2-D FDTD MODELING.
APPLIED TO DETECTION OF BURIED OBJECTS
IN STRATIFIED LOSSY MEDIUM

Tahar Ziani and **Mohamed Laour** [#]
[#] EMP: BP 17C. Bordj El Bahri. Algeria Fax : 233 0 21 86 32 04
Doctor: **Xavier Dérobert.** Head of LCPC- France
Professor: **Malek Benslama.** Member of IEEE

Abstract: ***In this paper we are interesting about a simple implementation of the Yee finite-difference in time domain (FDTD) to modelise electromagnetic differentiated Gaussian and sinus Gaussian pulses. The source is emitting from free space (in two dimensions) and hits a slab which formed by a lossy multilayer media. By using the MATLAB codes, stability criterion and the perfect matched layer (PML). Our objective is to asses the detection of buried objects by using the ground penetrating radar (GPR). The code is well commented, relatively easy to understand and can easily modified for user's specific purpose***.

Keywords: *finite-difference in time domain* **(***FDTD), ground penetrating radar (GPR), electromagnetic, pulse, lossy and multilayer media*.

I- INTRODUCTION

The GPR is a non-destructive technique used for high-resolution imaging of the shallow subsurface. Its performance is in a growing interest in the propagation of transient electromagnetic wave through the upper regions of earth's surface for detection and location of buried artefacts and structures within. The propagation of waves is resumed in figure 1.

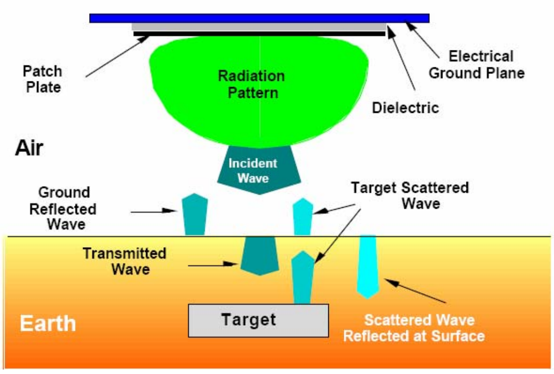

Figure 1: Waves in GPR unit

We are interesting in this paper in modeling the GPR for detecting objects in attenuating multilayer soil. Our work consist of emitting a differentiated Gaussian-pulse or a sinus Gaussian-pulse near the middle of a gird composed of Yee cells divided on two parts. The first one represents the free space and the other is representing the soil which is formed by three layers. Each one has a proper electric permittivity and conductivity as showed in figure 2. These parameters are taken to be near soil reality. A numerical modeling of complete GPR systems can be done by using the FDTD algorithms [1].

The FDTD method has prevailed in the computational electromagnetic area as an accurate numerical technique for the direct integration of Maxwell's equations. Its evolution has ensured from several technological developments, resulting in the emergence of various algorithms that extend the method's implementation to various modern applications. Representative examples are the simulation of light propagation in optical devices, soil modeling in GPR problems, and the study of potential effects of human tissue exposure to electromagnetic radiation [2], [3]. Because of the improved of computational resources of the 1990s made FDTD modeling of GPR feasible [4], [5]. Although FDTD simulations provide additional insight into antenna radiation mechanisms by visualizing the propagation of electric and magnetic fields as a function of time and space as the computations proceed [7]-[8] [9].

We choose MATLAB as our coding language because of its comprehensive library of graphics routines. It is relatively straight forward to produce animations using MATLAB; this is often critical to the understanding of a working FDTD algorithm. Due to the large amount of book-keeping required in any full 3D-FDTD code, it is common to reduce the dimensionality down to 2D for pedagogical purposes. Most of the equations in this report can be found in other texts; however, we have listed them here for the reason of continuity [10]. A working FDTD code must propagate waves properly, absorb waves at the edges of the modeling grid by implementing the perfectly matched layer (PML) absorbing boundaries, and calculate useful modelling results. This work addresses all of the above in a step by step process and has the following outline:

Model: To simplify the problem, the antenna emits a differential Gaussian wave near the soil which is presented as a stratified medium with different electrical permittivities and conductivities. And this model is presented in figure 2.

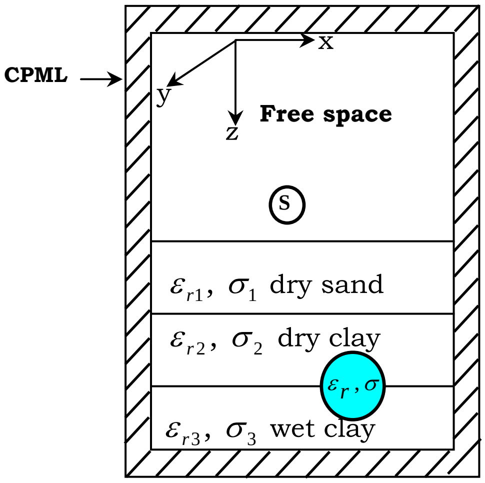

Figure 2: Geometry of subsurface model

II- THEORY

A- Governing equations

In deriving 2-D FDTD formulation, we choose between one of two groups of the three vectors each[11]:

1- The transverse magnetic (TM) mode, which is composed of: E_z, H_x, and H_y or

2- The transverse electric (TE) mode, which is composed of: E_x, E_y, and H_z.

We will work with the TM mode wave propagation. Expanding the following equations

$$\varepsilon \frac{\partial E}{\partial t} = \nabla \times H - J \qquad (1)$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0} \nabla \times E \qquad (2)$$

Where E and H are the electric and magnetic fields respectively and $J = \sigma E$ is the conduction current density with $\frac{\partial}{\partial z} = 0$, $E_x = E_y = H_z = 0$, we obtain the non zero components of $\frac{\partial E}{\partial t}$ and $\frac{\partial H}{\partial t}$:

$$\varepsilon \frac{\partial E_z}{\partial t} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) - \sigma E_z \qquad (3)$$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_z}{\partial y} \qquad (4)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_z}{\partial x} \qquad (5)$$

Where: The dielectric permittivity and the electrical conductivity are respectively represented as:

$\varepsilon = \varepsilon_0 \varepsilon_r$, $\sigma = \sigma_0 \sigma_r$: are respectively the dielectric permittivity and the electrical conductivity. The magnetic permeability is represented as: $\mu = \mu_0 \mu_r$.

B- FDTD appoximations

We used a differential Gaussian-pulse and sinus Gaussian-pulse.

The gird was composed of Yee cells (Yee, 1966), after substituting the appropriate finite-difference expressions into Eqs. (3) to (5) and solving for the updated electric and magnetic field components, we arrived to write the above three equations as:

$$\frac{E_z^{n+1/2}(i,j) - E_z^{n-1/2}(i,j)}{\Delta t} = \frac{1}{\varepsilon} \left(\frac{H_y^n(i+1/2,j) - H_y^n(i-1/2,j)}{\Delta x} - \frac{H_y^n(i,j+1/2) - H_y^n(i,j-1/2)}{\Delta y} - \sigma E \right) \qquad (6)$$

$$\frac{H_x^{n+1}(i,j+1/2) - H_x^n(i,j+1/2)}{\Delta t} = -\frac{1}{\mu_0} \frac{E_z^{n+1/2}(i,j+1) - E_z^{n+1/2}(i,j)}{\Delta y} \qquad (7)$$

$$\frac{H_y^{n+1}(i+1/2,j) - H_y^n(i+1/2,j)}{\Delta t} = \frac{1}{\mu_0} \frac{E_z^{n+1/2}(i+1,j) - E_z^{n+1/2}(i,j)}{\Delta x} \qquad (8)$$

In Eqs. (6)-(8), ε_0 and μ_0 differ by several orders of magnitude, E_z H_x and H_y will differ by several orders of magnitude. Numerical error is minimised by making the following change of variables as:

$$\tilde{E}_z = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_z \quad (9)$$

$$\tilde{E}_z^{n+1/2}(i,j) = \tilde{E}_z^{n-1/2}(i,j) + \frac{\Delta t}{\varepsilon_r \sqrt{\varepsilon_0 \varepsilon_r}} \left(\frac{H_y^n(i+1/2,j) - H_y^n(i-1/2,j)}{\Delta x} - \frac{H_y^n(i,j+1/2) - H_y^n(i,j-1/2)}{\Delta y} \right) \quad (10)$$
$$- \sigma \frac{\Delta t}{\varepsilon_r \sqrt{\varepsilon_0 \varepsilon_r}} \left(\frac{\tilde{E}_z^{n+1/2}(i,j) - \tilde{E}_z^{n-1/2}(i,j)}{2} \right)$$

des

$$H_x^{n+1}(i,j+1/2) = H_x^n(i,j+1/2) - \frac{\Delta t}{\mu_0 \Delta y} \left(\tilde{E}_z^{n+1/2}(i,j+1) - \tilde{E}_z^{n+1/2}(i,j) \right) \quad (11)$$

$$H_y^{n+1}(i,j+1/2) = H_y^n(i,j+1/2) + \frac{\Delta t}{\mu_0 \Delta x} \left(\tilde{E}_z^{n+1/2}(i,j+1) - \tilde{E}_z^{n+1/2}(i,j) \right) \quad (12)$$

C- Stability criterion

Working in discretization domain, dispersion of non physical signals appears in the lattice. This dispersion changes with frequency, the direction of propagation and the spatial discretization. To reduce its effect to acceptable values of precision the spatial discretization must be enough to sample the wave length of signals with an enough point number. This error fall about 0.3% when the spatial discretization is $\lambda_0 / 20.n$, however it is 1.2 % in all directions when it is about $\lambda_0 / 10.n$ (where λ_0 is the wave length of the wave and n index of propagation medium).In our work we have taken $\lambda_0 / 30$ in our Matlab programming of the FDTD [12] .

The FDTD is restricted by the gird size, because over one increment in the space gird, the electromagnetic field can not change. This means that in order to obtain meaningful result, it becomes necessary for the linear dimensions of the gird to be only a fraction of the wavelength. This requirement puts a restriction upon the time step (Δt) for the chosen gird dimensions ($\Delta x, \Delta y$). If ε and μ are allowed to be constant, this restriction, known as the stability criterion, can be expressed as,

$$\Delta t c_{\max} \le \left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} \right)^{1/2} \quad (13)$$

Where $c_{\max}$ is the maximum velocity through a given medium and this velocity can be defined as $c_{\max} = c / \sqrt{\varepsilon_r}$.

Where $c = 2.997 \times 10^8 \, m/s$ and ε_r is the relative permittivity of the medium

In our simulation we chosen Δt to satisfy the (14) equation taking from the works of Susan Hagness implementation in 2-D with Matlab codes[13].

Making use of equation (13) in equations (10-11-12), we obtain

$$\tilde{E}_z^{n+1/2}(i,j) = \tilde{E}_z^{n-1/2}(i,j) + \frac{\Delta t}{\varepsilon_r \sqrt{\varepsilon_0 \varepsilon_r}} \left(\frac{H_y^n(i+1/2,j) - H_y^n(i-1/2,j)}{\Delta x} \right)$$
$$- \frac{\Delta t}{\varepsilon_r \sqrt{\varepsilon_0 \varepsilon_r}} \left(\frac{H_y^n(i,j+1/2) - H_y^n(i,j-1/2)}{\Delta y} \right) \quad (14)$$
$$- \sigma \frac{\Delta t}{\varepsilon_0 \varepsilon_r} \left(\frac{\tilde{E}_z^{n+1/2}(i,j)}{2} \right) - \sigma \frac{\Delta t}{\varepsilon_0 \varepsilon_r} \left(\frac{-\tilde{E}_z^{n-1/2}(i,j)}{2} \right)$$

$$\tilde{E}_z^{n+1/2}(i,j) = \tilde{E}_z^{n-1/2}(i,j) \left(\frac{\left(1 - \frac{\sigma \Delta t}{2\varepsilon_0 \varepsilon_r}\right)}{\left(1 + \frac{\sigma \Delta t}{2\varepsilon_0 \varepsilon_r}\right)} \right) + \quad (15)$$
$$\frac{\frac{\Delta t}{\Delta \varepsilon_r \sqrt{\varepsilon_0 \varepsilon_r}}}{\left(1 + \frac{\sigma \Delta t}{2\varepsilon_0 \varepsilon_r}\right)} \left(\left(H_y^n(i+1/2,j) - H_y^n(i-1/2,j) \right) - \left(H_y^n(i,j+1/2) - H_y^n(i,j-1/2) \right) \right)$$

In this case we have assumed that: $\Delta x = \Delta y = \Delta$

III- MATLAB IMPLEMENTATION

The 2-D, TM-mode, finite-difference formulation will be presented, we will use matrices to store all field components and electrical properties

$dx = dy = 3.0\text{e-}3$ m

ε_r =[1 10 4 7 12]
σ =[0 10 5e-4 1e-2 0.4]
μ_r =[1 1 1 1 1]
The cylinder has ε_r =10 and σ =10

For the differential Gaussian pulse
$tw = 7.0 \times 26.53\text{e-}12$ and $t_0 = 4 \times tw$
source =
$$-2((T-t_0)/tw)\exp(-1((T-t_0)/tw)^2)$$
For the sinus Gaussian pulse
$$freq = 1.5GHz$$
$$omega = 2\pi.freq$$
$$rtau = 160.0e-12$$
$$tau = rtau/dt$$
$$delay = 3.tau$$
source=
$$\sin(omega.(n-delay)dt).\exp(-((n-delay)^2/tau^2))$$
For the two sources the frequency of the excitation is $1.5.10^9$ Hz.
For the thickness in of each medium is15 cm and the diameter of cylinder is 6 cm.
For the first the electric permittivity is ε_r =4, the second is 7 and the third is 12 however for the electric conductivity; the first σ =5e-4, the second is 1e-2 and the third is 0.4

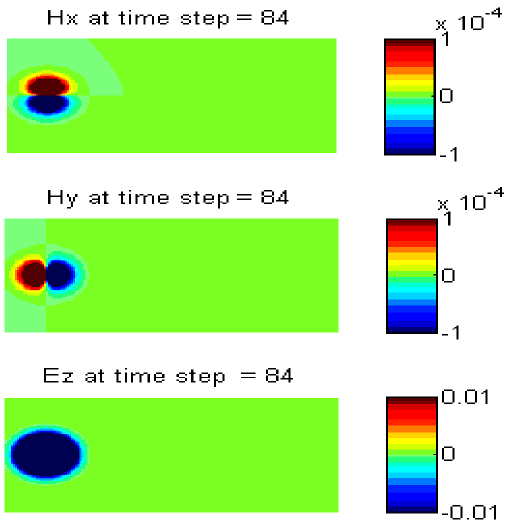

Figure 4: Sinus Gaussian pulse

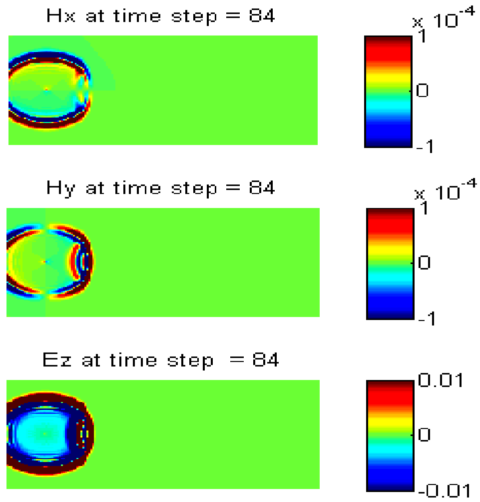

Figure 5: Differentiated Gaussian pulse

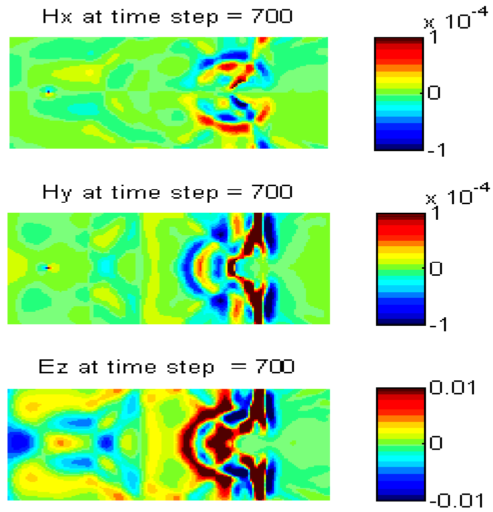

Figure 6: Sinus Gaussian pulse

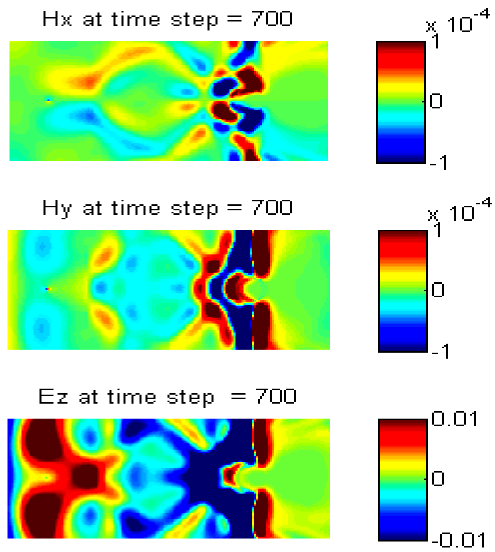

Figure 7: Differentiated Gaussian Pulse

IV-CONCUSION

For each source the cylinder appears clearly in spite of attenuation due the parameters of the soil and the results of simulation show that the two sources are adapted for this stratified medium and are comparable. We can notice that the Differentiated Gaussian pulse has high velocity than the sinus Gaussian one in free space; this is showed in figures (4) and (5); because at 84ns the first one hits the slab before the other.

References

[1] K.S.Yee. Numerical Solution of Initial Boundary Value Problems involving Maxwell's Equations in Isotropic Media. *IEEE Trans. Antennas Prop.* 14(5), pp.302-309, 1966.

[2] Carlos Bilich. A simple model to asses the feasibility of heart beat detection using commercial UWB communications devices. Technical report # DIT-03-033. January 2007.

[3] Terapss Jariyanorawiss and Nuttaka Homsup. Implementation of an electromagnetic perfectly matched layer in head model using FDTD.

[4] Stanley J Radzevicius, Chi-Chih Chen, Leon Peters., Jeffery J Daniels. Near-field dipole radiation dynamics through FDTD modeling. Journal of APPLIED GEOPHYSICS. October 2002.

[5] J.P.Berenger. Three-Dimensional Perfectly Matched Layer for the Absorbtion of Electromagnetic Wave. Journal of *Computational Physics*, Vol.127, pp.363-379.

[6] A.Taflove. Computational Electrodynamics. Artech House, 1995.

[7] Bernhard Lampe, Klaus Holliger and Alan. G.Green. A finite difference time domain simulation toll for ground-penetrating radar antennas. GEOPHYSICS Vol. 68 (May–june 2003); p.971-987.

[8] K.P.Prokopidis. T.D. Tsiboukis. Modeling of Ground Penetrating Radar for detecting buried objects in dispersive soils. ACES Journal, Vol.22 N°.2 July 2007.

[9] K.P.Prokopidis, E.P.Kosmidou and T.D. Tsiboukis. An FDTD Algirithm for wave propagation in dispersive media using high-order schemes. Journal of Electromagn. Waves and Appl., Vol.18 N°.9, 1171-1194, 2004.

[10] James Irving, Rosemary Knight. Numerical modeling of ground-penetrating radar in 2-D using Matlab. Computers and Geoscience 32. 1247-1258. 2006.

[11] Dennis. M. Sullivan. Electromagnetic simulation using the FDTD IEEE Press Series on RF and Microwave Technology.2000.

[12] Rafael PEREZ, Contribution à l'analyse théorique et expérimentale de radagrammes GPR. Performances des antennes : appoerts d'une configuration multistatique. Thése de Doctorat. Université de Limoge. 10 octobre 2005.

[13] Susan C Hagness.1-2-3D Matlab code. Department of Electrical and Computer Engineering. University of Wisconsin-Madison. Frebruary.2000.