} / \mathrm{m}]$ under the resonance condition, the Thevenin open circuit voltage $v_{o c}$ is found to be $-H_{0} / G[\mathrm{~V}]=-120 \lambda_{0} H_{0}[\mathrm{~V}]$ from the equivalent circuit in Fig. 2, where the minus sign is used to conform with conventional polarity markings (upper terminal positive). In order for the above equation to hold, $\vec{h}_{\text {eff }}$ should be defined as $\vec{h}_{\text {eff }}=\left(\lambda_{0} / \pi\right) \hat{\phi}$. This definition is compatible with the foregoing omni-directional receiving property that the power coupled into the input port and reradiated out of the output port is given as $\lambda_{0} / \pi$ times the incident power density $P_{i}$ at the input port irrespective of the incident angle $\phi$ and the actual slit width $d$ of the input port under the transmission resonance condition $Y_{i n}^{a-a^{\prime}}=2 G$ for $l_{\text {res }} \cong 0.5 \lambda_{0}\left[n-2 B /\left(\pi Y_{0}\right)\right]$. In this sense, the result of $\left(\lambda_{0} / \pi\right) \hat{\phi}$ has the same physical meaning as that of the transmission width in [2] and the transmission resonance phenomenon is essentially the same as that in [2].
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## 3. Conclusion

The problem of electromagnetic transmission through a narrow gap region between two parallel conducting half planes partially overlapped is studied, when the gap is illuminated by a TM polarized plane wave, by use of the simple equivalent circuit. It is found that the cavity structure formed by the overlapped two conducting planes with small gap is resonated when the cavity length approaches multiples of half wavelengths. Then the transmission width of the small gap amounts up to $1 / \pi$ wavelengths irrespective of the incidence angle of the plane wave. The cavity resonance in this letter is thought to be the same as that in the transmission through a narrow slit in a thick conducting screen [2] previously studied.

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Fig. 1 Two parallel conducting planes partially overlapped


$$
I_{i}=2 H_{0}, \quad Y_{r}=G+j B, \quad l=\text { transmission line length }
$$

Fig. 2 Equivalent circuit representation for the problem in Fig. 1

