Accurate Calculation of Input Power in the FDTD Algorithm

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Abstract: In this communication, we investigate the conservation of power in the FDTD method when a system is excited by using either voltage or current sources. Though both the hard and soft sources can be employed to excite a system in the FDTD method, the soft source is more widely used than the hard one since the former one is more realistic in the practical applications. We have validated the conservation of power for both the loss and lossless systems in a commercial code that is based on the FDTD technique by using the approach outlined in this paper.

Keywords: FDTD, Power conversation.

I. Introduction

It is very important to accurately calculate the input incident power of an excitation source into an EM system to calculate the gain and radiation efficiency in an antenna radiation problem, and to determine the signal level in a coupled line relative to the transmitted power when estimating the signal integrity of a package [1]. The sources in an electromagnetic system can either be the electric or the magnetic current density [2, 3].

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \sigma \vec{E} + \vec{J} \tag{1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M} \tag{2}$$

The electric current is located on a grid set up for the electric field, and, is usually added to the equation that governs that field. Since the distribution of the electric fields inside the FDTD computational domain is not continuous, the incident power of an excitation source is calculated based on the formula $\iiint_v \vec{E} \cdot \vec{J}^* dv$, where $\vec{E}$ is inside the source region and the total field that is generated by the excitation $\vec{J}$. The basic function for the electric field is a Haar function, and the excitation $\vec{J}$ is assumed to have a uniform distribution inside each FDTD cell. However, in both the subsection 5.3.2 of Taflove and Hagness's book [1], and in the reference [2], the actual source cross-section is reduced approximately by a factor 0.2 to match the analytical solution, which varies with the cell size used in the FDTD simulation. As a consequence, we cannot accurately calculate the incident power in the FDTD simulation when using a non-uniform mesh and dealing with objects.

In this paper, we investigate the relationship between the incident, radiated and dissipated power for the case example of dipole. Numerical experiments have been carried out to demonstrate that the power can be calculated accurately in the FDTD method.

II. Numerical Experiments

For the test case, we consider a short dipole, as shown in the Fig. 1, which is simulated by using the FDTD code [3]. We assume that the dipole arms are PEC. A voltage source is used to excite the dipole at the small gap between two dipole arms. A box encloses the dipole, which is used to calculate the radiation power from the source, by integrating the Poynting vector, shown below in (3), on the surface of the enclosed box. The input power should equal the radiation power for a lossless system [4].

$$\frac{1}{2}\mathrm{Re}\left(\iiint_v \vec{E}\cdot\vec{J}^* dv\right) = \frac{1}{2}\mathrm{Re}\left(\oiint_s \vec{E}\cdot\vec{H}^* d\vec{s}\right) \tag{3}$$

where, V is the source region and S is the surface of Huygens' box, respectively. Since we add the excitation to the electric field, the relationship between the E and J can be expressed as:

$$E^{n+1}(i,j,k) = \frac{\Delta t}{\varepsilon + 0.5\sigma\Delta t} J^{n+1/2}(i,j,k) \tag{4}$$

The incident and radiation power are plotted in Fig. 1.

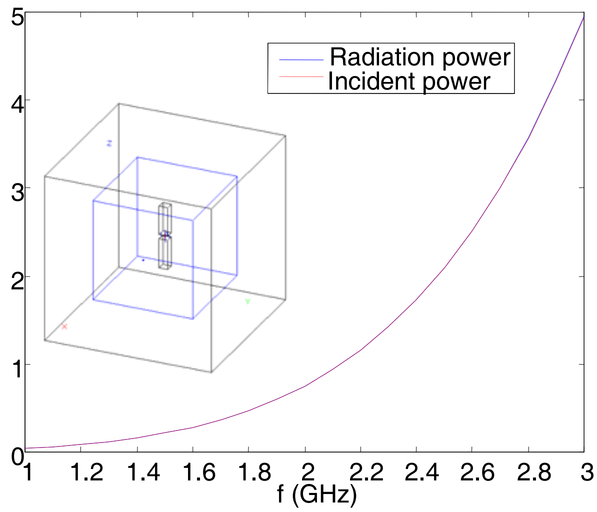

Figure 1. Radiation and incident power computed by using the FDTD code

Next, we inset a lumped element inside the gap between the two arms of dipole. The conductivity of this lumped element is calculated using the formula:

$$\sigma = \frac{LR}{D} \tag{5}$$

where, R, L and D are the resistance, length and size of cross section of the dipole, respectively.

For a lossy system, the input power should equal to the radiation plus the dissipated power.

$$\frac{1}{2}\mathrm{Re}\left(\iiint_v \vec{E}\cdot\vec{J}^* dv\right) = \frac{1}{2}\mathrm{Re}\left(\oiint_s \vec{E}\cdot\vec{H}^* d\vec{s}\right) + \frac{1}{2}\sigma|E|^2 \tag{6}$$

where the electric field, E, inside the integral of the left hand side is measured at the excitation point; the electric and magnetic fields, E and H, in the first term of the right hand side are measured on the closed surface; and the electric field E in the second term of the right hand side is measured inside the lumped element. The incident, radiated and dissipated power value are plotted in Fig. 2.

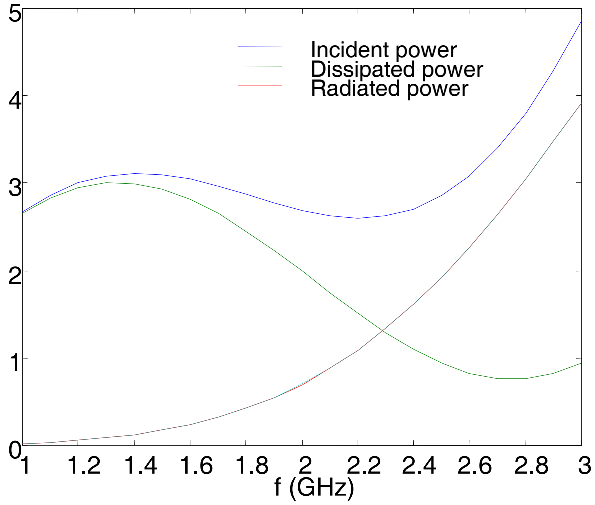

Figure 2. Radiation, dissipated and incident power simulated by using FDTD code

Next, we turn to the power conservation for the current source in the FDTD simulation. For a dipole includes one thin PEC with a square cross section, and is excited by a current loop at the center point. The following conservation condition should be satisfied for a lossless system:

$$\frac{1}{2}\mathrm{Re}\left(\iiint_v \vec{H}^* \cdot \vec{M}\ dv\right) = \frac{1}{2}\mathrm{Re}\left(\oiint_s \vec{E} \cdot \vec{H}^* d\vec{s}\right) \tag{7}$$

where V is the source region and S is a closed surface of box that encloses the volume. Since we add the excitation term to the equation for the electric field, the relationship between H and M can be expressed as:

$$H^{n+1/2}(i,j,k) = \frac{\Delta t}{\mu + 0.5\sigma_m \Delta t} M^n(i,j,k) \tag{8}$$

where, the μ and σ_m are the permeability and magnetic loss inside the source region, respectively. The incident and radiation power are plotted in Fig. 3.

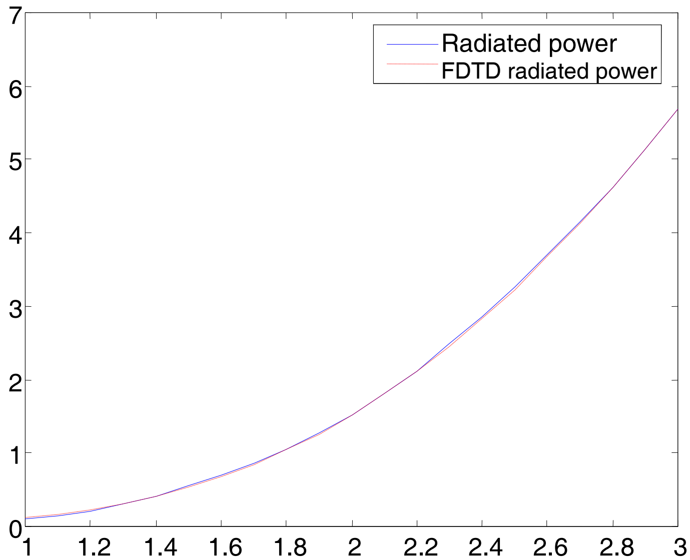

Figure 3. Radiation and incident power simulated by using FDTD code

For a lossy system, the input power should equal to the sum of the radiated and dissipated powers. Thus, we have

$$\frac{1}{2}\mathrm{Re}\left(\iiint_v \vec{H}^* \cdot \vec{M}\ dv\right) = \frac{1}{2}\mathrm{Re}\left(\oiint_s \vec{E} \cdot \vec{H}^* d\vec{s}\right) + \frac{1}{2}\sigma|E|^2 \tag{9}$$

where the magnetic field *H* inside the integral of the left hand side is measured at the excitation point; the electric and magnetic fields, *E* and *H*, in the first term of the right hand side are measured on the closed surface; and the electric field *E* in the second term of the right hand side is measured inside the lumped element. The incident, radiated and dissipated powers are plotted in Fig. 4.

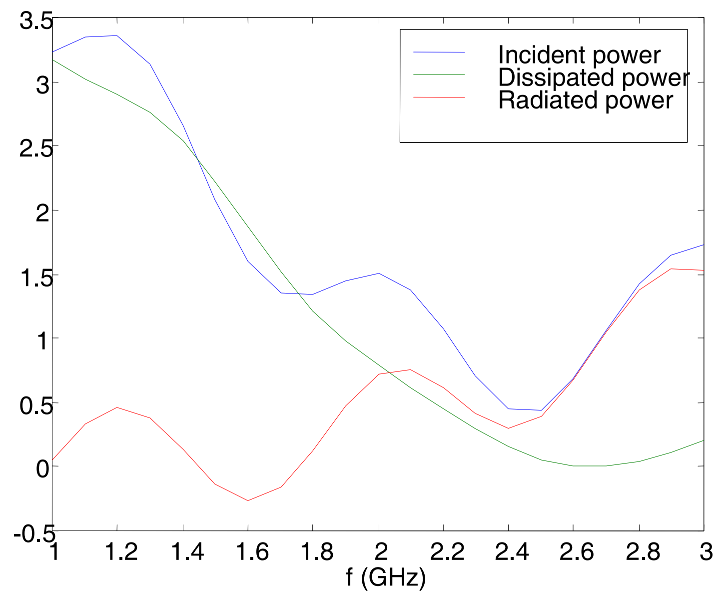

Figure 4. Radiation, dissipated and incident power simulated by using FDTD code

III. Conclusions

In this communication, we have investigated the power conservation in the FDTD simulation, and have shown that when the fields are measured correctly in the source region, the power conservation condition is satisfied for both the lossy and lossless systems without requiring a reduction in the size of FDTD cells.

This research reported herein was sponsored in part by Communication University of China under 111 Project.

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