# Artificial Surface Having Frequency Dependent Reflection Angle 

\# Kihun Chang, Jihwan Ahn, and Young Joong Yoon<br>Department of Electrical and Electronic Engineering, Yonsei University<br>134 Shinchondong, Seodaemungu, Seoul, 120-749, Korea<br>yjyoon@yonsei.ac.kr

## 1. Intruduction

The reflection coefficient phase of the electromagnetic bandgap (EBG) structure varies continuously from $180^{\circ}$ to $-180^{\circ}$ through $0^{\circ}$ at the surface resonant frequency, where the surface behaves as an artificial magnetic conductor (AMC). The EBG structure, first introduced and experimentally verified by Sievenpiper [1]- [2] has been realized by covering a metal ground plane with periodic metal-dielectric composite textures. By tuning the individual lattice of a periodic mushroom type texture in order to vary the surface impedance, the surface with reflection phase gradient as a function of position can be designed. Using a linear phase gradient across the face, the artificial surface having a steered reflection is proposed. Since the reflection phase curve is dependent on frequency, this metasurface reveals a frequency dependent reflection angle.

## 2. Theory and Design

The configuration of the two-layer structure with sub-wavelength lattices with different lengths is illustrated in Fig. 1. It consists of four parts: a ground plane, a dielectric substrate of $F R 4$ with $h=186$ mil and $\epsilon_{r}=4.5$, sub-wavelength metallic patches with identical 7 rows and 11 lattices with graded size in each row, and connecting vias with 0.5 mm in diameter. Longitudinal-section of the proposed structure and the equivalent circuit model is depicted in Fig. 2. For y-polarized wave, the proximities of the succeeding lattices in each column and the current loops within the structure generate the same capacitances and inductances, however, dependent on the column, that is, the cell index, $n$. The electromagnetic properties of each column can be reduced to an equivalent parallel LC resonant circuit [1], of which the impedance is given by Eq. (1). In a narrow band around the LC resonance given in Eq. (2), the impedance is very high.


Figure 1: Configuration of the artificial surface having a frequency dependent reflection angle


Figure 2: Longitudinal-section of the two-layer structure and the equivalent circuit model of $n$th column of the structure.


Figure 3: Reflection phase of each cell with length $L_{1-11}$ for $y$-polarized wave. By varying the length of lattices, we can shift this curve to left or right, thereby determining the reflection phase value for a fixed frequency of 2.4 GHz .

$$
\begin{gather*}
Z_{s_{n}}=\frac{1}{\frac{1}{j \omega L_{n}}+j \omega C_{n}}=\frac{j \omega L_{n}}{1-\omega^{2} L_{n} C_{n}}  \tag{1}\\
\omega_{n}=\frac{1}{\sqrt{L_{n} C_{n}}} \tag{2}
\end{gather*}
$$

The reflection coefficient of each column can be expressed as following Eq. (3). The reflection phase of each column can be calculated by Eq. (4).

$$
\begin{align*}
\Gamma_{n} & =\frac{Z_{s_{n}}-\eta}{Z_{s_{n}}+\eta}=\left|\Gamma_{n}\right| e^{j \phi_{n}}  \tag{3}\\
\phi_{n} & =\operatorname{Im}\left\{\ln \left(\frac{Z_{s_{n}}-\eta}{Z_{s_{n}}+\eta}\right)\right\} \tag{4}
\end{align*}
$$

where $Z_{s_{n}}$ is the surface impedance of $n$th column and $\eta$ denotes the wave impedance for the plane wave, defined as the ratio of the $E$ - and $H$-fields. In the resonant condition given in Eq. (2), the reflection phase goes to zero in Eq. (4). The phase crosses through $\pm \pi / 2$ when $Z_{s_{n}}$ is equal in magnitude to the wave impedance of free space.

Unlike the EBG structure with identical unit cells, this metastructure has sub-wavelength lattices with graded lengths along the unit cell index, in order to generate different surface impedance and reflection phase curves across the face for $y$-polarized wave. By shortening or lengthening the length of each lattice, the phase curve of the reflection coefficient goes right


Figure 4: Far-field geometry of Huygens' secondary source array of unit cells positioned along the $x$-axis.


Figure 5: The reflection angle of the proposed artificial surface having frequency dependent reflection angle
or left in frequency domain, respectively. Thereby we can determine the patch lengths of $n$th column to get a graded reflection phase along the unit cell index for a fixed frequency of 2.4 GHz . The reflection phase diagram of each cell with length $L_{1-11}$ is depicted in Fig. 3. The reflection phase of the surface becomes linearly graded from $\pi / 2$ to $-\pi / 2$ with $-\pi / 10$ step at 2.4 GHz. At the upper or lower frequency ranges, the phase differences between edge cells have smaller gradients. Thus this surface gets a weaker steering effect at the side band of the 2.4 GHz. Furthermore, at the frequency ranges having a homogeneous reflection phase like less than 1 GHz or more than 5 GHz , the surface provide no more steered reflection and satisfy the Law of Reflection.

## 3. Frequency Dependent Reflection Properties

The reflection pattern of the proposed surface is calculated using Huygens' Principle. Referring to the geometry of Fig. 4, all the cells reflect identical amplitude of the normally incident wave. The reflection pattern and direction can be obtained by assuming the cells to be Huygens' secondary sources which are reradiating with a phase difference $(\Delta \phi)$ in reflection coefficient of each succeeding cell along $x$-axis, and it is given by Eq. (5).

$$
\begin{align*}
R . P .= & e^{j\left(-5 k d \sin \theta+\phi_{1}\right)}+e^{j\left(-4 k d \sin \theta+\phi_{2}\right)}+e^{j\left(-3 k d \sin \theta+\phi_{3}\right)}+e^{j\left(-2 k d \sin \theta+\phi_{4}\right)} \\
& +e^{j\left(k d \sin \theta+\phi_{5}\right)}+1+e^{j\left(k d \sin \theta+\phi_{7}\right)}+e^{j\left(2 k d \sin \theta+\phi_{8}\right)}+e^{j\left(3 k d \sin \theta+\phi_{9}\right)}  \tag{5}\\
& +e^{j\left(4 k d \sin \theta+\phi_{10}\right)}+e^{j\left(5 k d \sin \theta+\phi_{11}\right)}
\end{align*}
$$



Figure 6: Reflected $E$-field distributions with a normally incident plane wave. Instantaneous $E$-fields from FDTD simulation at (a) 2 GHz , (b) 2.4 GHz , (c) 3 GHz , and (d) 4 GHz are shown.
where $k$ is a free-space wavenumber. The direction of the main lobe, $\theta_{\text {peak }}$, is plotted in Fig. 5.
Figure 6 shows the reflected $E$-field distributions with a normally incident plane wave. Instantaneous fields from FDTD simulation are depicted at $2 \mathrm{GHz}, 2.4 \mathrm{GHz}, 3 \mathrm{GHz}$, and 4 GHz . The frequency dependent steered reflection is observed obviously. Maximum steering is obtained as $21.6^{\circ}$ at 2.4 GHz , as shown in Fig. 5. The half power beam width (HPBW) of the reflected pattern, which depends on the physical dimension of the surface, is $37.2^{\circ}\left(4.1^{\circ} \sim 41.7^{\circ}\right)$.

## 4. Conclusion

In this work, we introduced an artificial surface having a frequency dependent reflection angle. Using theoretical calculations and numerical simulations, it was shown that a reflection steering can be obtained from the graded reflection phase across the face, and that the reflection steering is dependent on the phase differences across the surface. These results can lead the employment of physically flat but electromagnetically curved structure such as flat parabola or hyperbola systems.

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