Electric Field Measurement Using Multiple-Loaded Linear Scatterers

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1. Introduction

Modulated scatterer techniques (MST) based on 1D and 2D arrays of small loaded linear scatterers have been used extensively for rapid electric field distribution measurements at microwave frequencies [1]. In most applications, a compact array of closely-spaced small loaded dipoles is used in order to construct a representative map of the electric field of interest. The performance of the MST-based electric field measurement system depends on many factors. As stated in [2], "Six basic factors that can have an impact on measurement system performances are as follows: 1) dynamic range, 2) interelement mutual coupling, 3) interaction between the MST array and test antenna, 4) parasitic signals (modulated and/or unmodulated), 5) dispersion of element scattering characteristics, and 6) probe correction." These factors impose many tradeoff considerations that need to be resolved on per-application basis. For instance, the interelement mutual coupling combined with the fact that the utilized elements are commonly small scatterers (i.e., short dipoles) can adversely impact the system sensitivity, and hence reduce the overall system dynamic range. On the other hand, using stronger (loaded) scatterers can enhance the sensitivity when effective probe correction and compensation techniques are employed. Such a tradeoff was investigated in [3] where it was articulated that "For practical reasons, instead of trying to build noninterfering (and therefore more complicated and perhaps less sensitive) sensors, one can use common sensors (simpler, inexpensive, and possibly more sensitive) and include them in the reconstruction formulism itself." In line with this concept, we propose a new method for electric field distribution measurement based on utilizing multiple loaded linear scatterers (MLS) instead of the conventional arrays of discrete elements. It will be shown that, by utilizing a linear scatterer loaded at multiple discrete locations over its length, the electric field of interest can be reconstructed accurately from scattered field measurements taken under distinct modulation states. The feasibility of the proposed method for electric field distribution measurement is demonstrated via numerical simulations.

2. Multiple Loaded Linear Scatterer (MLS)

The proposed electric field measurement method is based on using a linear wire scatterer loaded with M controllable loads (e.g., active elements) of admittances $\{Y_m, m = 1, 2, \ldots, M\}$ located at $\{z_{Lm}, m = 1, 2, \ldots, M\}$ as shown in Fig. 1. The scatterer is illuminated by an unknown field $E_z^i(z)$ produced by a source (i.e., device-under-test (DUT)). The objective is to measure the incident electric field distribution over the wire length from scattered field measurements taken at an arbitrary single observation point $E_z^s(Q)$, as shown in Fig. 1. The general analysis of straight wire scatterers loaded with lumped linear elements can be found in [4]. An equivalent approach based on Maxwellian Circuits theory has been recently proposed as well [5]. The current induced on a linear wire scatterer of length L and radius a aligned along the *z*-axis due to the incident electric field

$E_z^i(z)$ can be found by solving Pocklington's integral equation [6, pp. 720] using the Method-of-Moment (MoM). Assuming the wire is modeled as N interconnected segments each of length l and centered at $\{z_i, i=1,2,\ldots,N\}$, the induced current solution is given as, $\mathbf{i}^{sc}=\mathbf{Ye}$, where $\mathbf{i}^{sc}=\left[I(z_1),I(z_2),\ldots,I(z_N)\right]^T$ is the induced "Short-Circuit" current distribution over the scatterer, $\mathbf{Y}=\mathbf{Z}^{-1}$ is the wire admittance matrix, and the excitation field vector is $\mathbf{e}=(-4j\pi k/\eta)\left[E_z^i(z_i),E_z^i(z_2),\ldots,E_z^i(z_N)\right]^T$. Following the approach outlined in [4], the total scattered field at the observation point can be found as,

$$E_z^s(Q)=\frac{-j\eta l}{8\pi^2 ak}\mathbf{g}\left[\mathbf{i}^{sc}+\left(\frac{-4j\pi k}{\eta l}\right)\mathbf{Y}\mathbf{v}^e\right]. \qquad (1)$$

where $\mathbf{g}$ can be found in [6, pp. 284], $\mathbf{v}^e$ is a vector of dimension $N\times 1$ with all zeros except at the M load positions where it is set to the load equivalent voltages found from,

$$\mathbf{v}_{Loads}^e=-\left[\mathbf{Y}_M+\mathbf{Y}_{Load}\right]^{-1}\mathbf{i}_{Load}^{sc} \qquad (2)$$

where $\mathbf{v}_{Load}^e=\left[v_1^e,v_2^e,\ldots,v_M^e\right]^T$ is the equivalent voltage vector, $\mathbf{i}_{Load}^{sc}=[I(z_{L1}),I(z_{L2}),\ldots,I(z_{LM})]^T$ is the short-circuit current at the load locations (samples of the distribution i^{sc} evaluated at the load locations), $\mathbf{Y}_{Load}=diag\{Y_1,Y_2,\ldots,Y_M\}$ is the load matrix, and finally, $\mathbf{Y}_M$ is the transfer admittance defined between the loaded ports.

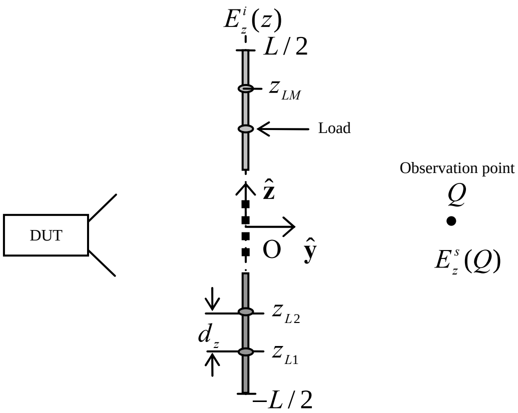

Figure 1: (a) Multiple loaded scatterer

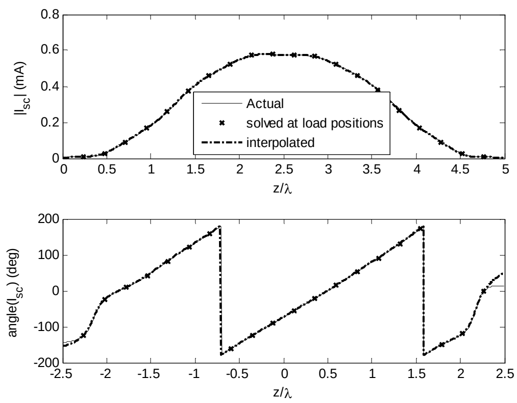

Figure 2: The actual, load, and interpolated short-circuit currents.

3. Field Mapping using MLS

In essence, the incident electric field distribution can be recovered if the short-circuit current distribution induced on the scatterer is measured (i.e., through the relation $\mathbf{i}^{sc}=\mathbf{Ye}$). The basic idea behind the proposed method is to first compute the short-circuit current at the load locations $\mathbf{i}_{Load}^{sc}$ indirectly from the scattered field measurements and subsequently estimate the distribution $\mathbf{i}^{sc}$. The short-circuit current, while dependent upon the wire geometry and the incident electric field, does not change as a function of the load values (see (1)). Hence, by changing the loads' values $\{Y_1,Y_2,\ldots,Y_M\}$ (e.g., realizing different loading conditions) and measuring the scattered field for each condition at the observation point Q, one can solve for the load short-circuit current. For this purpose, P distinctive loading conditions are realized by changing the load values electronically such that the p^{th} loading condition corresponds to a unique load matrix $\mathbf{Y}_{Loads}^p$ resulting in different load voltages $\mathbf{v}_{Load}^{ep}$ and $\mathbf{v}^{ep}$ (as evaluated from (2) using $\mathbf{Y}_{Load}^p$). Let us designate one of the P loading conditions as the *reference condition*, say the 1st loading condition ($p=0$). Under this condition, let the measured field be $E_z^{s0}(Q)$. Using (1), the difference between the field scattered under the reference and the p^{th} conditions is found as,

$$\delta^p = E_z^{sp}(Q) - E_z^{s0}(Q) = \frac{1}{2\pi a}\mathbf{gY}\left[\mathbf{v}^{e0} - \mathbf{v}^{ep}\right],\ p = 1, 2, \ldots, P-1. \quad (3)$$

Since the vectors $\mathbf{v}^{e0}$ and $\mathbf{v}^{ep}$ have nonzero components only at the load positions, the M corresponding positions only in the $N \times 1$ vector $\mathbf{gY}$ contribute to the right hand side of (3). Define a $1 \times M$ vector $\mathbf{b}$ which has these components sorted in order. The field difference in (3) becomes,

$$\delta^p = \frac{1}{2\pi a}\mathbf{b}\left[\mathbf{v}_{Load}^{e0} - \mathbf{v}_{Load}^{ep}\right] = \frac{1}{2\pi a}\mathbf{b}\left(\left[\mathbf{Y}_M + \mathbf{Y}_{Load}^{p}\right]^{-1} - \left[\mathbf{Y}_M + \mathbf{Y}_{Load}^{0}\right]^{-1}\right)\mathbf{i}_{Load}^{sc}. \quad (4)$$

Given the scatterer geometry, frequency of operation, and the load values for the loading conditions, the above equation has everything known except the loads short-circuit current vector which represents M unknowns. To solve for this current, M equations are formed using $P = M + 1$ loading conditions. By measuring the scattered field for all the P loading conditions and using the first loading condition as a reference, the load short-circuit current vector is found as,

$$\mathbf{i}_{Load}^{sc} = \mathbf{w}^{-1}\boldsymbol{\Delta} \quad (5)$$

where $\boldsymbol{\Delta} = \left[\delta^1, \delta^2, \ldots, \delta^M\right]^T$ is the vector of scattered field differences, and $\mathbf{w} = [\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^P]^T$ such that $\mathbf{w}^p = \frac{1}{2\pi a}\mathbf{b}\left(\left[\mathbf{Y}_M + \mathbf{Y}_{Load}^{p}\right]^{-1} - \left[\mathbf{Y}_M + \mathbf{Y}_{Load}^{0}\right]^{-1}\right)$. Before the incident electric field can be recovered, the computed load short-circuit current has to be interpolated over the length of the scatterer (that is; estimating $\mathbf{i}^{sc}$). Let $\hat{\mathbf{i}}^{sc}$ be the interpolated short-circuit current obtained from $\mathbf{i}_{Load}^{sc}$. Finally, the incident electric field over the length of the scatterer is found as,

$$E_z^i(z_i) = (j\eta / 4\pi k)\mathbf{Z}\hat{\mathbf{i}}^{sc}. \quad (6)$$

The proposed method can be summarized as follows. Step 1: Activate the reference loading condition and measure $E_z^{s0}(Q)$, Step 2: Activate the rest of the loading conditions and measure $E_z^{sp}(Q),\ p = 1, 2, \ldots, M$, Step 3: From the obtained measurements, compute field difference vector $\boldsymbol{\Delta}$ (off-line), Step 3: Compute $\mathbf{w}$ (no measurement is needed), Step 4: Solve for $\mathbf{i}_{Load}^{sc}$ and interpolate it, finally, Step 5: Solve for the incident electric field using (6).

4. Simulation Results and Discussions

To illustrate the efficacy of the proposed method, a MLS of length *L=5λ* and radius *a=0.001λ* loaded with 20 PIN diodes mounted along the wire with equal interspacing was considered. The actual incident electric field on the MLS was assumed to be, $E_z^i(z) = \cos^2(\pi z / 5)e^{j0.4kz}$ which represents more than 50 dB of field magnitude dynamic range/variation and more than two phase cycles over the length of the wire. The observation point was set at *Q(0,5λ,0)*. The proposed method was applied to solve for the loads short-circuit current using 21 loading conditions. The reference condition was when all PIN diodes were switched ON. The remaining 20 loading conditions were obtained by switching one diode ON at time while the rest are OFF. Fig. 2 shows the magnitude and phase of the actual short-circuit current induced on the wire. The short-circuit current found at the load locations from the scattered electric field (as per (5)) and the estimated short-circuit current on the MLS (after interpolation) are also shown in Fig. 2. The interpolated current was subsequently used to recover the incident electric field as per (6). Fig. 3 compares the actual incident field and the recovered field distribution using the proposed method. As shown in Fig. 3 there is a close match between the actual and recovered field distribution both in magnitude and phase. The discrepancies toward the edges of the MLS are mainly attributed to the small signal levels at these locations which are comparable to the interpolation error. To evaluate the performance of the proposed method in presence of uncertainties in the measured scattered electric field (i.e., due to system noise floor) a noise component with power proportional to the scattered field strength was added in simulation. Fig. 4 shows the mean-square error (MSE) between the actual and recovered incident fields as a function of the signal-to-noise ratio

(SNR). It is observed that the MSE decreases rapidly as the SNR increases. The MSE floor shown in the figure is due to the interpolation error. Fig. 4 suggests that the proposed method does not suffer from severe performance degradation due to noise in practical systems where the SNR is typically greater than 30 dB. The effects of static loads (i.e., fixed loads) that are not controlled dynamically in the measurement process such as dc-block capacitors and load biasing structure do not impact the accuracy of the proposed method since the differential scattered field is used in recovering the field of interest. This point was verified via simulation and it is conceived as an advantage of the proposed method since the effect of any fixed unintentional parasitic loads (e.g., due to component mounting) is nulled out in the process. In the simulation results presented here, the PIN diodes were assumed to have ideal load values in both ON and OFF states (e.g., short and open, respectively). The sensitivity of the proposed method to non-ideal load values where studied via simulations as well. These simulations showed that non-ideal load values do not impact the performance of the method significantly. However, it is emphasized that the load values should be known *a priori* in order to implement the proposed method. This is not a hard-limiting factor since the load values can be measured accurately.

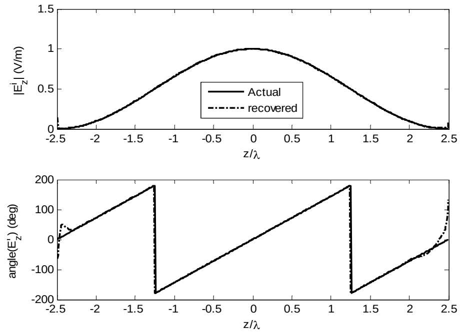

Figure 3: The actual and recovered incident electric field

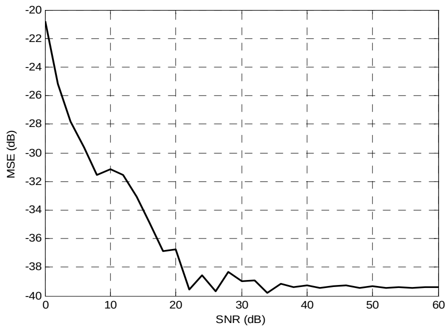

Figure 4: The MSE performance.

At high frequencies such as those in or near the millimeter wave region, the implementation of the conventional MST systems with array of discrete scatterers might become very challenging due the factors mentioned earlier. At these frequencies, the dipole, its load, and the biasing structure become appreciated portion of the wavelength. Consequently, the probe becomes highly perturbing and the biasing structure parasitics, which can be usually ignored at low frequencies, change the probe properties significantly. Thus, the factors pertaining to parasitic loads and element dispersion might become a significant issue in system design. Since probe parasitics are typically hard to model, most current compensation techniques do not deal with these factors on the basis that they can be ignored. Furthermore, due to the interaction among the array elements, highly sensitive receiver front-ends must be used to detect the signal of interest at these frequencies. This in turn results in higher system cost and complexity. The capability of the proposed MLS method to deal with these challenges, and hence establish a better alternative to the conventional MST, will be investigated in future.

References

[1] J.-C. Bolomey, G.E. Gardiol, "*Engineering applications of the modulated scatterer technique*," Norwood, MA: Artech house Inc. 2001.

[2] J-C. Bolomey, and *et al*, "Rapid near-field antenna testing via arrays of modulated scattering probes" *IEEE Trans. on Anten. and Propag.*, vol. 36, no. 6, pp. 804-814 , June 1988.

[3] O. Franza, N. Loachimowicz, and J-C Bolomey, "SICS: A sensor interaction compensation scheme for microwave imaging," *IEEE Trans. on Anten. and Propag.*, vol. 50, no. 2, pp. 211-216, Feb. 2002.

[4] R. Harrington and J. Mautz, "Straight wires with arbitrary excitation and loading," *IEEE Trans. on Anten. and Propag.*, vol. 15, no. 4, pp. 502-515, July 1967.

[5] M. Abou-Khousa, and R. Zoughi, "Maxwellian circuits-based analysis of loaded wire antennas and scatterers," *IEEE Antennas and Wireless Propagation Letters*, Accepted, p. 3, Feb. 2008.

[6] C. A. Balanis, Advanced Engineering Electromagnetic, New York: Wiley. 1989.