Impact of the Channel Estimation Error on the Array Processing Based decoder for QO-STBC

Haixia Zhang, Dongfeng Yuan
School of Information Science and Engineering
Shandong University
China
Email: (haixia.zhang,dfyuan)@sdu.edu.cn

Feng Zhao
Department of Science and Technology
Guilin University of Electronic Technology
China
Email: zhaofeng@guet.edu.cn

***Abstract*—An array processing based decoder for the quasi-orthogonal space-time block code (QO-STBC) has been analyzed in our former work. It is investigated in terms of complexity and symbol error ratio (SER) under the assumption that perfect channel state information (CSI) is known to the receiver. As in wireless communication scenarios, it is prohibitive to get the perfect knowledge of channel. So it is worth to investigate how the array processing based decoder will be effected by imperfect CSI. In this work, the impact caused by channel estimation error on the performance of the QO-STBC is evaluated. Based on the analysis and simulation results, we can see that the channel estimation error does degrade the SER performance a lot in high signal to noise ratio (SNR) regime. Mean square error (MSE) is employed to measure how the degradation depends on the correctness of the estimator. To make this method applicable, channel estimators should be probably chosen.**

***Index Terms*—Quasi-orthogonal space-time block code, channel state information, mean square error null space.**

I. Introduction

Orthogonal Space-Time Block Codes (O-STBCs) as a kind of simple space time code have been found wide applications because of their capability of offering higher diversity gain and the property that they can be decoded linearly, which means lower decoding complexity [1]–[3]. Unfortunately, this OSTBC codes suffer from a reduced code rate when employing complex signal constellations and more than two transmit antennas, which considerably constraint the application of them. In order to avoid this, Quasi-Orthogonal Space-Time Block Coding (QO-STBC) is proposed and investigated [4], [5]. The previous work has shown that QO-STBCs can provide higher data rate and in the same time offering partial diversity. Further, to have QO-STBCs with full diversity to ensure good performance, improved QO-STBCs through constellation rotation are proposed [6], [7].

Maximum-likelihood (ML) decoding is one of the best decoding scheme for QO-STBC, it works with pairs of transmitted symbols, which leads to a decoding complexity increasing with the square of the modulation level M, subsequently resulting in an increasing of system latency. To solve this, some new decoding methods were proposed, which can reduce the computational complexity [8], [9]. However, they are still too time consuming. To overcome this, in our former work we proposed a novel decoding approach which combines decoding together with array processing. Taking the advantage of the null space of a MIMO channel matrix, we divided a received symbol into independent parts, and then decode them separately. The complexity of the new approach was analyzed. It was shown that its decoding complexity increases with the modulation level M, greatly deduced compared with the traditional schemes [10]. In detail, the computational complexity is calculated in terms of how many times of multiplications and additions needed by the decoder, which shows that the decoder can considerably deduce the computational load without sacrificing the SER performance that much. As all the investigations in [10] are based on perfect channel state information (CSI), we in this work will focus on the performance when having only imperfect CSI.

We shall use the following notation: Matrices and vectors are denoted by the bold upper and lower case letter, respectively. $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ denote complex conjugation, transposition, complex conjugate transposition, respectively. $Re(\cdot)$ returns the real part of a complex value. $E[\cdot]$ is to calculate the expectation. $\|\cdot\|^2$ is to calculate the squared Frobenius norm.

II. Signal Model

As the work presented in this paper is a continue part of the content in [10], we should use the same system model taken in [10]. Therefore, we focus on a multiple-input and multiple-output (MIMO) system, which has N_T antennas at the transmitter and N_R antennas at the receiver. The system layout is shown in Fig. 1. The source bits are mapped to M-level Quadrature Amplitude Modulation (M-QAM). Then, the serial modulated symbols are transferred and stacked as a symbol vector $\boldsymbol{x}(t) = [x_t, x_{t+1}, \cdots, x_{t+N_T-1}]^T$, which afterwards will be transformed into a transmission matrix $\boldsymbol{X}$ with a QO-STBC encoder. The nth row elements of $\boldsymbol{X}$ are transmitted by the nth antenna. The tth column elements are transmitted at time slot t. We choose system with 4 transmit antennas. For simplify, the the transmitted symbol vector $\boldsymbol{x}(t)$ is written as $\boldsymbol{x} = [x_1, x_2, x_3, x_4]^T$ from now on. We assume that the MIMO channel is a Rayleigh fading channel, denoted by $\boldsymbol{H} \in \mathbb{C}^{N_R \times N_T}$, the element h_{mn} of which represents the path gain between the nth transmit and mth receive antennas.

Then, the received signal vector $\boldsymbol{r}$ can be written as

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{n}, \tag{1}$$

where the vector $\boldsymbol{n} \in \mathbb{C}N_R$ is complex noise vector, the elements of which are independent identically distributed (i.i.d.) Gaussian random variables of zero mean and unit variance. $E[\boldsymbol{n}\boldsymbol{n}^H] = \boldsymbol{I}_{N_R}$.

A. Quasi-Orthogonal Space-Time Block Code

As in paper [10], we take the QO-STBC described in [4] as the coding scheme of concern. The symbol matrix $\boldsymbol{X}$ is constructed by two O-STBC code matrices in the way

$$\begin{aligned} \boldsymbol{X} &= \begin{bmatrix} \boldsymbol{X}_{12} & -\boldsymbol{X}_{34}^* \\ \boldsymbol{X}_{34} & \boldsymbol{X}_{12}^* \end{bmatrix} \qquad (2) \\ &= \begin{bmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{bmatrix}, \end{aligned}$$

where

$$\boldsymbol{X}_{12} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix},$$

and

$$\boldsymbol{X}_{34} = \begin{bmatrix} x_3 & -x_4^* \\ x_3 & x_4^* \end{bmatrix},$$

are the well known Alamouti O-STBC encoding matrix with x_1, x_2 and x_3, x_4, respectively. More information on how to generate the matrices, readers are referred to [1].

B. Channel Estimation Error

Channel estimation methods are widely explored [11], [12]. Mean square error (MSE) is often employed to evaluated the correctness of a channel estimator [12]. In this paper MSE is employed to measure how correctly the channel is estimated. Let $\hat{\boldsymbol{H}}$ denote the estimated channel matrix. The MSE of the channel estimation is written as

$$MSE = \frac{1}{N_R}\sum_{m=1}^{N_R} E[(\boldsymbol{h}_m - \hat{\boldsymbol{h}}_m)(\boldsymbol{h}_m - \hat{\boldsymbol{h}}_m)^H], \tag{3}$$

where the $\boldsymbol{h}_m$ and $\hat{\boldsymbol{h}}_m$ are mth row of $\boldsymbol{H}$ and $\hat{\boldsymbol{H}}$, respectively. They represent the true and estimated channel vectors seen from the mth receive antenna.

C. Decoding Scheme

As mentioned above, the array processing based decoding scheme proposed in our former work is employed [10]. The Least square (LS) estimator is employed to obtain the channel knowledge at the receiver offering CSI needed for array processing and decoding. As we can see from the transmission matrix $\boldsymbol{X}$ shown in (2), $\boldsymbol{X_{12}}$ is transmitted from the 1st and 2nd antennas and $\boldsymbol{X_{34}}$ is transmitted from the 3rd and 4th antennas at the first time slot. Then, in the next two slots their transformed versions are transmitted, separately. Thus, if we divide the transmitted signals into two parts which will be transmitted by antenna group 1 including the 1st and 2nd antennas and group 2, including the 3rd and 4th antennas, then the linear decoding scheme can be employed. In this way, the decoding computational load can be considerably deduced.

To separate the signals, array processing shown in [13] is employed. After the division, the estimated MIMO channel $\hat{\boldsymbol{H}}$ can be rewritten as $\hat{\boldsymbol{H}} = [\hat{\boldsymbol{H}}_1, \hat{\boldsymbol{H}}_2]$. $\hat{\boldsymbol{H}}_1$ and $\hat{\boldsymbol{H}}_2$ are defined by

$$\hat{\boldsymbol{H}}_1 = \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \vdots & \vdots \\ \hat{h}_{N_{R1}} & \hat{h}_{N_{R2}} \end{bmatrix}, \tag{4}$$

and

$$\hat{\boldsymbol{H}}_2 = \begin{bmatrix} \hat{h}_{13} & \hat{h}_{14} \\ \vdots & \vdots \\ \hat{h}_{N_{R3}} & \hat{h}_{N_{R4}} \end{bmatrix}. \tag{5}$$

The null space of a matrix $\boldsymbol{A}$ is composed of all vectors $\boldsymbol{y}$, which satisfy

$$\boldsymbol{A}\boldsymbol{y} = 0. \tag{6}$$

It is worth mentioning that for single user case, there should be more than two antennas at the receiver to ensure the existence of null space.

Let $\boldsymbol{\Phi}_1$ denote the null space of $\hat{\boldsymbol{H}}_1^T$ and $\boldsymbol{\Phi}_2$ denote the null space of $\hat{\boldsymbol{H}}_2^T$. With the help of (6), we have

$$\hat{\boldsymbol{H}}_1^T\boldsymbol{\Phi}_1 = \boldsymbol{\Phi}_1^T\hat{\boldsymbol{H}}_1 = 0, \tag{7}$$

and

$$\hat{\boldsymbol{H}}_2^T\boldsymbol{\Phi}_2 = \boldsymbol{\Phi}_2^T\hat{\boldsymbol{H}}_2 = 0. \tag{8}$$

Multiplying (1) by $\boldsymbol{\Phi}_1^T$ and $\boldsymbol{\Phi}_2^T$, respectively, we can get

$$\boldsymbol{\Phi}_1^T\boldsymbol{r} = \boldsymbol{\Phi}_1^T\boldsymbol{H}\boldsymbol{X} + \boldsymbol{\Phi}_1^T\boldsymbol{n}, \tag{9}$$

and

$$\boldsymbol{\Phi}_2^T\boldsymbol{r} = \boldsymbol{\Phi}_2^T\boldsymbol{H}\boldsymbol{X} + \boldsymbol{\Phi}_2^T\boldsymbol{n}. \tag{10}$$

Therefore, taking (7) and (8) into account, we can write the noise free parts of the two above equations in the way

$$\begin{aligned} \boldsymbol{\Phi}_1^T\boldsymbol{H}\boldsymbol{X} &= \begin{bmatrix} \boldsymbol{\varepsilon}_1 & \boldsymbol{\Phi}_1^T\boldsymbol{H}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_{12} & -\boldsymbol{X}_{34}^* \\ \boldsymbol{X}_{34} & \boldsymbol{X}_{12}^* \end{bmatrix} \qquad (11) \\ &= \begin{bmatrix} \boldsymbol{\Phi}_1^T\boldsymbol{H}_2\boldsymbol{X}_{34} + \boldsymbol{\varepsilon}_1\boldsymbol{X}_{12} & \boldsymbol{\Phi}_1^T\boldsymbol{H}_2\boldsymbol{X}_{12}^* - \boldsymbol{\varepsilon}_1\boldsymbol{X}_{34}^* \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} \boldsymbol{\Phi}_2^T\boldsymbol{H}\boldsymbol{X} &= \begin{bmatrix} \boldsymbol{\Phi}_2^T\boldsymbol{H}_1 & \boldsymbol{\varepsilon}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_{12} & -\boldsymbol{X}_{34}^* \\ \boldsymbol{X}_{34} & \boldsymbol{X}_{12}^* \end{bmatrix} \qquad (12) \\ &= \begin{bmatrix} \boldsymbol{\Phi}_2^T\boldsymbol{H}_1\boldsymbol{X}_{12} + \boldsymbol{\varepsilon}_2\boldsymbol{X}_{34} & -\boldsymbol{\Phi}_2^T\boldsymbol{H}_1\boldsymbol{X}_{34}^* + \boldsymbol{\varepsilon}_2\boldsymbol{X}_{12}^* \end{bmatrix}. \end{aligned}$$

Herein, $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ is the error brought by the incorrectness of the channel estimation.

$$\boldsymbol{\varepsilon}_i = \boldsymbol{\Phi}_i^T\boldsymbol{H}_i - \boldsymbol{\Phi}_i^T\hat{\boldsymbol{H}}_i \text{ for } i = 1,\ 2. \tag{13}$$

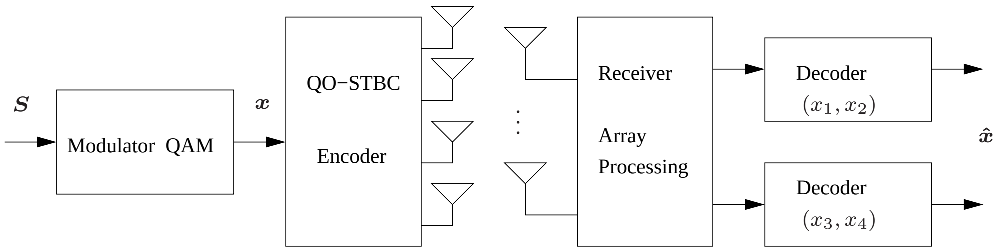

Fig. 1. Block diagram of QO-STBC coded MIMO system

Define $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ as the parts of received signals related to $\boldsymbol{X}_{12}$ in $\boldsymbol{\Phi}_1^T\boldsymbol{r}$ and $\boldsymbol{\Phi}_2^T\boldsymbol{r}$, respectively. Let the corresponding columns of $\boldsymbol{\Phi}_1^T\boldsymbol{n}$ and $\boldsymbol{\Phi}_2^T\boldsymbol{n}$ related to X_{12} are denoted by $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ accordingly. Then, the received symbols are decoded in a parallel way [10]. According to (9) and (10), we can further write

$$\begin{aligned}\left[\begin{array}{cc}\boldsymbol{R}_1^* & \boldsymbol{R}_2\end{array}\right] = & \left[\begin{array}{cc}\boldsymbol{\Phi}_1^H\boldsymbol{H}_2^* & \boldsymbol{\Phi}_2^T\boldsymbol{H}_1\end{array}\right]\boldsymbol{X}_{12} \\ & + \left[\begin{array}{cc}-\varepsilon_1^*\boldsymbol{X}_{34} & \varepsilon_2^*\boldsymbol{X}_{34}\end{array}\right] + \left[\begin{array}{cc}\boldsymbol{n}_1^* & \boldsymbol{n}_2\end{array}\right].\end{aligned} \quad (14)$$

Let the corresponding columns of $\boldsymbol{\Phi}_1^T\boldsymbol{n}$ and $\boldsymbol{\Phi}_2^T\boldsymbol{n}$ related to X_{34} are denoted by $\boldsymbol{n}_3$ and $\boldsymbol{n}_4$ accordingly. Obviously, in this way, $\boldsymbol{X}_{12}$ can be decoded as an O-STBC with 2 transmit and two receive antennas. Similarly, we can see that $\boldsymbol{X}_{34}$ can be decoded in the same way by letting $\boldsymbol{R}_3$ and $\boldsymbol{R}_4$ be the parts related to $\boldsymbol{X}_{34}$ in $\boldsymbol{\Phi}_1^T\boldsymbol{r}$ and $\boldsymbol{\Phi}_2^T\boldsymbol{r}$, respectively. Then we have

$$\begin{aligned}\left[\begin{array}{cc}\boldsymbol{R}_3 & \boldsymbol{R}_4^*\end{array}\right] = & \left[\begin{array}{cc}\boldsymbol{\Phi}_1^T\boldsymbol{H}_2 & -\boldsymbol{\Phi}_2^H\boldsymbol{H}_1^*\end{array}\right]\boldsymbol{X}_{34} \\ & + \left[\begin{array}{cc}\varepsilon_1\boldsymbol{X}_{12} & \varepsilon_2^*\boldsymbol{X}_{12}\end{array}\right] + \left[\begin{array}{cc}\boldsymbol{n}_3 & \boldsymbol{n}_4^*\end{array}\right].\end{aligned} \quad (15)$$

From the analysis made above, we can see that the QO-STBC with 4 transmit antennas can be decoded in two steps. However, the channel estimation errors make it impossible to separate $\boldsymbol{X}_{12}$ and $\boldsymbol{X}_{34}$ perfectly. They both will be noise to each other, resulting inter-symbol interference. How serious the noise will be depends on how big the channel estimation error is.

III. Performance Evaluation and discussion

As it is analyzed afore, the system SER performance will be decayed if there is no perfect CSI at the receiver when employing the array processing based QO-STBC decoder. And that how much the SER will decay depends on the error of the channel estimation. In this section, we evaluate the relationship of system SER performance and the channel estimation error denoted by the MSE in dB. As we mentioned earlier, without loss of generality we choose the system with a 4×4 antennas array. Thus, the coding rate is one. At transmitter, the source bits are mapped to M-QAM constellations to form transmitted symbols. In this paper, 16-QAM, 64-QAM and 256-QAM are employed. After QO-STBC coder, the symbol words are launched over a MIMO channel with Rayleigh fading. The channel is estimated employing LS channel estimator. More detailed parameters are shown in Table I.

TABLE I
Parameters used to perform the system evaluation

Number of antennas	$N_T = N_T = 4$
M-QAM	$M \in \{16,\ 64,\ 256\}$
QO-STBC	coding rate=1, array processing based decoder
Channel estimator	Least square estimator
MSE	MSE $\in \{-9, -11, -13, -15, -20\}$ dB

The simulation results when employing different modulation constellations are shown in Fig. 2, Fig. 3 and Fig. 4, separately. For sake of comparison, the SER curves when perfect CSI is known to the receiver are also given. We can see from these results that in lower signal to noise ratio(SNR) regime there are almost no SER decay caused by the channel estimation error. The degradation of SER becomes obviously when SNR increasing for all case. Systems with small-size modulation constellations suffer slight from the incorrectness of the channel estimation. Using 16-QAM makes the system robust to the MSE of estimator, while system using 256-QAM is more sensitive. Anyway, the decay caused can be ignored when the MSE of channel estimation goes below -13 dB which is easy to fulfill by taking use of a proper channel estimator. Besides, the channel estimation error normally goes down when the SNR goes up. That means at high SNR regime we can expect more accurate channel estimation leading to smaller MSE, which thereby results in a robust array processing based decoder with low complexity. If the MSE is below -20 dB, we cannot see any SER decay even when 256-QAM is utilized in our system. Therefore, the low-complexity array processing based coding scheme still performs well even there is no perfect CSI.

IV. Conclusion

In this paper, we investigate the impact of the channel estimation error on the SER performance of a QO-STBC system with a new decoding scheme based on the array processing. The receiver gather the CSI by employing LS

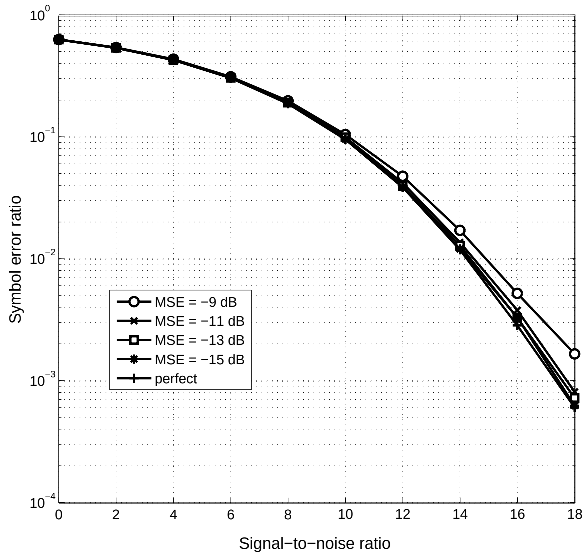

Fig. 2. SER performance of QO-STBC with 16-QAM.

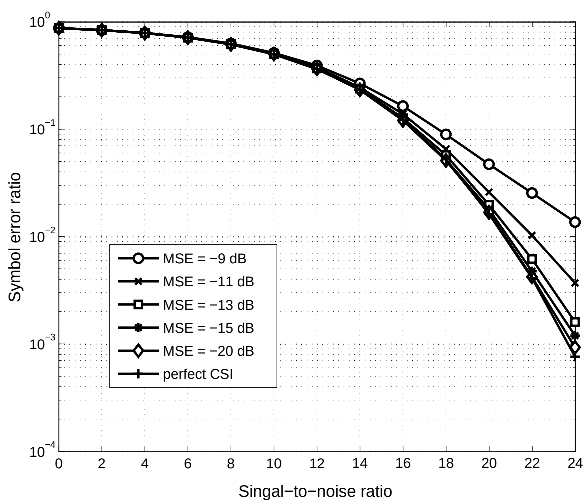

Fig. 3. SER performance of QO-STBC with 64-QAM.

channel estimator. System SER performance is simulated for different modulation constellations and different MSE of the estimator. In lower SNR regime, without perfect CSI won't cause SER decay, while the decay becomes obvious in high SNR region. However, when the MSE is below -20 dB, almost no difference can be found between the SER obtained with and without perfect CSI. Considering the fact that when SNR goes high the MSE of channel estimator will be easily lower than 20 dB, we can say that the array processing based decoding scheme depends not so much on the correctness of the channel estimation and works well if proper channel estimator is employed.

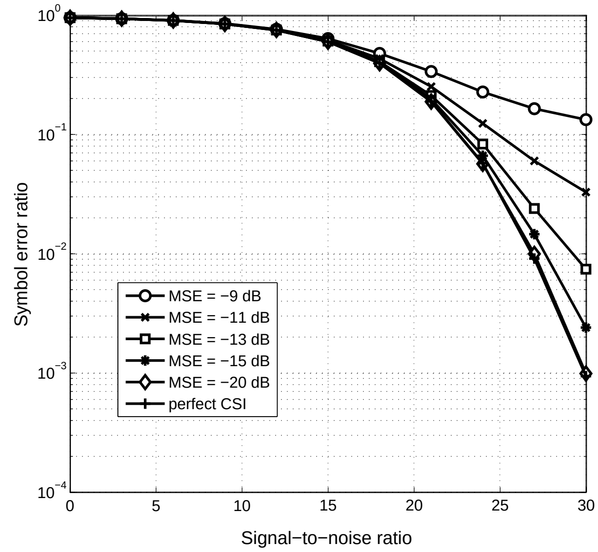

Fig. 4. SER performance of QO-STBC with 256-QAM.

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References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16 no. 8, pp. 1451–1458, Oct. 1998.

[2] B. Vucetic, J. Yuan, "Space-time coding," *Wiley*, 2003

[3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Information Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.

[4] H. Jafarkhani, "A Quasi-Orthogonal Space-Time Block Code," *IEEE Trans. Commu.*, vol. 49, no. 1, pp. 1–4, Jan. 2001.

[5] C. B. Papadias and G. J. Foschini, "A space-time coding approach for systems employing four transmit antennas," *Proc. IEEE ICASSP'01*,Salt Lake City, USA, 2001, vol. 4, pp. 2481–2484.

[6] W. Su, and X. G. Xia, "Signal constellations for quasi-orthogonal space-time block codes with full diversity," *IEEE Trans. Inform. Theory*, vol. 50 pp. 2331–2347, Oct. 2004

[7] N. Sharma and C. B. Papadias, "Improved Quasi-Orthogonal Codes through Constellation Rotation," *IEEE Trans. Commun.*, vol. 51, pp. 332–335, Mar. 2003

[8] M. Le, V. Pham, L. Mai and G. Yoon, "Low-Complexity Maximum-Likelihood Decoder for Four-Transmit-Antenna Quasi-Orthogonal Space-Time Block Code," *IEEE Trans. Commu.*, vol. 53, no. 11, pp. 1817–1821, Nov. 2005.

[9] L. He and H. Ge, "Fast maximum-likelihood decoding of quasiorthogonal codes," Proc. Asilomar Conf. Signals, Syst., Comput., vol. 1, pp. 1022–1026 Nov. 2003.

[10] C. Jiang, H. Zhang, D. Yuan amd HH Chen, "A Low Complexity Decoding Scheme for Quasi-Orthogonal Space-Time Block Coding," *Accepted for publication in the fifth IEEE sensor array and multi-channel signal processing workshop.*

[11] S. Crozier, D. Falconer, and S. mahmoud, "Least Sum of Squared Errors (LSSE) channel estimation.," *IEE Proceedings*, vol. 138, pp. 371–378, May 1991.

[12] I. Perisa, J. Egle, and J. Lindner "Channel Estimation with Pilot-Symbols over WSSUS Channels," *European Wireless*, 2002.

[13] V. Tarokh, A. Naguib, N. Seshadri and A. R. Calderbank, "Combined Array Processing and Space-Time Coding," *IEEE Trans. Information Theory*, vol. 45, no. 4, pp. 1121–1128, May 1999.