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The Fundamental Characteristic of Hysteresis-Divided Optimization

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Abstract—In this paper, we propose a hysteresis divided optimization algorithm (HDO) which can be classified into one of meta-heuristic optimization algorithms. The searching area is divided into some region which corresponds to the number of particles. The algorithm consists with plural particles which search the optimal value of the given evaluation function. Each particle is placed in each region, and the particle searches the optimum value within the corresponding region. The size of each region is determined by the adjacent best informations. Also, each search region is discretized in order to reduce the computational amount of search process. The moving direction of the particle is determined by the output of the bipolar hysteresis. Namely, the particles continue to explore in each region. By using these properties of the HDO, we apply the HDO to the multi-solution problems (MSP). We confirm the search performance of the HDO by using well-known benchmark function of MSP. Based on the numerical simulation results, the HDO exhibits the good performance.

1. Introduction

Optimization problems are very important problems for engineering fields. Therefore, many researchers have investigated the solving algorithm of the optimization problems. In such optimization problems, there is a case that the evaluation function has multiple feasible solutions. Such problem is called Multi-Solution Problems (MSP) [1],[2]. The MSP is inevitable in practical/potential applications, therefore, several interesting methods have been studied. Especially, some researchers proposed the methods which applied the particle swarm optimization algorithms to the MSP [2]-[7]. The PSO is an excellent search algorithm for optimization problems, however, it is difficult to solve the MSP. The reason why it is difficult is that the PSO has a redundancy search process. Namely, plural particles searched the same area. Thus the particles might find the same solutions.

To overcome such problems, we propose a novel solving algorithm named Hysteresis Divided Optimization (HDO) algorithm which is based on the improved PSO. The search space is divided into some region by the HDO. The particles search the feasible solution in each region with parallel. In order to reduce the computational amount, the search space is discretized. Thus, the proposed system can be regarded as a discrete system. The interval of the search region is controlled by the thresholds of the bipolar hysteresis. In addition, the proposed system changes the thresholds which are divided each region. By changing the thresholds adaptively, the system improves searching ability effectively. We focus on a deterministic system to analyze the dynamics theoretically. That system does not contain any stochastic factors. By using a benchmark function of the MSP, we investigate the capability of our proposed algorithm.

2. Hysteresis divided optimization

In this section, we propose a hysteresis divided optimization algorithm. The hysteresis divided optimization algorithm is one of meta-heuristic optimization algorithm. This algorithm does not require the derivative information of the evaluation function. The algorithm consists with plural particles which search the optimal value of the given evaluation function. In our hysteresis divided optimization, the searching area is divided into some region which corresponds to the number of particles. Therefore, each particle is placed in each region, and the particle searches the optimum value within the corresponding region.

The dynamics of the HDO is described by

$$\begin{aligned} \boldsymbol{x}_i(t+1) &= \boldsymbol{x}_i(t) + \boldsymbol{\Delta}_i H(\boldsymbol{x}_i(t), \boldsymbol{Th}_{min_i}, \boldsymbol{Th}_{max_i}) \\ H(X) &= \begin{cases} +1 & \text{for } \boldsymbol{x}_i(t) \geq \boldsymbol{Th}_{min_i} \\ -1 & \text{for } \boldsymbol{x}_i(t) \leq \boldsymbol{Th}_{max_i} \end{cases} \end{aligned} \tag{1}$$

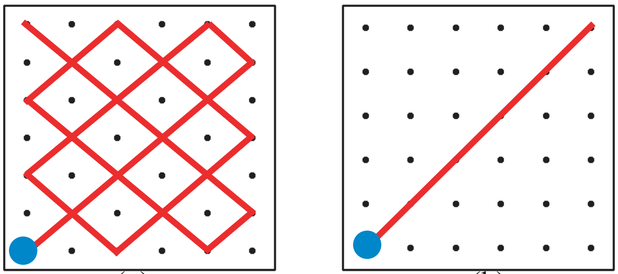
where x_i denotes the location vector of the *i*-th particle. Th_{min_i} and Th_{max_i} denote the threshold vector of the *i*-th particle. The search region of each particle is defined by the threshold vector. $H(X)$ is a bipolar hysteresis. When $H(X) = -1$, $H(X)$ is switched from -1 to +1 if the X reaches the positive threshold $\boldsymbol{Th}_{max_i}$. When $H(X) = +1$, $H(X)$ is switched fron +1 to -1 if the X reaches the negative threshold $\boldsymbol{Th}_{min_i}$.

In order to reduce the computational amount of search process, we discretize each region which is defined by $\boldsymbol{Th}_{max_i}$ and $\boldsymbol{Th}_{min_i}$. At first, we define the number of dvision of each region. $\boldsymbol{LP}$ denotes the number of lattice points of each region. Therefore, we can define an interval of the lattice points as

$$\boldsymbol{\Delta}_i = \frac{\boldsymbol{Th}_{max_i} - \boldsymbol{Th}_{min_i}}{\boldsymbol{LP}} \tag{2}$$

Based on this interval, the searching point of the particles is changed on every iterations as Eq. 1. The moving direction of the particle is determined by the output of the bipolar hysteresis. Namely, the particles continue to explore between $\boldsymbol{Th}_{min_i}$ and $\boldsymbol{Th}_{max_i}$.

The number of lattice points $\boldsymbol{LP}$ parameter plays an important role for searching. The aspect of the trajectory of each particle is depended on the number of lattice points in each dimension. Figure 1(a) and (b) illustrate the trajectory when the number of lattice points in each dimension is changed. Since we consider the MSP in 2-dimensional objective functions, the system has two kinds of number of lattice points; LP_x and LP_y. LP_x is the number of lattice points of the first dimension, and LP_y is the number of lattice points of the second dimension.



(a) (b)

Figure 1: (a)The trajectory of the case where each number of lattice points is the different.($LP_x = 6, LP_y = 7$) (b)The trajectory of the case where each the number of lattice points is the same.($LP_x = 6, LP_y = 6$)

Table 1: The number of trajectories of the 2-dimensional HDO for each combination of the lattice points

		LP_x				
		6	7	8	9	10
LP_y	6	6	2	2	2	2
	7	2	7	2	3	4
	8	2	2	8	2	2
	9	2	3	2	9	2
	10	2	4	2	2	10

Figure 1(a) shows the case where each the number of lattice points is the different, namely $LP_x = 6$ and $LP_y = 7$. In this case, the system exhibits the complex trajectory as shown in red line in Fig. 1(a). The searching region consists with 42 lattice points, and the system explores 21 lattice points. On the other hand, Fig. 1(b) shows the case where each the number of lattice points is the same, namely $LP_x = 6$ and $LP_y = 6$. The searching region consist with 36 lattice points, however, the system explores only 6 points.

Table 1 shows the number of trajectories of 2-dimensional HDO for each combination of the lattice points. The case where the number of lattice points are the same, some trajectories are co-existed, and each trajectory explores only a limited lattice points. On the other hand, almost all cases where the number of lattice points have the different values, the system has only two trajectories. In this case, the trajectory explores many lattice points compared with the same values as shown in Fig. 1, namely such case has a significant searching capability.

Table 2: Exploring rate of trajectories of the 2-dimensional HDO for each combination of the lattice points

		LP_x				
		6	7	8	9	10
LP_y	6	10/36	21/42	24/48	27/54	30/60
	7	21/42	12/49	28/56	21/63	23.5/70
	8	24/48	28/56	14/64	36/72	40/80
	9	27/57	21/63	36/72	16/81	45/90
	10	30/60	23.5/70	40/80	45/90	18/100

Table 2 shows the exploring rate of trajectories of the 2-dimensional HDO for each combination of the lattice points. The exploring rate means the rate at which the trajectory passes through the lattice points within the search region. In this case where some trajectories coexist in the same parameters, the average rate is represented in Table 2. The denominator of each rate in Table 2 denotes the total number of points in the searching region. This table indicates a potential to improve search performance if the region consists with the different number of lattice points.

In addition, the thresholds of the HDO are changed when a period of search has passed. The period is depended on the number of lattice points on the trajectory. The operation of the changing thresholds is described as

$$\begin{cases} Th_{min_{x_i}} = \dfrac{PB_{x_{i-1}} + PB_{x_i}}{2} \\ \quad \text{except for } \ i = 1, N+1, \ldots, N^2 - N + 1 \\ Th_{max_{x_i}} = \dfrac{PB_{x_i} + PB_{x_{i+1}}}{2} \\ \quad \text{except for } \ i = N, 2N, \ldots, N^2 \\ Th_{min_{y_i}} = \dfrac{PB_{y_{i-N}} + PB_{y_i}}{2} \\ \quad \text{except for } \ i = 1, 2, \ldots, N \\ Th_{max_{y_i}} = \dfrac{PB_{y_i} + PB_{y_{i+N}}}{2} \\ \quad \text{except for } \ i = N^2 - (N-1), N^2 - (N-2), \ldots, N^2 \end{cases} \tag{3}$$

$\boldsymbol{PB}_i = (PB_{x_i}, PB_{y_i})$ means the best location vector which gives the best evaluation value of i-th particle. $Th_{min_{x_i}}$ and $Th_{max_{x_i}}$ means the threshold of the divided region of the first dimension. $Th_{min_{y_i}}$ and $Th_{max_{y_i}}$ means the threshold of the divided region of the second dimension. Based on Eq. (3) the thresholds which divide the search region are changed. The meaning of Eq. (3) is the threshold is determined by the best information of the adjacent region. Note that the operation of Eq. (3) changes only the range of each search region. The number of the lattice points in the region is not changed. Thus, the interval of the lattice points is changed. Since the interval of the lattice points is changed, the system can search the new lattice points which are different from the previous lattice points. Thus, the system may find the new best location. For two or higher dimensional system, it is possible to overlap the search region since the thresholds of each region are determined by the adjacent best informations.

3. Experiments

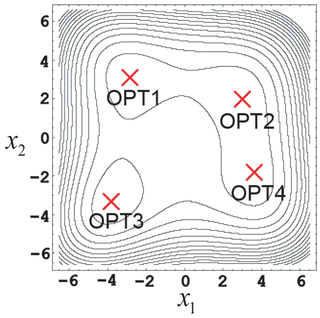


Figure 2: Contour map of the Himmelblau function where the red crosses denote the multi-solution.

In order to confirm the search performance of the HDO, we carry out a numerical simulation by using well-known MSP benchmark function. The benchmark function is Himmeiblau function as the following:

$$f_{\text{Himmelblau}}(\boldsymbol{x}) = \left(x_1^2 + x_2 - 11\right)^2 + \left(x_1 + x_2^2 - 7\right)^2 \quad (4)$$

where $\boldsymbol{x} = (x_1, x_2)^{\mathrm{T}} \in [-6, 6]^2$. This function has 4 global optima (solutions) as illustrated in Fig. 2 For the numerical simulation of the function, we define that the criterion value for successful search is $f_{\text{Himmelblau}}(\boldsymbol{x}) < 0.1$. Figure. 3 shows a typical example of the HDO search process. Figure. 3(a) shows the case $N = 4$ and (b) shows the case $N = 6$, then we set the lattice points as $LP_x = 7$ and $LP_y = 8$. m denotes the interval period for change the threshold by Eq. (3). Therefore $m = 0$ denotes the system does not change the thresholds. In the case of $N = 4$(see Fig. 3(a)), all regions don't satisfy the criteria on the three periods to change the thresholds. On the other hand, in the case of $N = 6$(see Fig. 3(b)), some regions satisfy the criteria. In this case, the HDO finds four feasible solutions which satisfy the criteria.

Table 3 shows the number of regions that satisfy the criterion $C1$ when the thresholds of the divided region are not changed. If the number of divided regions is increased or the number of lattice points of each region is increased, the number of the regions which satisfy the criteria is increased. Namely the resolution of the search space becomes high since the interval of the lattice points of the search region becomes fine. However the computational amount is increased when the resolution becomes high. Thus, there exists a trade-off between the search performance and the computational amount. To optimize such problem is one of our future problems.

Table 4 shows the number of regions that satisfy the criterion when the thresholds are changed. In this case, the maximum number of the changes is three. The red number indicates that the system can find all four feasible solutions. Comparing with the case where the thresholds are not changed, the system that the thresholds are changed can find all feasible solutions. Based on these results, we confirmed the search performance of the adaptive threshold system.

4. Conclusions

In this paper, we introduced the HDO, and we confirmed its fundamental performance. The algorithm consists with plural particles which search the optimal value of the given evaluation function. The searching area is divided into some region which corresponds to the number of particles. Each particle is placed in each region, and the particle searches the optimum value within the corresponding region. The size of each region is determined by the adjacent best informations. Therefore the HDO can be regarded as a kind of adaptive systems. Also, each search region is discretized in order to reduce the computational amount of search process. The number of the lattice points of each dimension of the search region exhibited very important role. We clarified that the trajectory in the region becomes long if each dimension of the search region has the different number of the lattice points.

We applied the HDO to the MSP. The results indicates that the HDO exhibits good searching performance. Especially, in the case where the size of the search region is varied, the searching performance is improved.

Finally, we enumerate some future problems of our study.

1. Theoretical analysis of the dynamics of the HDO
2. Estimation of the computation amount of the HDO
3. Improvement of the threshold change procedure

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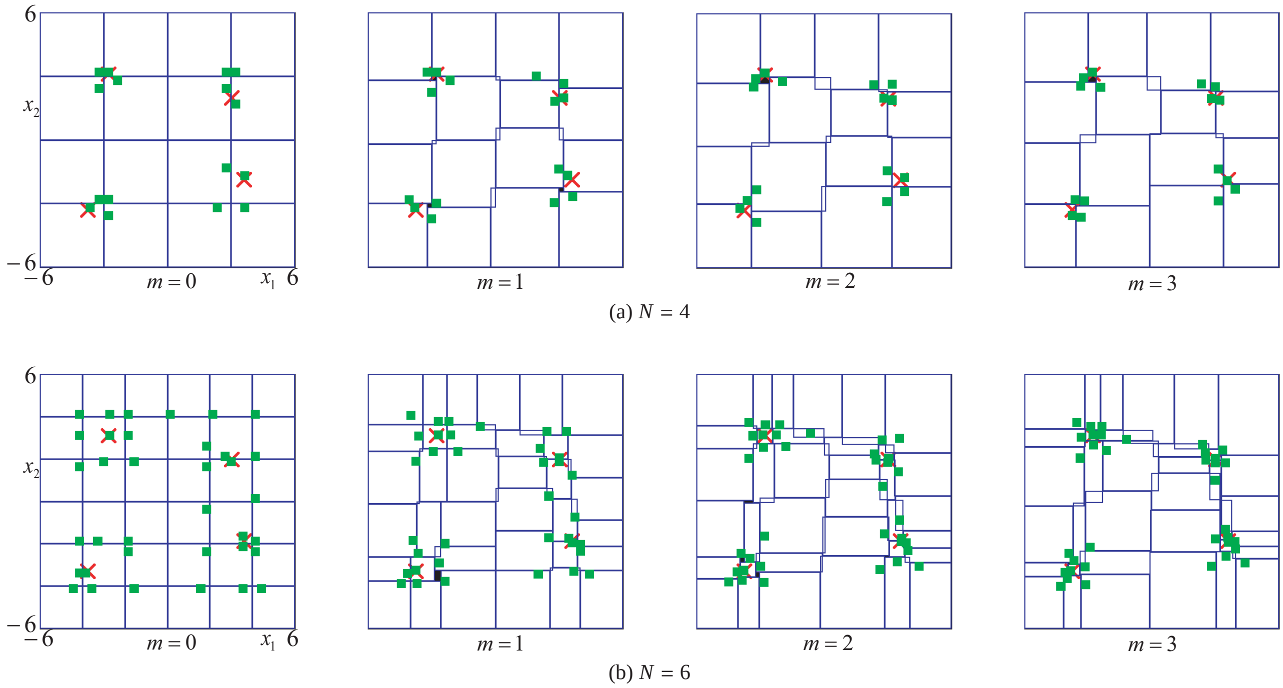

Figure 3: Search process of Himmelblau function. Blue squares denote the divided regions. Green square is best location of each divided regions.

Table 3: The number of regions that satisfy the criterion $C1$ when not changed threshold of the divided regions.

		$N(R)$			
		5(25)	6(36)	7(49)	8(64)
$LP_x \times LP_y$	6×7	1	0	0	0
	7×8	0	0	0	0
	8×9	0	0	0	0
	9×10	0	1	1	1
	10×11	1	0	0	0

Table 4: The number of areas that satisfy the criterion $C1$ when changed threshold of the divided regions.(Red number shows found the four solutions)

		$N(R)$			
		5(25)	6(36)	7(49)	8(64)
$LP_x \times LP_y$	6×7	3	4	4	5
	7×8	2	4	4	5
	8×9	2	3	4	5
	9×10	2	5	4	5
	10×11	3	5	4	6

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