An approximate UTD ray solution of an oblique EM wave diffraction at a junction between two different thin planar material slabs on ground plane

#Titipong Lertwiriyaprapa[1], Prabhakar H. Pathak[2], and John L. Volakis[2]

[1] Department of Teacher Training in Electrical Engineering,
King Mongkut's University of Technology North Bangkok
1518 Piboonsongkarm Rd., Bangkok, Thailand, Email:ttp@kmutnb.ac.th
[2] Electroscience Laboratory, The Ohio State University
1320 Kinnear Rd., Columbus, Ohio 43212, USA

1 Introduction

In this paper, it is of interest to extend the normal incidence solution as discussed in [1] in order to treat the more general case of skew (or oblique) incidence (three-dimensional 3-D). Plane wave (for oblique or skew incidence) and spherical wave illumination are considered here. The geometry of the problem is shown in Fig. 1(a). Previous works dealing with the analytical solutions via the Wiener-Hopf (W-H) solution to diffraction by a junction between two different thin planar material slabs on a perfect electric conductor (PEC) ground plane [2, 3] generally replace the original coated metallic surfaces or material slabs by approximate impedance boundary condition. The latter approximation allows one to arrive at a rigorous analytical solution to the resulting approximate problem configuration. These previous works primarily address the scattering problem in which the illumination is a uniform plane wave that is incident on the thin material discontinuity. In contrast, the present work is expected to be very useful not only to the analysis of scattering situations but also to antenna problems which are equally importance from a practical standpoint. Unlike W-H solution, the solutions developed in this work recover the proper, local plane wave Fresnel reflection and transmission coefficients (FRTCs), and surface wave constants, respectively, for the actual material, and they also allow the material to be both double positive (DPS) or double negative (DNG). DPS materials are those which exhibit positive values of electrical permittivity and permeability while DNG materials are supposed to exhibit negative values for these quantities. The present works provides solutions for finite sources on or near such structures. In addition, it is important to note that the expressions present in this paper are appropriately approximated via physical reasoning so that they can be made free of the complicated integral forms of the W-H split (or factorization) functions.

2 Formulation

The solutions to corresponding 3-D problems (skew incidence) in Fig. 1(a) can be obtained by extending the two dimensional (2-D) solution [1] via an approach similar to that in [2]. It is known that the normal field components E_y and H_y satisfy the Helmholtz scalar equation and impedance boundary conditions independently. This leads to a decoupled solution separately for E_y and H_y. Thus it is convenient to start an ansatz, based on the simplification of a related effective 2-D W-H solution [3] for the normal field components in the case of a unit amplitude, plane wave at skew incidence when it is applied to the special case in Fig. 1(b) where the n-face ($x < 0$, $y = 0$, z) is assumed to be a PEC. In particular, the plane wave spectral (PWS) integral for the diffraction of an obliquely incident plane wave by a two part grounded material slab is first constructed from the ansatz provided by the W-H solution [3]. By using the vector potentials, the tangential field components E_z and H_z can thus be obtained. This allows one to have an ansatz for solving the spectral function occurred in a spherical wave spectral (SWS) integral for a point source illumination. The Fourier transformation, described in [4], can be used to synthesize SWS integral in terms of the PWS integral so a point source illumination can be accommodated. After solving the integral asymptotically, the expression for the UTD first order diffracted field is then found to have the general form as

$$\bar{U}_z^d = \bar{U}_z^i(Q_e) \cdot \bar{\bar{D}}(\phi, \phi') A(s, s') e^{-jks} \tag{1}$$

where $\bar{\bar{D}} = \bar{\bar{D}}^{go} + \bar{\bar{D}}^{sw}$. The $\bar{U}_z^i(Q_e)$ represents the incident field at the point of diffraction Q_e, and $A(s, s')$ is a spread factor given by $A(s, s') = \sqrt{\frac{s'}{s(s+s')}}$. Here s is the distance from Q_e to an observation point, and the s' is the distance from Q_e to the source point. The $\bar{\bar{D}}^{go}$ is based on the Pauli-Clemmow method while $\bar{\bar{D}}^{sw}$ is based on the Van der Waerden method; they are given by

$$\bar{\bar{D}}^{go} = \frac{1}{\Delta(\phi)\Delta(\phi') \sin\beta_o} [C(\phi, \phi') \overline{\overline{T}}(\phi) \cdot \bar{\bar{D}}^c(\phi, \phi') \cdot \overline{\overline{T}}(\phi') + \overline{\overline{W}}], \tag{2}$$

and

$$\bar{\bar{D}}^{sw} = \frac{1}{\Delta(\alpha_{sw}^o)\Delta(\phi') \sin\beta_o} [C(\alpha_{sw}^o, \phi') \overline{\overline{T}}(\alpha_{sw}^o) \cdot \bar{\bar{D}}^{csw}(\alpha_{sw}^o, \phi') \cdot \overline{\overline{T}}(\phi')] \tag{3}$$

where the C, Δ, and α_{sw}^o can be found in [5] and the $\bar{\bar{D}}^c = \begin{bmatrix} D_e^c & 0 \\ 0 & D_h^c \end{bmatrix}$ and $\bar{\bar{D}}^{csw} = \begin{bmatrix} D_e^{sw} & 0 \\ 0 & D_h^{sw} \end{bmatrix}$ with $D_{e,h}^c$ and $D_{e,h}^{sw}$ are given by

$$D_{e,h}^c(\phi, \phi') = \mp \frac{e^{-j\pi/4}}{2\sqrt{2\pi k}} \left[\Gamma_{e,h}^o(\phi, \phi') - \Gamma_{e,h}^n(\phi, \phi')\right] \left[\sec\left(\frac{\phi - \phi'}{2}\right) F_{KP}\left(kLa_{go}^-\right) \right.$$
$$\left. \pm \sec\left(\frac{\phi + \phi'}{2}\right) F_{KP}\left(kLa_{go}^+\right)\right], \tag{4}$$

and

$$D_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^o) = \mp \frac{e^{-j\pi/4}}{2\sqrt{2\pi k}} \left[\frac{R_{e,h}^{swo}(\alpha_{sw}^o, \phi')}{\sin\left(\frac{\alpha_{sw}^o - \phi}{2}\right)} [1 - F_{KP}(kLa_{sw}^o)] + d_{e,h}^{swo}(\phi, \phi'; \alpha_{sw}^o)\right] \tag{5}$$

$$d_{e,h}^{swo}(\phi, \phi'; \alpha_{sw}^o) = \frac{P_{e,h}^o(\alpha_{sw}^o)}{Q_{e,h}^o(\phi)} \left[\sec\left(\frac{\alpha_{sw}^o - \phi'}{2}\right) \pm \sec\left(\frac{\alpha_{sw}^o + \phi'}{2}\right)\right]. \tag{6}$$

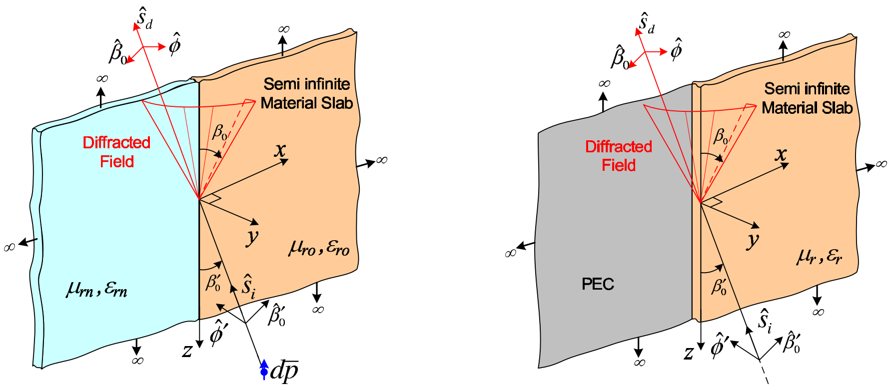
The $\overline{\overline{W}}$ is the unknown constant introduced to suppress the spurious residues which have no physical meaning and may occur during the process of finding the tangential field components. It is given by $\overline{\overline{W}} = -\frac{e^{-j\pi/4}}{\sqrt{2\pi k}} \left[\overline{\overline{T}}_u(\phi) \cdot \overline{\overline{U}} + \overline{\overline{T}}_v(\phi) \cdot \overline{\overline{V}}\right] \cdot \overline{\overline{T}}(\phi')$. The $\overline{\overline{T}}$, $\overline{\overline{T}}_u$, and $\overline{\overline{T}}_v$ are coordinate transformation matrix defined in [5]. The $\overline{\overline{U}}$ and $\overline{\overline{V}}$ can also be found in [5]. The $\Gamma_{e,h}^o$ is an ad hoc modification so as to preserve reciprocity. It is given by

$$\Gamma_{e,h}^o(\phi') = \frac{2\sin\frac{\phi}{2}\sin\frac{\phi'}{2} - \delta_{e,h}^o/\sin\beta_o}{2\sin\frac{\phi}{2}\sin\frac{\phi'}{2} + \delta_{e,h}^o/\sin\beta_o} \tag{7}$$

where $\delta_e^o = -j\mathcal{Y}_d N\cot(N\tau k_d)$ and $\delta_h^o = j\mathcal{Z}_d N\tan(N\tau k_d)$ with $N = \sqrt{1 - \eta 4\sin^2\beta_o \sin^2\frac{\phi}{2}\sin^2\frac{\phi'}{2}}$. The $\Gamma_{e,h}^n = \mp 1$ because the n-face is PEC. It is important to note that the UTD solutions for a junction between two different planar material slabs on a PEC ground plane at skew incidence as shown in Fig. 1(a) can be easily given in the same form as (1)-(4) except the $\Gamma_{e,h}^n(\phi')$ is now

$$\Gamma_{e,h}^n(\phi') = \frac{2\sin\frac{\phi}{2}\sin\frac{\phi'}{2} - \delta_{e,h}^n/\sin\beta_o}{2\sin\frac{\phi}{2}\sin\frac{\phi'}{2} + \delta_{e,h}^n/\sin\beta_o} \tag{8}$$

with the proper substitution of n-face electrical permittivity and permeability, ϵ_{rn} and μ_{rn}, respectively.



(a) 3-D junction between two different, thin, planar DPS/DNG material slabs on a PEC ground plane illuminated by a $\hat{z}$-directed current moment.

(b) Thin, planar DPS/DNG material half plane on an entire PEC ground plane illuminated by a skew incident plane wave excitation.

Figure 1: Canonical of interest

3 Numerical Results

Numerical results for a DPS material junction shown in Fig. 1(a) based on the work presented in this paper referred to as UTD are compared with the results of the Maliuzhinets (MZ) solution [6]. There is a very good agreement, with less than $\pm$ 1 dB differences. In Figs. 3 and 4, only the UTD solutions developed in this paper are shown. It is noted that the excitation for the problem in Fig. 4 is a current moment ($d\bar{p}_e = \hat{z}dp_{ez}$ or $d\bar{p}_m = \hat{z}dp_{mz}$), which produces a spherical wave, whereas the excitation is a skew incident plane wave for Figs. 2 and 3. It is important to note that the surface wave effects are neglected in these plots in order to clearly test if the boundary conditions on the first order UTD diffracted fields are properly satisfied as compared to reference MZ solutions; otherwise the surface waves would have masked the behavior of the diffracted fields near the boundaries.

References

[1] T. Lertwiriyaprapa, P. H. Pathak, and J. L. Volakis, "A UTD for Predicting Fields of Sources near or on Thin Planar Positive/Negative Material Discontinuities," *Radio Science*, vol. 42, 2007.

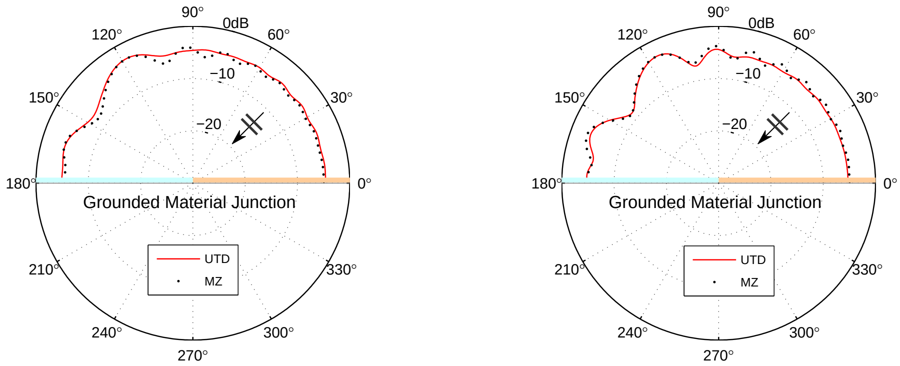
[2] R. G. Rojas, "Wiener-Hopf Analysis of the EM Diffraction by an Impedance Discontinuity in a Planar Surface and by an Impedance Half-Plane," *IEEE Trans. Antenna Propagat.*, vol. 36, pp. 71–83, Jan. 1988.

[3] R. G. Rojas and P. H. Pathak, "Diffraction of EM Waves by a Dielectric/Ferrite Half-Plane and Related Configurations," *IEEE Trans. Antenna Propagat.*, vol. 37, pp. 751–763, June 1989.

[4] P. H. Pathak and R. G. Kouyoumjian, "The Dyadic Diffraction Coefficient for a Perfectly Conducting Wedge," Tech. Rep. 2183-4, The Ohio State University; prepared under Contract AF19(628)-5929 for Air Force Cambridge Research Laboratories, June 1970.

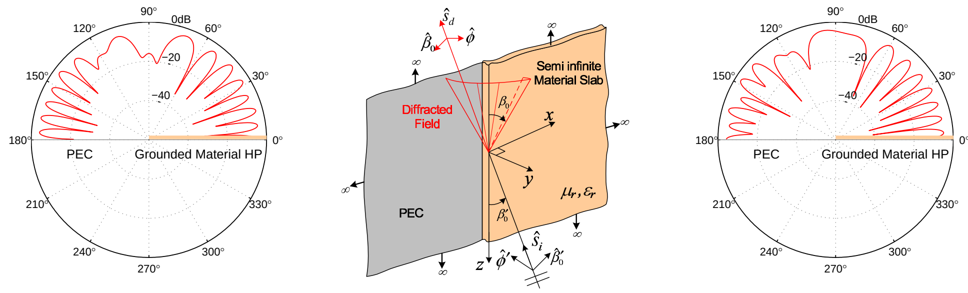
[5] T. Lertwiriyaprapa, *An Approximate UTD development for the Radiation by Antennas near or on Thin Material Coated Metallic Wedges*. PhD thesis, The Ohio State University, 2007.

[6] R. G. Rojas, "Electromagnetic Diffraction of an Obliquely Incident Plane Wave Field by a Wedge with Impedance Faces," *IEEE Trans. Antenna Propagat.*, vol. 36, pp. 956–970, July. 1988.



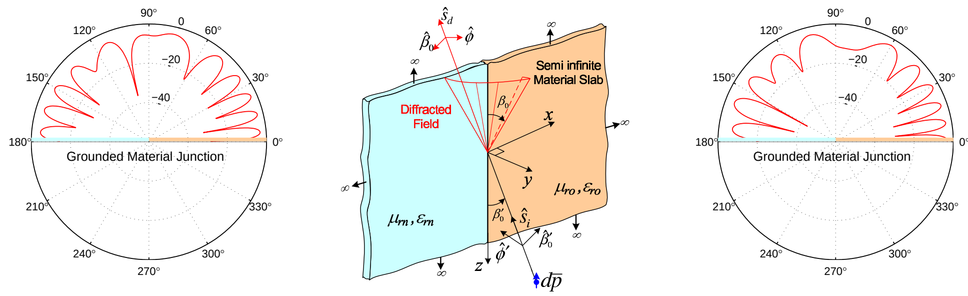
(a) (b)

Figure 2: Comparison of scattered fields of UTD and MZ solutions for a DPS material junction excited by a uniform skew incident plane wave shown in Fig. 1(a)(a) TE and (b) TM at $\phi' = 45°$ and $\beta_o' = 65°$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with ($\epsilon_{ro} = 4, \mu_{ro} = 2$) and ($\epsilon_{rn} = 5, \mu_{rn} = 1$).



(a) (b) (c)

Figure 3: Total field of UTD solution for a grounded DPS material half plane with PEC ground plane excited by a uniform skew incident plane wave (a) TE and (c) TM at $\phi' = 60°$ and $\beta_o' = 120°$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/10$ thick with ($\epsilon_{ro} = 4, \mu_{ro} = 2$).



(a) (b) (c)

Figure 4: Total field of UTD solution for a DPS material junction excited by (a) a $\hat{z}$-directed electric current moment dp_{ez} and (c) a $\hat{z}$-directed magnetic current moment dp_{mz} at $r' = 7\lambda$, $\phi' = 45°$ and $\theta' = 55°$. The fields are observed at $r = 15\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with ($\epsilon_{ro} = 12, \mu_{ro} = 8$) and ($\epsilon_{rn} = 1, \mu_{rn} = 4$).