Electromagnetic Modeling and Analysis of 2D Comb Structured MEMS Capacitor with Accelerated Motion

Shafrida Sahrani[1], Michiko Kuroda[2]
[1]Faculty of Engineering, Universiti Malaysia Sarawak,
94300 Kota Samarahan, Sarawak, Malaysia, sshafrida@feng.unimas.my
[2]School of Computer Science, Tokyo University of Technology,
1404 Katakura, Hachioji, Tokyo 192-0982 Japan, kuroda@cs.teu.ac.jp

1. Introduction

The applications in RF MEMS especially for wireless and microwave devices have seen an amazing growth in the past 10 years. The nature of RF MEMS technology and its diversity of useful applications make it potentially a far more pervasive technology than even integrated circuits microchips. As compared to the semiconductors switches, PIN diode and FET, RF MEMS have many excellent advantages such as high isolation, high bandwidth and low power consumption [1], [2], [3]. Ever since RF MEMS technology is still new in these decades, the numerical and dynamic analysis of RF MEMS components seems to be critical issues that need to be solved immediately. To solve this problem, an improved and efficient knowledge of the electromagnetic field distribution around a moving or rotating body is required. But due to the limitations of the conventional numerical techniques for the time changing boundaries, it is computationally tedious and time consuming to solve these problems numerically for the electromagnetic fields.

In this paper, an efficient numerical approach, which is a combination of FDTD method [4] and the body-fitted grid generation method with moving boundaries [5], is presented for the analysis of 2D comb structured MEMS variable capacitors with accelerated motions.These comb structured variable capacitors consists of two plates, which is the static plate and the movable plate that suspended with the spring. By using this numerical technique, it is possible to analyze the combined effect of the mechanical and electrical forces. The relation of the acceleration of the plate and the bias voltage are derived and the theoretical and the numerical results are agreed very well. The relation between the oscillations of the frequency with the acceleration is also shown.

2. Modeling of The Proposed 2D MEMS Capacitor

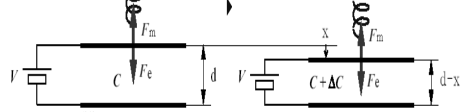


Fig 1. Model of MEMS

A schematic diagram of the proposed MEMS capacitor is shown in Fig. 1. The upper plate was suspended by a spring. A bias voltage has been applied between the two plates and it generates an electrostatic force, F_e. Under the combined effect with mechanical force, F_m, the upper plate is deflected towards the bottom plate until the equilibrium between the two forces is reached and acceleration x'' can be derived as [1]

$$x'' = \frac{1}{m}\left(\frac{1}{2}\frac{\partial C}{\partial x}V^2 - bx' - kx\right) \qquad (1)$$

where b is the damping coefficient, m is the mass of the plate, k is the spring constant while, V is the bias voltage.

3. Coordinate Transform Technique

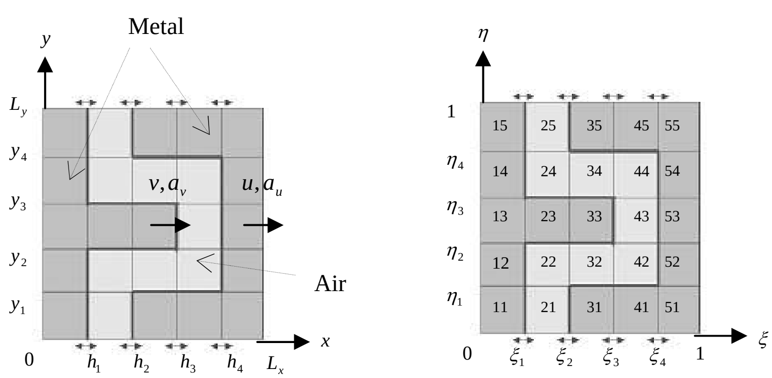
Based on the modeling of MEMS variable capacitor structure in Fig. 1, the physical region is transformed into a computational region as shown in Fig. 2. Here, the plates are assumed to move for x direction with velocities v and u, and the acceleration a_v and a_u respectively. For 2D TM-propagation case, $H_x = 0$ and $\partial/\partial z = 0$, there are only components with a time variation given by the following equations

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \qquad (2)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - J_x\right) \qquad (3)$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} + J_y\right) \qquad (4)$$

where ε and μ are the constitutive parameters of the respective medium.



(a) (b)

Fig. 2 (a) Physical region and (b) Computational region

The transform equation between the physical region and the computational regions are

$$\xi = \frac{x - h_n(t)}{h_{n+1}(t) - h_n(t)} \quad (5)$$

$$\eta = \frac{y - y_m}{y_{m+1} - y_m} \quad (6)$$

$$\tau = t \quad (7)$$

where $n = 1,2,3$ $m = 1,2,3$ and the transformation functions are written as follows.

$$h_1(t) = x_1 + vt + \frac{1}{2} a_v t \quad (8)$$

$$h_2(t) = x_2 + ut + \frac{1}{2} a_u t \quad (9)$$

$$h_3(t) = x_3 + vt + \frac{1}{2} a_v t \quad (10)$$

$$h_4(t) = x_4 + ut + \frac{1}{2} a_u t \quad (11)$$

assuming that the plate accelerations, velocities remain time changing values for the whole time of their motion. The functions $h_1(t)$, $h_2(t)$, $h_3(t)$, $h_4(t)$, describe the movement along the x axis and allow for the realization of a rectangular grid with stationary boundary conditions. To ensure the accuracy of computed results and the stability of FDTD method, the stability criterion is chosen as $c\Delta t \le \frac{\delta}{\sqrt{2}}$, where the space increment, δ must be chosen the minimum cell size.

4. Numerical Result

In order to illustrate the new approach, the left plate is assumed to move due to the coupling of the electrostatic and the mechanical forces. The grid includes 200×200 cells, input frequency is $f = 20GHz$, grid length is $L_x = L_y = L = 5\lambda$, space interval is $\Delta x = \Delta y = L/200$, and time interval is $\Delta t = 3.125 \times 10^{-13}$ (sec). The grid is terminated with Mur's absorbing boundary conditions [6].

In this numerical analysis, initial values of the velocity are assumed as $u_0 = 0$, $v_0 = 0$ and $a_u = 0$. The numerical results are shown in Fig. 3 and Fig. 4. Fig.3 shows the acceleration of the plate when the plate moves away each other. The mechanical resonant frequency is $\omega = \sqrt{k/m} = 1.0 \times 10^{10} (rad/\text{sec})$, and normalized bias voltage is $V^2/m = 1.0 \times 10^{10}$ with no damping co-efficient, $b = 0$. The numerical results are compared with the theoretical results and it shows that both are agreed well. Fig.4 presents the acceleration versus time for a sinusoidal modulation at a frequency values in the range of $\omega = 1.0 \times 10^9$ to $\omega = 1.8 \times 10^9$ with normalized bias voltage $V^2/m = 1 \times 10^{13}$ when the plate moves away each other. At low moving frequency $\omega = 1.0 \times 10^9$, the oscillations of the accelerations are slower than in high moving frequency $\omega = 1.8 \times 10^9$. This shows that the greater frequency of the moving plates, the accelerations become faster and larger with the same normalized bias voltage.

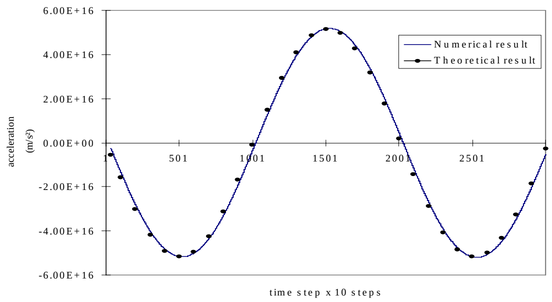


Fig. 3 The comparison of the numerical and the theoretical result for the acceleration of the plate.

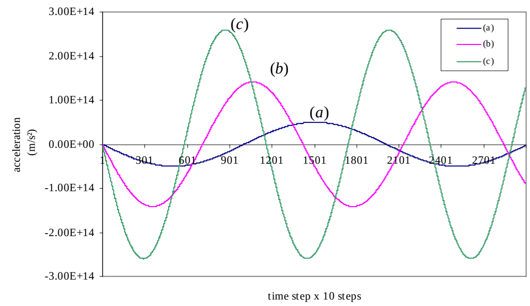


Fig. 4 Time dependence of the acceleration for each frequency, $V^2 / m = 1 \times 10^{13}$

(a) $\omega = 1.0 \times 10^9$ (b) $\omega = 1.4 \times 10^9$ (c) $\omega = 1.8 \times 10^9$

5. Conclusions

The advancement in new numerical technique is the key to success of newer generation RF MEMS devices. In this paper, a novel time-domain modeling technique that has the capability to accurately simulate the transient effect of RF MEMS variable capacitors with accelerated motion controlled by the coupling of the electrostatic and mechanical forces is presented. The relation between the sinusoidal modulations of the frequency with the acceleration is shown. Its validity has been demonstrated in comparison between the computational results of the displacement with the theoretical results. Both results are in very good agreement. The next work will consider to include the damping coefficient. Due to its numerical efficiency, the proposed technique can be a useful technique, which makes it suitable for the numerical analysis of the moving boundary problem in the near future.

6. References

[1] Gabriel M. Rebeiz, *RF MEMS Theory, Design and Technology* ,John Wiley & Sons Publication, 2003

[2] Vijay K. Varadan K.J. Vinoy, K.A. Jose, *RF MEMS and Their Applications*, John Wiley & Sons Publication, 2003

[3] Laurent Dussopt, Gabriel M. Rebeiz, "Intermodulation Distortion and Power Handling in RF MEMS Switches, Varactors and Tunable Filters", *IEEE Transactions on Microwave Theory and Techniques*, Vol. 51, No.4, April 2003

[4] Dennis M. Sullivan, *Electromagnetic Simulation Using the FDTD Method*, IEEE Press Series on RF and Microwave Technology

[5] Michiko Kuroda, Noriyuki Miura, Manos Tentzeris, "A Novel Numerical Approach for the Analysis of 2D MEMS-Based Variable Capacitors Including the Effect of Arbitrary Motions ", *ACES Journal* , vol19, no.1b, pp133-138, March 2004

[6] Gerrit Mur, "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic-Field Equations", *IEEE Transactions on Electromagnetic Compatibility*, Vol. EMC-23, No.4, November 1981