Applicability of Quasistatic Approximation for Exposure Assessment of Wireless Power Transfer

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***Abstract*—Magnetic resonant coupling between two coils allows effective wireless transfer of power over distances in the range of tens of centimetres to a few metres. The resonant magnetic field also extends to the surroundings of the power transfer system. When a person is exposed to this magnetic field, electric fields are induced in the body. It is necessary to evaluate whether these fields satisfy the human exposure limits specified in international guidelines and standards. The objective of this work is to investigate the effectiveness of the quasistatic approximation for modelling human exposure to the magnetic fields of wireless power transfer systems. When valid, this approximation can greatly reduce the computational resources needed for the assessment of human exposure.**

I. Introduction

Wireless power transfer (WPT), which is based on magnetic resonant coupling between two coils, allows effective wireless transfer of power over distances in the range of tens of centimetres to a few metres [1], [2]. When users or bystanders are moving in the electromagnetic field produced by a WPT system, electric fields and currents are induced in the body. This raises concerns about the safety of WPT for general public use. Open questions about the exposure of humans to the fields of WPT need to be solved before the technology can be adopted widely.

Several international guidelines and standards limit the human exposure to electromagnetic fields [3]–[5]. In the guidelines developed by the International commission on non-ionizing radiation protection (ICNIRP) [3], [4], the *reference levels* for exposure are given in terms of the strength of the external electromagnetic fields, and the *basic restrictions* are defined in terms of the specific energy absorption rate (SAR) at frequencies higher than 10 MHz. It is notable that the magnitudes of the magnetic and electric fields used in WPT in the 10 MHz band considerably exceed the reference levels [6], [7]. Therefore, it is necessary to investigate whether the SAR induced in the body satisfies the basic restrictions. This investigation requires the use of computational dosimetry of the electromagnetic fields in the human body.

Until now, few studies have computationally investigated human exposure to electromagnetic fields of WPT systems [6]–[9]. A feature of the frequency band of WPT is that it falls between the low- and high-frequency regimes. At high frequencies, full-wave computational methods are used. These methods numerically solve the complete Maxwell equations, but they can be very intensive computationally, especially at lower frequencies. In contrast, at low frequencies, computationally effective methods, which are based on the quasistatic approximation, are used. The applicability of the quasistatic approximation for dosimetry of WPT is unclear, because the fields of WPT are highly resonant and the operation frequencies are much higher than the frequencies for which the quasistatic approximation has been previously used. This study discusses the applicability of the quasistatic approximation for the evaluation of human exposure to the fields of WPT. The quasistatic approximation can lead to an extreme reduction of computational requirements compared to full wave methods, and, when valid, it could greatly facilitate the exposure assessment of WPT.

II. Theory

A. Quasistatic approximation

Consider the scenario where a body consisting of biological tissue is exposed to an incident magnetic field $\mathbf{B}_0 = \nabla \times \mathbf{A}_0$ and an incident electric field $\mathbf{E}_0$ that are produced by a WPT system.

Under the quasistatic approximation, the electromagnetic fields are assumed to change so slowly that at each instant, the fields can be considered to be at equilibrium. In this work, the quasistatic approximation consists of the following assumptions. The first assumption is that the displacement current term in Maxwell's equations is set to be zero. The second assumption is that the secondary magnetic field induced by the currents flowing in the body is ignored. This is a valid assumption because the conductivities of biological tissues are much smaller than those of metals. With these assumptions, the electromagnetic problem splits into two separate parts: the magnetoquasistatic and electroquasistatic problems.

For the magnetoquasistatic problem, the induced electric field is solenoidal, i.e., there is no accumulation of electrical charges, and the electric current flows in closed loops. The electric field induced by the magnetic field is $\mathbf{E}_{\mathrm{MQS}} = -\nabla\phi_M - \frac{\partial}{\partial t}\mathbf{A}_0$, where ϕ_M is the electric scalar potential, which satisfies the following elliptic equation:

$$\nabla \cdot \sigma\nabla\phi_M = -\nabla \cdot \sigma\frac{\partial}{\partial t}\mathbf{A}_0 \quad (1)$$

with the boundary condition

$$\mathbf{n} \cdot \mathbf{J} = \sigma \mathbf{n} \cdot \left(-\nabla \phi_M - \frac{\partial}{\partial t} \mathbf{A}_0 \right) = 0. \quad (2)$$

The electroquasistatic electric field is irrotational. Its source is a slowly pulsating surface charge distribution that is induced on the surface of the body by the external electric field. The induced electric field is $\mathbf{E}_{\mathrm{EQS}} = -\nabla \phi_E$, where the electric scalar potential ϕ_E satisfies the homogeneous elliptic partial differential equation

$$\nabla \cdot \sigma \nabla \phi_E = 0 \quad (3)$$

with the boundary condition

$$\mathbf{n} \cdot \mathbf{J} = -\sigma \mathbf{n} \cdot \nabla \phi_E = -\frac{\partial}{\partial t} \varrho_s, \quad (4)$$

where $\varrho_s = \epsilon_0 \mathbf{n} \cdot \mathbf{E}_{\mathrm{ext}}$ is the surface charge distribution induced by the external electric field $\mathbf{E}_{\mathrm{ext}}$. The electroquasistatic approximation results in more complicated calculations, as determining the external electric field $\mathbf{E}_{\mathrm{ext}}$ from the incident electric field $\mathbf{E}_0$ is a separate nontrivial task that requires the use of numerical methods.

In this work, we define

$$\mathbf{E}_{\mathrm{FQS}} = \mathbf{E}_{\mathrm{MQS}} + \mathbf{E}_{\mathrm{EQS}}, \quad (5)$$

which is the electric field by the "full quasistatic" approximation, i.e., it includes the contribution from both incident magnetic and electric fields. Possible phase differences [1] between the magneto- and electroquasistatic electric fields were ignored in this work to consider the worst case scenario.

B. Full-wave analysis

In this work, full-wave analysis means analysis of the electric and magnetic fields using "full" Maxwell's equations, taking into account displacement current and the secondary magnetic and electric fields. Full-wave analysis also takes into account the effects of the presence of the body on the power transfer characteristics. In full-wave analysis, the magnetic and electric fields are coupled.

III. Applicability of QS approximation

A. Assessment of human exposure

In [9], we investigated the applicability of the quasistatic approximation for SAR calculations for a WPT system that consisted of two identical perfectly electrically conducting helical coils [10]. The dimensions of the coils were the following: diameter 30 cm, width 20 cm, number of turns 5, and wire diameter 2 mm. The odd resonance mode (11.36 MHz) was considered. The transmitting coil was excited by a voltage source located at the midpoint of the wire. Our investigation was limited to only one frequency, as the geometry of the system would need to be altered for each operating frequency.

A cylindrical human phantom whose dielectric properties were equal to 2/3 of those of the muscle tissue [11] was placed next to the coils. Some of the cases that we considered are shown in Fig. 1. Due to the presence of the cylinder, the electric field is pertubated, resulting in mismatch of the impedance or lowered transfer efficiency. To correct this, for each case, the resonant frequency was kept constant at 11.36 MHz by adding a suitable capacitance to the input voltage source. This simulated a realistic power transfer system, where an active feedback circuit controls that the transfer frequency stays unchanged when humans and objects move in the vicinity of the system.

The electric field and the magnetic vector potential near the WPT system and inside the cylinder were first determined using full-wave analysis (FEKO, EMSS, South Africa). The calculated magnetic vector potential was used for the magnetoquasistatic analysis. For the electroquasistatic analysis, we calculated the surface charge distribution ϱ_s on the surface of the cylinder from the normal component of the external electric field by the Gauss law (while neglecting the full-wave electric field inside the cylinder). The purpose of performing the full-wave analysis in advance of the quasistatic analysis was to make sure that the field sources were identical for both approaches, which allowed direct comparison of the results. In practical simulations, one would determine the magnetic vector potential/surface charge density using methods other than full-wave analysis (because we would already know all induced quantities after the full-wave analysis has finished). For numerically solving equations (1) and (3), an in-house solver that implements the scalar-potential finite-difference method [12] was used.

The specific absorption rate (SAR) was calculated from the induced electric fields by

$$SAR = \frac{\sigma}{2\rho} |\mathbf{E}|^2, \quad (6)$$

where $\rho = 1000$ kg/m^3, and it was averaged over 10 g cubical volumes [13]. The SAR calculated with magnetoquasistatic approximation was compared with the SARs calculated with the full quasistatic approximation. In addition, the full quasistatic SAR was compared to the SAR determined using full wave analysis. The errors in the SAR were defined as

$$\text{error I} = \frac{SAR_{\mathrm{MQS}} - SAR_{\mathrm{FQS}}}{SAR_{\mathrm{FQS}}} \times 100\% \quad (7)$$

$$\text{error II} = \frac{SAR_{\mathrm{FQS}} - SAR_{\mathrm{FW}}}{SAR_{\mathrm{FW}}} \times 100\%, \quad (8)$$

where SAR_{MQS}, SAR_{FQS}, and SAR_{FW} are the peak 10 g averaged SARs calculated using magnetoquasistatic, full quasistatic and full-wave analysis, respectively. For simplicity, we only compared the peak SAR values. This comparison is valid because the locations of the peak SARs for MQS, FQS, and full wave solutions were located close to each other in each case (less than 2 cm difference).

Table I shows the error of the magnetoquasistatic solution compared with the full quasistatic solution. The magnitude of the error decreases when the distance between the cylinder and the coils increases. This is due to the fact that the external electric field is more concentrated near the coils

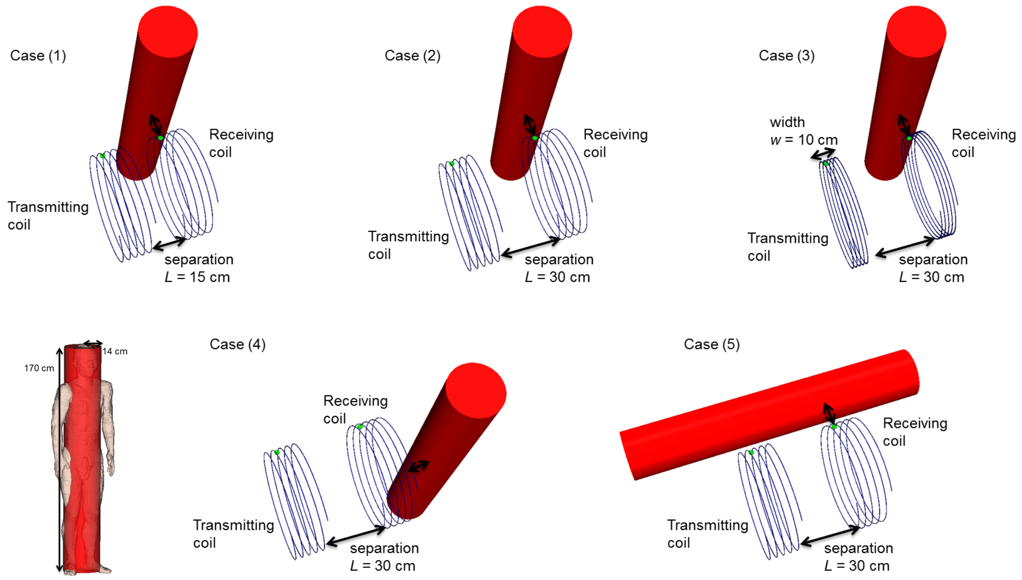

Fig. 1. Investigated scenarios. In all cases except in case (3), the width of the coils was 20 cm. Bottom left: the dimensions of the cylinder compared to those of an average Japanese adult male.

than the magnetic field. The errors are the largest in case (5). For this case, the cylinder effectively "short circuits" the two coils. In summary, it seems that it is acceptable to ignore the contribution of the electroquasistatic electric field on the SAR. For exposure assessment, the primary advantage of this observation is that it is sufficient to determine the magnetic field distribution of the WPT system—modelling the external electric field, which can be complicated and depends on the position and shape of the body phantom, is not needed. Previously, it has been shown that the magnetic field is negligibly disturbed by the body [6], [7], [9], [14]. Therefore, the magnetic field can be first determined in free space, for instance, using method of moments, and then the same field can be used for magnetoquasistatic SAR calculations. There is no need to recalculate the magnetic field if the position or the shape of the body phantom changes.

Table I also shows the comparison between the SAR calculated using the full quasistatic approximation and full wave analysis. The difference between the two SARs is typically in the range $-10\ldots+10\%$, and no clear pattern can be observed. These relatively "random" differences are likely due to different computational methods that were used: the finite-element method with tetrahedral elements was used for full-wave analysis, but quasistatic calculations used the scalar-potential finite-difference method.

B. Power transfer for medical implants

The fact that the magnetic field is not perturbed by the presence of the body enables another application for WPT: transferring power from outside to inside the body for charging of batteries of medical implants. The effects of biological tissue on the self and mutual lumped inductances of magnetically coupled coils were analysed in [15]. One of the investigated scenarios is shown in Fig. 2. The analysis was performed numerically using the full-wave finite-element method.

TABLE I
ERROR OF THE QUASISTATIC APPROXIMATIONS IN THE PEAK 10 G AVERAGED SAR. D IS THE DISTANCE BETWEEN THE CYLINDER AND THE COILS. NEGATIVE VALUES MEAN THAT THE SAR IS SMALLER THAN THE REFERENCE VALUE.

	Case (1)	Case (2)	Case (3)	Case (4)	Case (5)
Error I (%, magnetoquasistatic versus full quasistatic)					
D = 1.0 cm	−16.7	−1.9	−5.0	−9.2	−28.9
D = 3.0 cm	−7.5	−1.0	−4.5	−4.9	−21.3
D = 5.0 cm	−4.1	−0.7	−3.9	−4.9	−18.2
D = 10 cm	−0.6	−0.2	−3.1	−4.9	−14.2
Error II (%, full quasistatic versus full wave)					
D = 1.0 cm	−11.9	−9.4	14.1	6.4	0.0
D = 3.0 cm	−0.5	−8.3	11.2	2.5	−2.4
D = 5.0 cm	4.1	−5.0	10.1	1.7	−3.5
D = 10 cm	−0.7	−11.9	5.8	0.6	−12.1

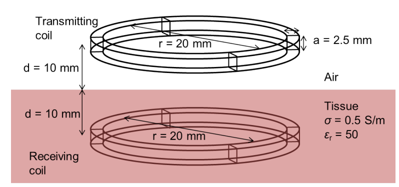

Fig. 2. Configuration of the coils. One of the coils is located in air and the other is embedded in biological tissue.

If the quasistatic approximation is valid, the inductances of the two coils should remain the same whether the coil is embedded in biological tissue or located in air. As shown in Table II, the presence of the biological tissue does not noticeably alter the coil inductances for frequencies lower than or equal to 10 MHz. This means that the magnetic field penetrates unobstructed into biological tissues which further

TABLE II
THE FREQUENCY DEPENDENCY OF THE SELF AND MUTUAL LUMPED COMPLEX INDUCTANCES OF THE TRANSMITTING AND RECEIVING COILS IN THE SCENARIO SHOWN IN FIG. 2. THE REAL RESISTANCE AND INDUCTANCE CAN BE OBTAINED FROM THE COMPLEX INDUCTANCE L^* AS $R = -\omega \mathrm{Im}\{L^*(\omega)\}$ AND $L = \mathrm{Re}\{L^*(\omega)\}$.

	Inductance (nH)		
Frequency	Transmitting coil	Mutual	Receiving coil
1 kHz	55.9	7.9	55.7
10 kHz	55.9	7.9	55.6
100 kHz	55.8	7.9	55.6
1 MHz	55.8	$7.9 - 0.2i$	$55.6 - 0.1i$
10 MHz	55.8	$7.9 - 0.5i$	$55.7 - 0.6i$
100 MHz	55.7	$8.1 - 2.1i$	$56.7 - 6.6i$

supports the validity of the quasistatic approximation.

IV. DISCUSSION

The effectiveness of the quasistatic approximation for human exposure assessment was investigated using a cylindrical human phantom placed near a WPT system operating at 11.36 MHz. Comparison with the full wave analysis showed that the quasistatic approximation leads to an error of about ±10% in the SAR. It was also shown that the SAR is primarily induced by the incident magnetic field. The SAR due to the external electric field of the WPT system is much smaller and can be ignored.

The magnetic field distribution around the coupled coils stays almost unaltered independent of the position of the body with respect to the coils [6], [7], [9]. The requirement for the above is that the shift in the resonant frequency/impedance mismatch for each body position is corrected by adjusting the input capacitance appropriately. Then the magnetic field distributions for the original (free space) and adjusted resonance modes are almost identical. This is likely to be true for any realistic WPT system. Therefore, for human exposure assessment, it is sufficient to determine magnetic field of the WPT system in free space, and then use this magnetic field for magnetoquasistatic analysis of the induced electric field.

The observation that the external electric field can be ignored seems to conflict with some recent studies [8], [16]. Namely, for the exposure to *uniform* magnetic and electric fields, analytical calculations show that the effect of the incident electric field cannot be ignored [16]. However, in this study, the sources of the field are located close to body, not at an infinite distance. Therefore, the presence of the body alters not only the external electric field but also the *sources* of the field. After the resonant frequency is tuned so that it stays constant by a feedback circuit, the resulting magnetic field remains almost unchanged from the case of free space. However, the external electric field is altered in a way that reduces its impact on the electric field induced inside the body.

As discussed above, the magnetic field is not perturbed by humans or objects placed near the system. We have previously shown that there is a strong correlation between the incident magnetic field and the electric fields induced in the body [17]. Therefore, magnetic field measurements are a sufficient and practical way for assessing human exposure to WPT. Another consequence of the validity of the quasistatic approximation is that embedding one of the coils in biological tissue does not degrade the magnetic coupling performance, which shows that the technology is applicable for use for implanted or on-body devices.

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