# Survival Traffic Ratio Analysis for Cascading Failure in Interdependent Networks


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***Abstract*— This paper proposes a new measure for the impacts of cascading failures in interdependent networks. This measure adds information about traffic paths, expressing the routes conveying traffic on a telecommunication network, to the existing model. The survival traffic ratio of this improved model is defined as the ratio of the surviving capacity of traffic which telecommunication network can ensure after the cascading failure has occurred. While the existing measure is unreasonable because it focuses on only the topology of networks, our measure is advantageous in terms of considering the traffic capacity degradation caused by cascading failure, which affects customer satisfaction. We implemented a software program to compute both the traditional measure and the survival traffic ratio, and the numerical results demonstrate usefulness of our measure as it surely clarified what nodes should be watched and given high priority in guarding against cascading failures.**



***Keywords*— Cascading failures, Interdependent networks, Telecommunications networks, Power networks, Survival traffic ratio**


## I. INTRODUCTION

Cascading failures cause damage over a wide area to telecommunications networks. They are successive failures of elements triggered by only a failure of one element.

Many researchers have studied how to protect users from cascading failures. Almost all of their approaches are based on quantitatively analyzing the impact of a cascading failure so that it can be minimized by designing an appropriate telecommunication network. For quantitative analysis, cascading failures must be modeled mathematically. Such models include two major types. One is a single network model for telecommunication networks [1][2][3]. The other is interdependent model expressing the cascading mechanism in interactions between telecommunication networks and power networks [4][5].

We focus on the latter type because such a failure actually occurred in Italy [4], and it is a serious concern that this type may be experienced in other countries or even world-wide.

Reference [4] proposed an analysis method for this type, where the ratio of surviving nodes to original nodes, $N'/N$, expresses the impact of the cascade mechanism. Reference [5] proposed an improved model and analysis using a similar measure. However, both are problematic for measuring the impact on telecommunication networks, because they are estimated using only the topology of the network and do not include any information on capacity degradation.

The degradation of traffic capacity should be considered when we measure the impact of cascading failures in a standard reliability design scheme for telecommunications networks [6].

This paper proposes a new measure to express the impact of cascades in interdependent networks that considers capacity degradation. We call this measure the survival traffic ratio.

## II. PRELIMINARY

A graph is defined as a set of nodes and links, where a link is a pair of nodes. We denote $G$ by $G = (V, E)$, where $V$ is the set of nodes of $G$ and $E$ is the set of nodes of $G$. If a graph $G_0 = (V_0, E_0)$ satisfies $V_0 \subseteq V$ and $E_0 \subseteq E$, then $G_0$ is called a subgraph of $G$.

If the end nodes of link $e$ are $i$ & $j$, we write $e = (i, j)$. If node $i$ is an end node of link $e$, then $i$ is said to be connected to $e$. The path between two nodes $i_1$ & $i_m$ is defined by the alternating sequence of nodes & links, $i_1$ - $(i_1, i_2)$ - $i_2$ – $(i_2, i_3)$ - … - $(i_{m-1}, i_m)$ - $i_m$. If a path includes $L$ links the length of this path is taken to be $L$. If there is a path between $i_1$ & $i_m$, we say that $i_1$ & $i_m$ are connected. If the length of a path between $i_1$ & $i_m$ is the minimum among all paths between $i_1$ & $i_m$, it is called the shortest path between $i_1$ & $i_m$.

If $G_M$ is a subgraph of $G$ and it is a maximal subgraph in which every pair of nodes is connected, then it is called a 'connected component'. The connected component having the maximum number of nodes among all connected components is called the maximum connected component.

In this paper, we suppose two graphs, $G_1 = (V_1, E_1)$ and $G_1 = (V_2, E_2)$, satisfying $V_1 \cap V_2 = \phi$ and $E_1 \cap E_2 = \phi$, where $\phi$ means the empty set. We assume $|V_1| = |V_2|$, where $|S|$ is defined as the number of elements in any finite set $S$. We assume that there is a bijective mapping, denoted by $f$, from $V_1$ to $V_2$. We call $f(v_1) \in V_2$ *the corresponding node of* $v_1 \in V_1$ and we call $f(v_2)^{-1} \in V_1$ *the corresponding node of* $v_2' \in V_2$. An illustrative example is shown in Fig. 1, where the dotted lines show the above correspondences.

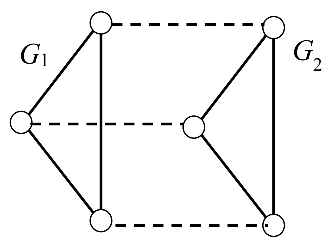


Fig. 1 Example of $G_1$ & $G_2$.

## III. EXISTING RESEARCH

There are two major types of cascading failure. One type is in a single network [1][2][3]. The other type is caused by interactions between two interdependent networks [4][5].

A typical model for the former type involves transferring congestion from one component to other components. If a component stops functioning (the trigger failure), then all traffic going through this component must be conveyed by other reconfigured routes. If the rerouting causes congestion stopping several other components, this series of events results in a cascading failure.

A typical model for the latter type is when the failure of a function in one component (a trigger failure) in telecommunication network causes a function of the corresponding component of a power network to stop by error control, and this stoppage then leads to stoppage of another component in the telecommunication network. The repetition of this series of events is a cascading failure. (Sometimes, the trigger failure occurs in the power network.)

This paper focuses on the latter model, because we agree with the statement of refs. [4][5] that the recent trend of telecommunications and power network architectures is toward increasing interdependency.

The scope of our research should be clarified though, because different models, such as in refs. [4][5], have been proposed for interdependent networks. Here, we decided to focus on the model of ref. [4], because it reflects detailed research about a notorious cascading failure really occurred in Italy 2003. We believe there are strong chances that a similar cascading failure may occur in Japan or some other country in the future. Its effect may even be world-wide.

In Sections A, B and C below, we explain the model of ref. [4], the conventional measure, and its problems.

### *A. Model for cascading failures in interdependent networks*

We suppose two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, where $G_1$ represents a telecommunication network and $G_2$ represents a power network.

The following is the model of cascading failure proposed in ref. [4], where $v_0 \in V_1$ is the trigger node implying that the cascading failures are caused by the deletion (implying failure) of this node.

Cascading failure mechanism

1.Delete $v_0$ from $V_1$, and delete all links connected to $v_0$ in $V_1$.

Let the graph obtained by this operation on $G_1$ be $G_1' = (V_1', E_1')$.

2. Delete $f(v_0)$ from $V_2$ and delete all links connected to $v_0$ in $V_2$.

Let the graph obtained by this operation on $G_2$ be $G_1' = (V_2', E_2')$.

3. For each link $e_a = (i_a, j_a)$ in $E_2'$, if $f^{-1}(i_a)$ and $f^{-1}(j_a)$ are not connected in $G_1'$,then link $e_a$ is deleted.

4. For each link $e_b = (i_b, j_b)$ in $E_1'$ if $f(i_b)$ and $f(j_b)$ are not connected in $G_2'$, then link $e_b$ is deleted.

5. Repeat 3 & 4 until there are no links which can be deleted.

An illustrative example of steps 1-5 is given in Fig. 2, where the cross indicates the trigger node.

$G_1^{\Delta}$ is the graph finally obtained from $G_1$ by the cascading failure procedure.

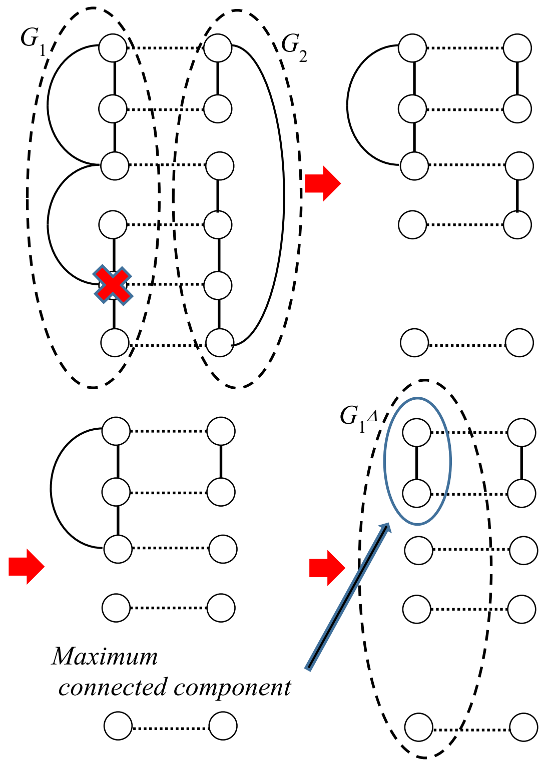


Fig. 2 Illustrative example for cascading failures.

### *B. Existing measure for impact of cascading failure*

Reference [4] defined $N$ as the number of nodes included in $G_1$, and $N'$ as the number of nodes included in maximum connected component of $G_1^{\Delta}$. It defined $N'/N$ as the measure of impact of a cascading failure.

For example in Fig. 2, $N'/N$ is 2/6 = 0.333…. The bigger $N'/N$ is, the smaller its impact becomes.

### *C. Problems of existing research*

$N'/N$ is determined using only information on the topology of the surviving network $G_1^{\Delta}$, However, it is a poor measure of the impact on telecommunication networks, because it does not consider traffic-capacity degradation.

Traditionally, reliability design schemes for telecommunication networks, as in refs. [6] & [7], emphasize that users experience inconvenience when the traffic capacity falls to half even if the topology is connected. Therefore, a measure of impact of a cascading failure should also consider the traffic-capacity degradation.

## IV. PROPOSAL

### *A. Additional information to model*

To define a measure expressing the traffic-capacity degradation, we give additional information to $G_1$ about traffic paths [7] composing routes to convey traffic.

We define $m$ to be the number of traffic paths $T_1$, $T_2$,…, $T_m$ on $G_1$, where each path is defined as a specific subset of $V_1 \cup E_1$. The elements included in a traffic path are the nodes and links the path goes through. For example, there are four traffic paths $T_1$, $T_2$, $T_3$, and $T_4$ in Fig. 3, where $T_1 = \{v_1, e_1, v_2\}$, $T_2 = \{v_1, e_1, v_2, e_2, v_3\}$, $T_3 = \{v_1, e_3, v_3\}$ $T_3 = \{v_2, e_2, v_3\}$, as shown by the two-way arrows.

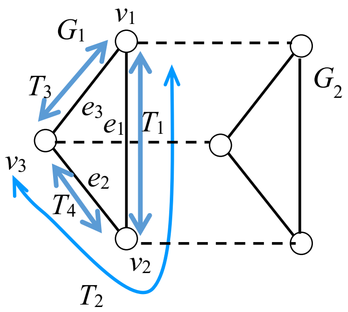


Fig. 3 Example of traffic paths.

We say that traffic path $T_i$ is working if and only if any element included in $T_i$ is not deleted by the cascading failure.

Let $Z$ be $Z = \{T_1, T_2, \dots, T_m\}$. For any $Z_0 \subseteq Z$, mapping $C_0(Z_0)$ outputs a non-negative real number expressing the capacity under the assumption that all traffic paths included in $Z_0$ are working.

We never assume simple additivity, i.e., $C_0(\{T_i, T_j\}) = C_0(\{T_i\}) + C_0(\{T_j\})$, because $C_0(\;)$ has different values for different policies in traffic path assignments.

An illustrative example is shown below.

Assume $C_0(\{T_2\}) = C_0(\{T_3\}) = 100$ in Fig. 3. If each of $T_2$ & $T_3$ ensures half the capacity (load share) of the traffic between nodes $v_1$ & $v_3$, then $T_2$ & $T_3$ both working ensures 200 between these nodes. In this case, $C(\{T_1, T_2\}) = C_0(\{T_1\}) + C_0(\{T_2\}) = 200$ is true. However, if $T_2$ is a complete backup for $T_3$, then both paths working ensures only 100 because $T_2$ being a complete back up implies that it does not work if $T_3$ is working. In this case, $C_0(\{T_1, T_2\}) = C_0(\{T_1\}) = C_0(\{T_2\}) = 100$.

Additionally, we assume $C_0(Z_1) \leq C_0(Z_2)$ if $Z_1 \subseteq Z_2 \subseteq Z$, to avoid the unreasonable case that increasing the working traffic paths leads to a degradation of capacity.

An example of $C_0(\;)$ that does not show simple additivity, but satisfies the above assumption is described below.

<u>Example of $C_0(\;)$</u>

$C_0(\phi) = 0$, $C_0(\{T_1\}) = C_0(T_2) = C_0(T_3) = C_0(T_4) = 100$, $C_0(\{T_1, T_2\} = C_0(\{T_1, T_3\}) = C_0(\{T_1, T_4\}) = C_0(\{T_2, T_4\}) = C_0(\{T_3, T_4\}) = 200$, $C_0(\{T_2, T_3\}) = 100$, $C_0(\{T_1, T_2, T_3\}) = C_0(\{T_2, T_3, T_4\}) = 200$, $C_0(\{T_1, T_2, T_4\}) = C_0(\{T_1, T_3, T_4\}) = C_0(\{T_1, T_2, T_3, T_4\}) = 300$

Let $H$ be a graph $G_1$ or graph obtained by deleting some nodes or links from $G_1$. $Z_H$ is defined as the set of all working traffic paths of $H$.

Accordingly, $C(H)$ is defined as $C(H) = C_0(Z_H)$.

In the example shown in Fig. 3 with $C_0$ defined as 'Example of $C_0$', if $H$ is obtained by deleting $e_1$ from $G_1$, then $C(H) = C_0(\{T_3, T_4\}) = 200$.

### *B. Survival traffic ratio*

The measure expressing the impact of a cascading failure is $C(G_1^{\Delta})/C(G_1)$; we call it the 'survival traffic ratio'. This measure implies the extent of surviving capacity as for traffic volume after the occurrence of a cascading failure.

The survival traffic ratio in the case of Fig. 2 is 100/1500 = 0.0666…. , when we have a single traffic path between each pair of nodes of $G_1$ and $C_0(Z_0)$ is defined as 'the number of traffic paths included in $Z_0$' × 100.

We emphasize that the survival traffic ratio fits real telecommunications networks through appropriate assignment of traffic paths and giving an appropriate function for $C_0(\;)$.

## V. Analysis Example

We developed a software program to compute $N'/N$ and the survival traffic ratio; its inputs are $G_1$, $G_2$ and information on traffic paths.

We ran the program in the following environment.

OS: Windows 10 Home 64bit
CPU: Intel® Core™ i5-4590@3.30GHz
Memory DDR3-1333 8GB
Language: Python 3.6

This section shows examples of analyzing our proposal.

### *A. Target Model*

The two analysis targets are the topologies of $G_1$ & $G_2$ illustrated in Fig. 4 (Target 1) and $G_1$ & $G_2$ illustrated in Fig. 5 (Target 2).

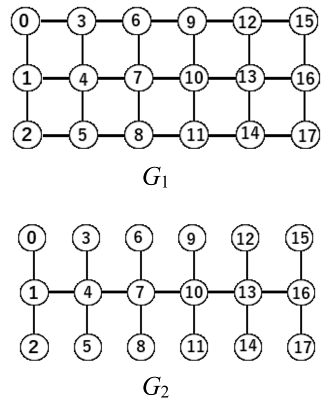


Fig. 4 Topologies of $G_1$ & $G_2$ (Target 1)

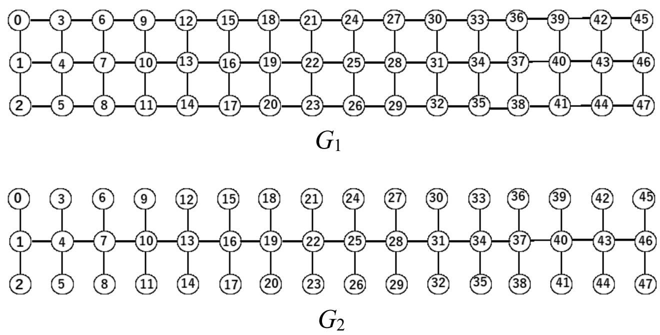


Fig. 5 Topologies of $G_1$ & $G_2$ (Target 2).

The numbers in these figures are node identifiers. Nodes having the same number correspond to each other, where corresponding nodes are defined by $f$ in Subsection II.

The following assumptions were applied to both targets.

A1. A single traffic path is assigned to every pair of nodes in $G_1$. The shortest path between each pair of nodes is selected as this traffic path. If there are two or more shortest paths, one of them is randomly selected.

A2. $C_0(Z_0)$ is defined as 'the number of traffic paths in $Z_0$' × 100.

A3. We do not assume multiple trigger-nodes-failures, like ref. [4].

### *B. Analysis Results*

This subsection shows analysis results for different trigger nodes.

Figure 6 (7) shows the result for Target 1 (2), where the vertical axis depicts $N'/N$ (red dotted line) and the survival traffic ratio (blue solid line) after occurrence of a cascading failure caused by trigger nodes noted by the number on the horizontal axis.

The plotted data for $N'/N$ are results sorted in ascending order from left to right.

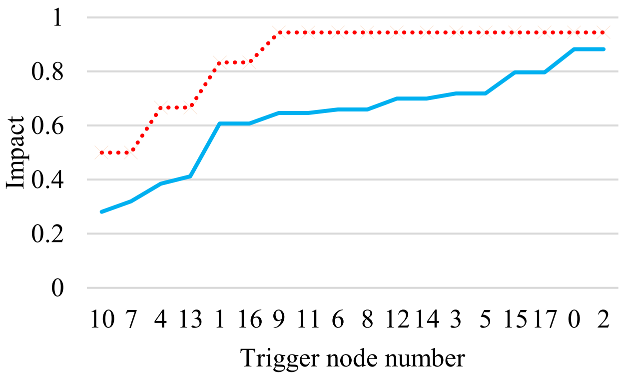


Fig. 6 Result for Target 1.

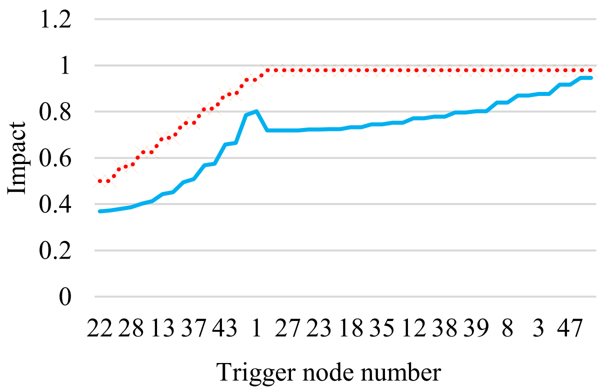


Fig. 7 Result for Target 2.

The total computation time needed for plotting Fig. 6 was 8.53 seconds. It was 408.72 seconds for Fig. 7.

### *C. Discussion*

The survival-traffic-ratio computation does not cause a heavy computational burden even for quite large networks.

The fluctuations of the two curves in each figure are quite different. In particular, while the curve of $N'/N$ at trigger node 1 in Fig. 7 is flat, the survival traffic ratio shows a sudden dip. Accordingly, while the value of $N'/N$ being nearly 1 for nodes 27, 23, 18, 35, 12, 38, 39, 8, 3, 47 in Fig. 7 suggests that these nodes don't have to be monitored for failures, the survival traffic ratio indicates otherwise; hence, we must monitor and maintain almost of all nodes from this viewpoint.

## VI. Conclusion

This paper proposed a new measure of impact of cascading failures in interdependent networks. The new measure adds information about the routes conveying traffic on telecommunications networks to the existing model of cascading failures of interdependent networks and defines the survival traffic ratio for this improved model.

We implemented a software program to compute both $N'/N$ and the survival traffic ratio. The numerical results demonstrate the usefulness of our proposal by showing that we can identify which nodes should be monitored and given high priority to protect them against cascading failures.

Our future work will be to clarify the above findings by conducting more numerical experiments using the software program and devise models for more accurately describing cascading failures that have occurred in various countries.

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