Diffraction by Composite Wedge Composed of Perfect Conductor and Lossy Dielectric: Physical Optics Solution

Se-Yun Kim
Imaging Media Research Center, Korea Institute of Science and Technology
P.O.Box 131 Cheongryang, Seoul, Korea, ksy@imrc.kist.re.kr

1. Introduction

One of canonical structures in high-frequency electromagnetic scattering problems is penetrable wedge. In this paper, we present the physical optics (PO) solution to the E-polarized diffraction by a composite wedge consisting of perfect conductor and lossy dielectric. This diffraction problem is formulated into the dual integral equations [1]. Using the modified propagation constants for non-uniform plane wave transmission through conducting media [2], one may obtain an analytic expression on the geometrical optics (GO) field including multiple reflections inside the lossy dielectric part. Replacing the exact field by the GO field on the boundaries of the composite wedge generates the PO field, which is expressed by an asymptotic integral of the PO diffraction coefficients. And the corresponding PO diffraction coefficients are expressed by a finite series of cotangent functions. Deforming the integral path into the SDP (steepest decent path) [3], one may express the PO field by sum of the GO and the edge-diffracted fields. One may find the one-to-one correspondence [4] between the ordinary rays of the GO field and the cotangent functions of the PO diffraction coefficients. Hence the PO diffraction coefficients of the composite wedges can also be constructed in the same analytic form by employing only the ray-tracing data.

But it is well recognized that the PO diffraction coefficients cannot satisfy the boundary condition at wedge interfaces and the edge condition at wedge tip. According to the formulation of the dual integral equations, the error posed in the PO solution can be checked by showing how closely the PO solution satisfies the null-field condition in the complementary region [4]. For a typical case, the PO diffraction coefficients and field patterns are plotted here.

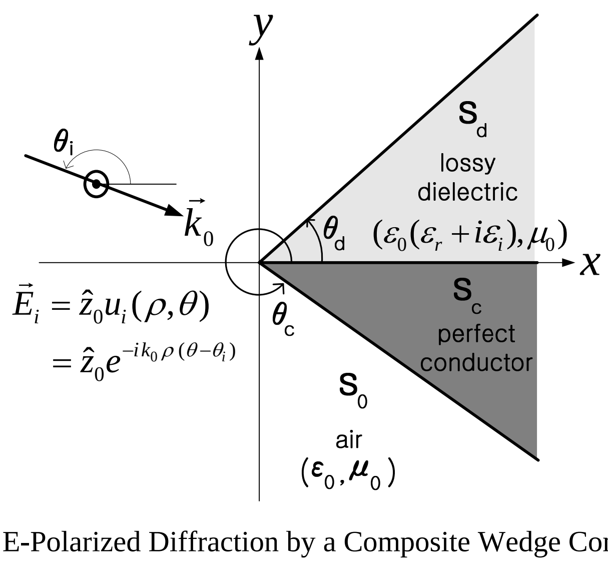
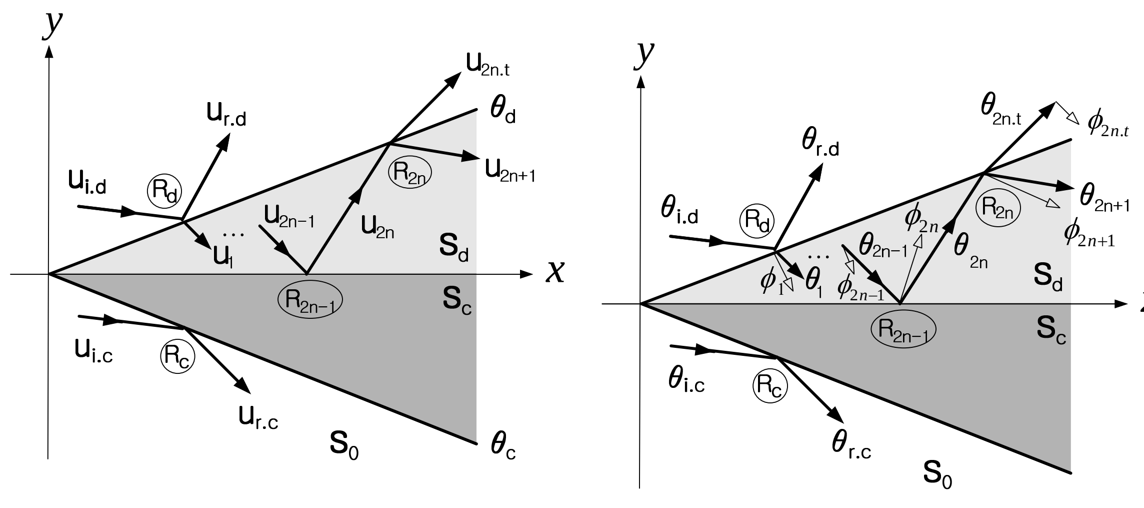


Figure 1: E-Polarized Diffraction by a Composite Wedge Composed of Perfect Conductor and Lossy Dielectric

2. Physical Optics Approximation

Fig. 1 shows the geometry of a composite wedge consisting of perfect conductor in S_c and lossy dielectric with complex relative dielectric constant $\varepsilon^* = \varepsilon_r + i\varepsilon_i$ in S_d. When an E-polarized unit plane wave $u_i(\rho,\theta)$ with an arbitrary angle θ_i is incident on the wedge, the z-component of the total electric field $u(\rho,\theta)$ may be written into the dual integral equations [1]. But there is no systematic way to solve the dual integral equations exactly. As a detour, the conventional PO approximation is employed here.

At first, ordinary rays are easily traced by employing the usual principle of GO in the physical region even if the dielectric may be lossy. The modified propagation constants for non-uniform plane wave transmission through conducting media [2] provide the GO field including the multiple reflections inside the lossy dielectric part. Figure 2(a) shows ray trajectory in case that an E-polarized unit plane wave u_i is incident on both interfaces of the composite wedge, in which the propagation angles of the incident field $u_{i.c}$ and the reflected field $u_{r.c}$ with reflection coefficient R_c are denoted by $\theta_{i.c}$ and $\theta_{r.c}$, respectively. When the incident ray $u_{i.d}$ with angle $\theta_{i.d}$ impinges on the lossy dielectric boundary at $\theta = \theta_d$, one may easily trace the reflected ray $u_{r.d}$ with angle $\theta_{r.d}$. However, as shown in Figure 2(b), the refracted ray u_1 propagates non-uniformly inside the lossy dielectric with real angle θ_1 and imaginary angle ϕ_1. Let u_{2n} denote the ray undergoing the (2n-1)-th internal reflection from u_1. The ray u_{2n} with real angle θ_{2n} and imaginary angle ϕ_{2n} impinges on the dielectric boundary, and then generates the reflected ray u_{2n+1} and the refracted ray $u_{2n.t}$. The corresponding real and imaginary angles of the reflected and refracted rays are easily determined by the Snell's law.



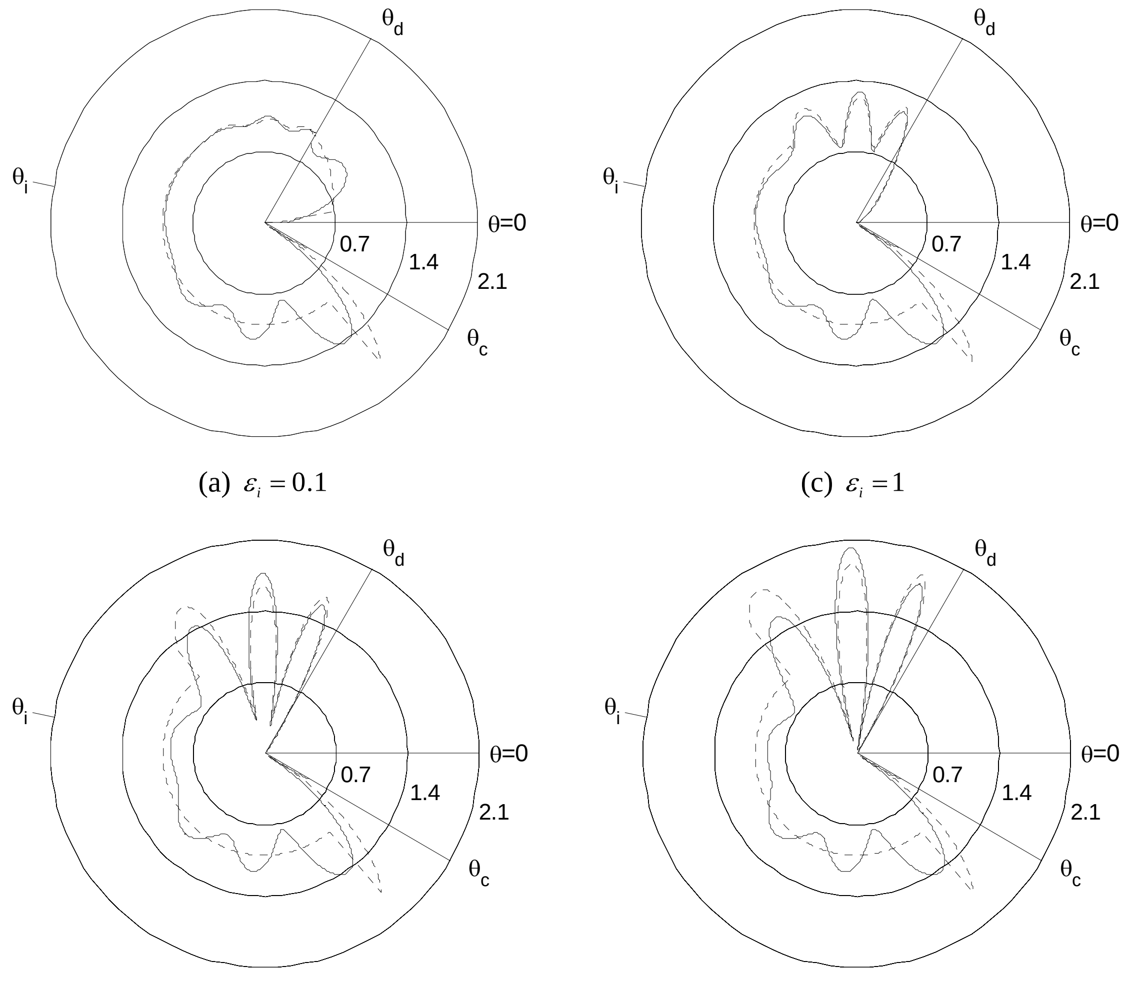
(a) GO Rays (b) Complex Propagation Angles of GO Rays

Figure 2: Ordinary Ray-Tracing

The uniform currents induced on the wedge interfaces are derived from GO field. The complete PO solution could be obtained analytically by applying the uniform currents into the dual integral equations approximately. The PO diffraction coefficients are given by finite series of the cotangent functions, which correspond to the ordinary rays of GO field one by one. In a reverse sense, the construction of the PO diffraction coefficients from the ordinary ray-tracing data can also be implemented according the following rule [4]. At first, ordinary rays are traced by employing the usual principle of GO. And then, if an ordinary ray $u_n(\rho,\theta)$ with complex propagation angle θ_n^* and amplitude K_n exists, the corresponding component of the PO diffraction coefficients can be expressed directly by $(-1)^{\tau} \frac{1}{2} K_n \cot(\frac{w-\theta_n^*}{2})$. It should be noted that τ is taken by 0 or 1 in case that the normal direction from the boundary illuminated by the geometrical ray $u_n(\rho,\theta)$ to the physical region is positive or negative θ , respectively. It implies that the PO diffraction coefficients of penetrable wedges can be constructed in analytic form only by employing the ray-tracing.

3. Diffraction Coefficients and Field Patterns

As a typical example in Fig. 1, the composite wedge composed of lossy dielectric ($\theta_d = 60^\circ$, $\varepsilon_r = 1.01$ and $\varepsilon_i = 0.1 \sim 100$) and perfect conductor ($\theta_c = 330^\circ$) is illuminated by an E-polarized unit plane wave ($\theta_i = 170^\circ$). Fig. 3 shows the amplitude patterns of GO and PO fields at 1.5λ (wavelength) away from the edge. The dotted lines in Figs. 3(a)-(d) denote the GO fields, which become continuous at the dielectric boundary but suffer from abrupt jump at the transition angles. The PO field patterns may be easily calculated by evaluating the uniform asymptotic integral [4] for the PO diffraction coefficients. The PO field patterns, plotted by bold lines in Figs. 3(a)-(d), become smooth near the transition angles. But it is well known that PO fields cannot satisfy the boundary condition at wedge interfaces.



(a) $\varepsilon_i = 0.1$ (c) $\varepsilon_i = 1$

(c) $\varepsilon_i = 10$ (d) $\varepsilon_i = 100$

Figure 3: Field Patterns at 1.5λ away from the Wedge Tip for
$\theta_d = 60^\circ$, $\theta_c = 330^\circ$, $\theta_i = 170^\circ$, and $\varepsilon_r = 1.01$
(Dotted: GO, Bold: PO)

Fig. 4 shows the PO diffraction coefficients in the air regions. The broken and dotted lines marked by $\varepsilon_i = 0$ and ∞ denote the exact diffraction coefficients of the perfectly conducting wedges with $\theta_d = 0^\circ$ and 60°, respectively. As expected, the PO diffraction coefficients cannot satisfy the boundary condition at the conducting boundary. And the PO patterns intersect the exact solution near the dielectric boundary and transition angle. Based on the formulation of the dual integral equations, the accuracy of the diffraction coefficients can be verified by showing how well the null-field condition is satisfied in the complementary region [1]. According to this criterion, Fig. 4 shows PO diffraction coefficients erroneous in the complementary air regions $S_d^{(0)}$ and $S_c^{(0)}$.

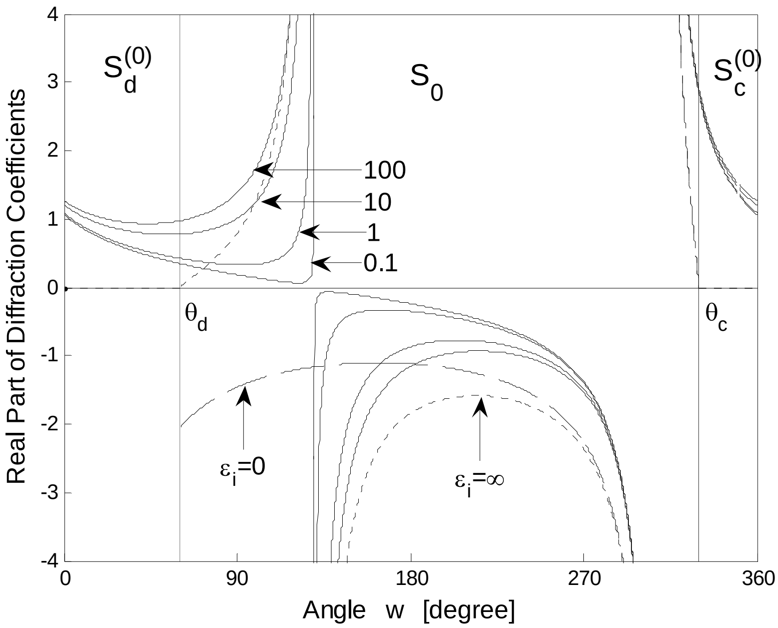


Figure 4: PO Diffraction Coefficients in the Physical and Complementary Air Regions for $\theta_d = 60^\circ$, $\theta_c = 330^\circ$, $\theta_i = 170^\circ$, $\varepsilon_r = 1.01$, and $\varepsilon_i = 0.1 \sim 100$
(Broken: Exact for $\varepsilon_r = 1$ and $\varepsilon_i = 0$, and Dotted: Exact for $\varepsilon_i = \infty$)

4. Conclusion

For an E-polarized diffraction by a composite wedge consisting of perfect conductor and lossy dielectric, the PO diffraction coefficients were constructed by employing only the ordinary ray-tracing data. The PO solution provides smooth behavior near the transition angles, but violates the boundary condition at the wedge interfaces. The error posed in the PO diffraction coefficients was accounted by showing how closely the null-field condition is satisfied in the complementary region. The method of hidden rays proposed in [4] may play an important role in correcting the error posed in the PO diffraction coefficients.

References

[1] S.-Y. Kim, J. W. Ra, S. Y. Shin, “Diffraction by an arbitrary-angled dielectric wedge”, IEEE Trans. Antennas Propagat., Vol. 39, No. 9, pp.1272-1292, 1991.

[2] R. D. Radcliff, C. A. Balanis, “Modified propagation constants for nonuniform plane wave transmission through conducting media”, IEEE Trans. Geosci. Remote Sensing, Vol. 20, No. 3, pp. 408-411, 1982.

[3] L. B. Felsen, N. Marcuvitz, Radiation and Scattering of Waves, Prentice-Hall, 1973.

[4] S.-Y. Kim, “Hidden rays of diffraction”, IEEE Trans. Antennas Propagat., Vol. 55, No. 3, pp. 892-906, 2007.