Microwave Multi-Level Band-Pass Filter Using Discrete-Time Yule-Walker Method

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***Abstract* — The modified Yule-Walker scheme is employed to define arbitrary transmission scattering parameter of a multiple-level band-pass filter in the discrete-time domain. Equal-length microstrips including single- and double-section stubs are used to implement the multiple-level band-pass filter. Experimental results are presented to illustrate the validity of this design method.**

***Index Terms* — Yule-Walker scheme, multiple-level filter, Z domain.**

I. INTRODUCTION

Microwave filters [1-5] are two-port networks used to allow transmission of signals over the pass-band and reject all signals over the stop-band. Typical frequency-domain filters include low-pass, high-pass, band-pass and band-stop filters. Conventional methods to design and implement microwave filters begin with lumped-element prototypes. Richard's transformation [2] and Kuroda's identities [3], are then used to transform lumped-element values to the corresponding transmission-line circuits. Butterworth, Chebyshev and elliptical filters are three popular prototypes used in filter design. Each of three has its distinct characteristic that meets specific applications. The motivation of this work is to explore the design of filter having arbitrary frequency response. In other words, the goal is to develop a filter design method for a given arbitrary frequency-domain response.

The modified Yule-Walker equation [6-7] is commonly used in statistics for estimating the time domain parameters of an autoregressive process. But there is still no filter design using modified Yule-Walker equation. The modified Yule-Walker scheme defines explicitly arbitrary responses of filters. In this letter, we limit our focus on the multiple-level band-pass filter. We begin with a multiple-level band-pass filter having arbitrary frequency-domain response. The Yule-Walker scheme converts the frequency-domain response into the Z-domain response [8]. To implement a filter having transfer function in the Z domain, an equal-length

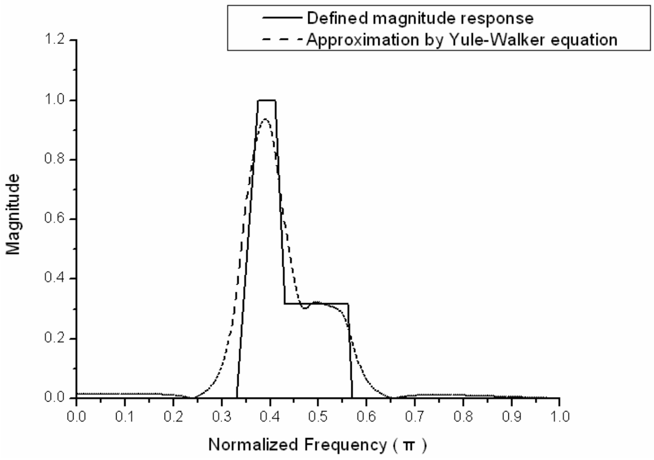


Fig 1: Defined magnitude response of two-level band-pass filter and its Yule-Walker approximation.

transmission-line configuration is proposed to emulate the multiple-level filter. Both numerical and measured results are presented to illustrate the validity of this design method.

II. TWO-LEVEL BAND-PASS FILTER USING MODIFIED YULE-WALKER EQUATION

Fig. 1 depicts a two-level band-pass filter with amplitude responses as m = [0, 0, 1, 1, 0.317, 0.317, 0, 0] at the normalized frequencies f = [0, 0.33, 0.375, 0.41, 0.43, 0.56, 0.57, 1]. As shown in Fig.1, two pass-bands occur at 0.375-0.41 and 0.43-0.56. In particular, two pass-bands represent a difference of 10dB in amplitude response. Upon using the command statement [8],

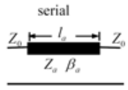
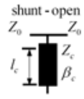
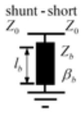
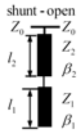
[b,a] = yulewalk (f,m);

we obtain the system function of the band-pass filter in the Z domain, which is as follows

$$F(z)=\frac{\sum_{j=0}^{9} b_j z^{-j}}{\sum_{i=0}^{9} a_i z^{-i}}, \quad (1)$$

Table I Chain-Scattering-Parameter Matrices of Basic Transmission lines

Table I Basic Transmission Line

	$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_z$	
serial 	$\frac{1}{z^{-1/2}(1-\Gamma^2)}\begin{bmatrix} 1-\Gamma^2 z^{-1} & -(\Gamma-\Gamma z^{-1}) \\ \Gamma-\Gamma z^{-1} & -\Gamma^2+z^{-1} \end{bmatrix}$	where $\Gamma=\frac{Z_a-Z_0}{Z_a+Z_0}$
shunt - open 	$\frac{1}{1+z^{-1}}\begin{bmatrix} (1+c)+(1-c)z^{-1} & c-cz^{-1} \\ -c+cz^{-1} & (1-c)+(1+c)z^{-1} \end{bmatrix}$	where $c=\frac{Z_0}{2Z_c}$
shunt - short 	$\frac{1}{1-z^{-1}}\begin{bmatrix} (1+c)-(1-c)z^{-1} & c+cz^{-1} \\ -c-cz^{-1} & (1-c)-(1+c)z^{-1} \end{bmatrix}$	where $c=\frac{Z_0}{2Z_b}$
shunt - open two - section 	$\frac{1}{D(z)}\begin{bmatrix} \frac{Z_0C(z)+2Z_2D(z)}{2Z_2} & \frac{Z_0C(z)}{2Z_2} \\ -\frac{Z_0C(z)}{2Z_2} & \frac{-Z_0C(z)+2Z_2D(z)}{2Z_2} \end{bmatrix}$	where $C(z)=1-z^{-2}$, $D(z)=1+2\gamma z^{-1}+z^{-2}$, $\gamma=\frac{Z_1-Z_2}{Z_1+Z_2}$

where $\{b_j, 0 \leqq b_j \leqq 9\}$ = {0.0269, -0.0097, 0.0207, -0.0020, 0.0069, 0.0185, -0.0105, 0.0215, -0.0152, 0.0015 } and $\{a_i, 0 \leqq a_i \leqq 9\}$ = {1.0000, -1.2446, 3.5043, -2.9885, 4.4439, -2.5480, 2.4624, -0.8013, 0.4962, -0.0248 }. Fig. 1 also shows the amplitude response of equation (1), which deviates slightly from the original amplitude response. We may select higher-order values of (*i,j*) to obtain *F(z)* so that *F(z)* will fit better the original amplitude response. However, higher-order values of (*i,j*) will lead to a more complex circuit when *F(z)* is implemented with transmission lines. Equation (1) represents the system function of arbitrary two-level band-pass filter. Notice that the amplitude response $|F(z)|$ in Fig. 1 is symmetric with respect to $\Omega = \pi$ with Ω being the normalized frequency. $|F(z)|$ is also a periodic function with a period of 2π.

III. EQUAL-LENGTH TRANSMISSION LINES IN THE Z DOAMIN

Table I [5] shows the chain-scattering parameters matrices of basic transmission lines, namely, series line, shunt-open stub, shunt-short stub, and shunt-open two-section stub. We assume that each of finite sections has the same electric length with $\beta_i\ l_i = \omega\tau$, where ω is the angular frequency and τ is the propagation delay time of each finite line. The discrete-time domain response of transmission-line configuration is obtained by setting $z^{-1} = exp\ (-j2\beta_i\ l_i)$, where Z_i, β_i, and l_i (i=1, 2, a, b, c) are the characteristic impedance, propagation constant, and physical length, respectively. Notice that Z_0 is the reference characteristic impedance, which is assumed to be 50Ω.

The chain scattering matrix of a cascaded network consisting of series-shunt transmission lines is the sequential multiplication of the chain scattering matrix of each fundamental element. It is given by

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{network} = \prod_{i=1}^{P} \begin{bmatrix} T_{11}^{i} & T_{12}^{i} \\ T_{21}^{i} & T_{22}^{i} \end{bmatrix}, \qquad (2)$$

where P is number of the fundamental elements, and T_{11}^{i}, T_{12}^{i} ,T_{21}^{i}, T_{22}^{i} are the matrix elements representing the i-th element. If a network consists of K two-section open-circuited stubs, L series sections, M short stubs, and N open stubs, the transfer function $T_{11,network}(z)$ of the overall network is given as follows in equation (3), where all a_i are real and are determined by the characteristic impedances of all series and shunted components. When the output port of the cascade network is properly terminated, the transfer function $T(z)$ of such a network becomes equation (4), where $z^{-L/2}$ represents time delay of series lines and

$$A_i = \frac{a_i}{\prod_{l=1}^{L}(1-\Gamma_m^2)},\ 0 \leqq i \leqq 2K+L+M+N \qquad (5)$$

is a function of the characteristic impedances of all components involved. Both system function $F(z)$ of two-level band-pass filter and transfer function $T(z)$ of transmission-line network are expressed in terms of Z parameters. In order to implement a two-level band-pass filter with a transmission–line network, we set

$$F(z) = T(z). \qquad (6)$$

Notice that A_i in $T(z)$ is a function of characteristic impedances of transmission lines. The task is, therefore, to adjust characteristic impedances of transmission lines so that $T(z)$ is as close to $F(z)$ as

$$T_{11,network}(z) = \frac{\sum_{i=0}^{2K+L+M+N} a_i z^{-i}}{\prod_{k=1}^{K}(1+2\gamma\, kz^{-1}+z^{-2})\prod_{l=1}^{L} z^{-L/2}(1-\Gamma_l^2)\prod_{m=1}^{M}(1-z^{-1})\prod_{n=1}^{N}(1+z^{-1})} \qquad (3)$$

$$T(z) = \frac{1}{T_{11,network}(z)} = z^{-L/2}\frac{\prod_{k=1}^{K}(1+2\gamma\, z^{-1}+z^{-2})\prod_{m=1}^{M}(1-z^{-1})\prod_{n=1}^{N}(1+z^{-1})}{\sum_{i=0}^{2K+L+M+N} A_i z^{-i}} \qquad (4)$$

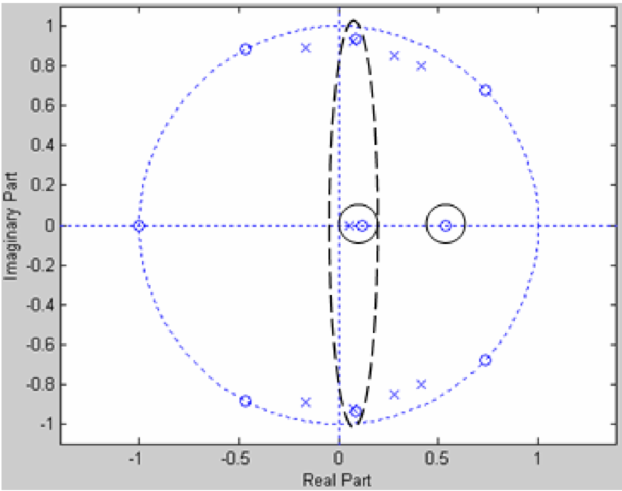


Fig 2: Pole-zero locations of band-pass filter in the z plane.

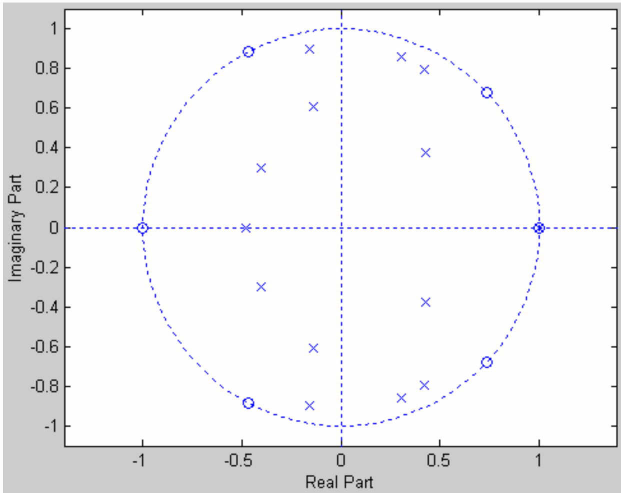


Fig 3: Pole-zero locations after the removal of zeros not located on the unit circle.

possible in the sense of least square error [5].

IV. IMPLEMENTATION OF FILTER

Fig. 2 shows pole-zero locations of equation (1) in the complex z plane. There are four zeros not located on the unit circle, which are z = 0.53531, 0.12039 and 0.087959±0.93833i. We rearrange equation (1) and the system function of the two-level filter is as follows in equation (7). Notice that all zeros located on the unit circle can be realized with shunt stubs [4]. On the other hand, zeros not located on the unit circle cannot be implemented with simple transmission lines. In order to remove these zeros from the numerator, we divide both numerator and denominator of equation (7) with the factor $[(1-0.53531z^{-1})\times(1-0.12039z^{-1})\times(1-0.175918z^{-1}+0.8882z^{-2})]$. In addition, to introduce a zero at DC, we multiply both numerator and denominator of equation (7) with the factor $(1-z^{-1})$. After the algebraic manipulation, we obtain the system function of two-level band-pass filter as follows

$$F(z)=\frac{\sum_{j=0}^{6} b_j z^{-j}}{\sum_{i=0}^{14} a_i z^{-i}}, \qquad (8)$$

where $\{b_j, 0\leqq b_j\leqq 6\}$ = {0.0269, -0.0142, -0.0101, 0.0000, 0.0098, 0.0145, -0.0268} and $\{a_i, 0\leqq a_i\leqq 14\}$ = {1.0000, -1.4130, 2.5059, -2.3061, 1.9421, -1.3451, 0.3049, -0.2884, -0.1778, -0.1028, -0.0595, -0.0287, -0.0112, -0.0081, -0.0084}. Fig.3 depicts the pole-zero locations of equation (8) after the removal of zeros not located on the

$$F(z)=\frac{b(z)}{a(z)}=\frac{\sum_{j=0}^{5} b_j z^{-j}(1\text{-}0.53531z^{-1})(1\text{-}0.12039z^{-1})(1\text{-}0.175918z^{-1}+0.8882z^{-2})}{\sum_{i=0}^{9} a_i z^{-i}} \qquad (7)$$

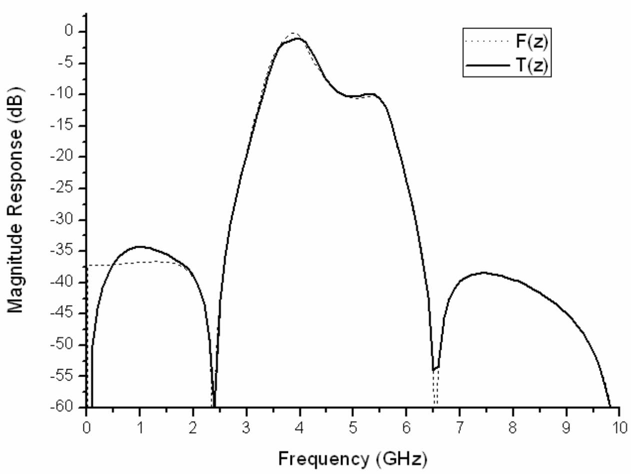


Fig. 4: Amplitude responses of F(z) and T(z) for the two-level filter.

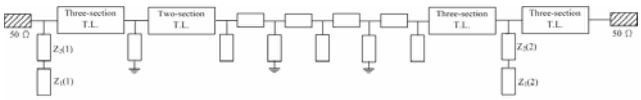


Fig 5: Configuration of two-level band-pass filter.

unit circle. Notice that the amplitude response of equation (8) is the same as that of equation (7). Fig. 4 shows the amplitude response of equation (8) when the normalizing frequencies is 10 GHz. The modified system function of equation (8) is employed to implement the two-level band-pass filter by employing equal-length transmission lines. Fig. 5 shows the configuration of transmission-line network used to synthesize the filter, of which K = 2, L = 15, M = 3, and N = 3. There are two two-section open stubs, namely, ($Z_2(1)$, $Z_1(1)$) and ($Z_2(2)$, $Z_1(2)$), which are used to implement transmission zeros at 2.43 GHz and 6.6 GHz, respectively [4]. Three one-section short stubs and three one-section open stubs are used to produce transmission zeros at DC and 10 GHz, respectively.

Upon using the requirement set by equation (6) and optimization process [5], we obtain the characteristic impedances of transmission lines. The characteristic impedances of short stubs, from left side to right side in Fig. 5, are 160, 27.282,

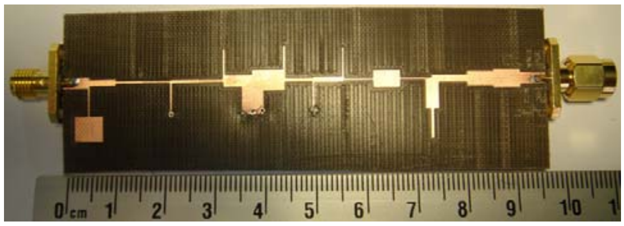
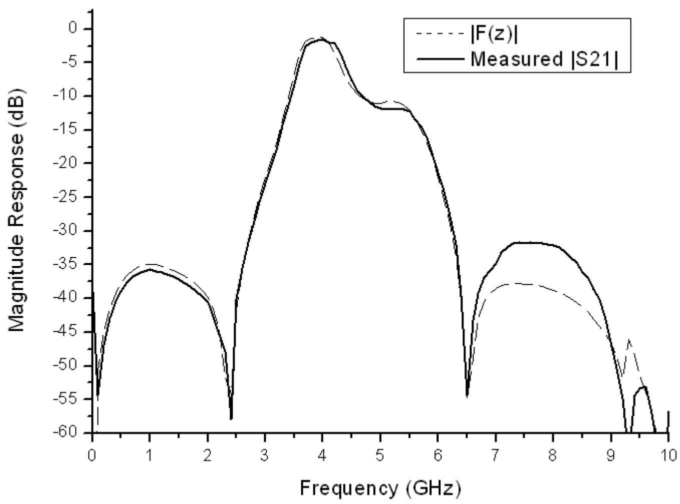
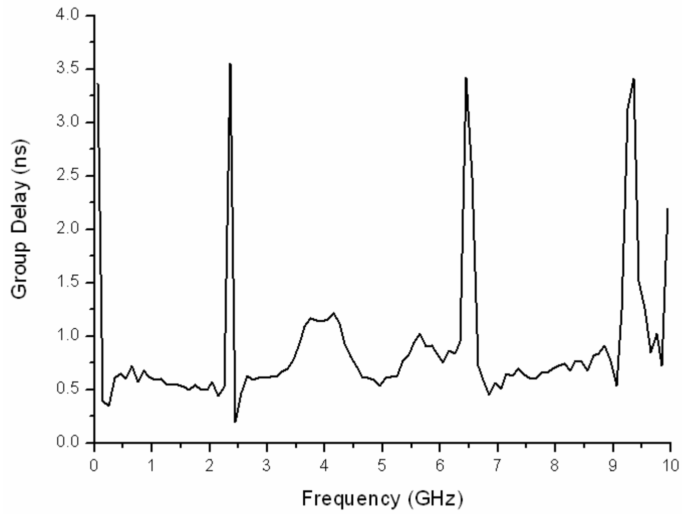


Fig 6: Photograph of two-level band-pass filter.



(a)



(b)

Fig. 7: (a) |F(z)| and measured S_{21} and (b) measured group delay of the band-pass filter in Fig. 6.

and 160 Ω. Three open stubs have impedance profile, from left side to right side, as 160, 160, and 139.22 Ω. The characteristic impedances of two two-section stubs are $Z_2(1)$=160 Ω, $Z_1(1)$=24.64 Ω, $Z_2(2)$=42.06 Ω, and $Z_1(2)$=115.54

Ω. In addition, the characteristic impedances of fifteen series lines are 74.83, 125.68, 142.03, 110.1, 109.46, 50.13, 22.21, 97.59, 70.49, 160, 38.76, 144.26, 70.55, 43.40, and 35.90 Ω. Fig. 4 shows the simulation result of frequency response *T(z)* when the normalizing frequency is 10 GHz, i.e., each finite section of transmission line has a electrical length of 90^o at the normalizing frequency of 10 GHz. Fig. 6 shows the photograph of two-level band-pass filter, which is built on the UL2000 substrate with thickness of 30 mil (0.762 mm), relative dielectric constant of 2.45, and loss tangent of 0.0025. The total length of the filter excluding reference lines on both sizes is 85.10 mm. Fig. 7(a) shows both measured result and *F(z)* of the band-pass filter. The insertion losses for band 3.62 GHz- 4.17 GHz and band 4.64 GHz-5.34 GHz are 0.8 dB and 10 dB, respectively. The measured result deviates slightly from postulated two-level filter *F(z)*. Fig. 7(b) depicts the group delay of two-level filter. The group delay is about 1.2 ns in the transmission band.

V. CONCLUSION

A method to design the microwave filter with multiple-level response was proposed in this letter. The modified Yule-Walker scheme was employed to define arbitrary transmission scattering parameter of a multi-level band-pass filter in the discrete-time domain. A two-level band-pass filter with 10dB difference in amplitude response was implemented to illustrate the validity of this design method.

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