A Distributed Localization with Unknown Attenuation Coefficient in Wireless Sensor Networks

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***Abstract*—Recently, a localization attracts much attention in sensor networks. One of major localization algorithms uses Received Signal Strength Indicator (RSSI). This algorithm needs some prior knowledge of an environment and thus requires prior experiments. To avoid troublesome prior measurements, this paper proposes a decentralized localization algorithm with unknown attenuation coefficient. The proposed algorithm has no need to measure radio propagation characteristics, because the algorithm estimates attenuation coefficient besides a source location and a signal energy at unit-distance away from the source. The proposed algorithm has three calculation methods, because they estimate three unknown parameters. We evaluate the computational complexity and localization accuracy of the proposed calculation methods. Finally, we show location estimation accuracies of the proposed algorithm by results of computer simulation and experiment.**

I. INTRODUCTION

In recent years, a localization in sensor networks is attracting much interests for many applications, and many localization algorithms have been proposed [1]. A localization algorithm using Received Signal Strength Indicator (RSSI) is one of the major localization algorithms [2]. This algorithm has two steps to estimate a source location. In the first step, a radio propagation characteristic is investigated by measurements of a relation between RSSI and a distance. In the second step, a source location is estimated by the radio propagation characteristic and RSSI from the source. As above, this algorithm using RSSI needs some prior knowledge of an environment, that is a radio propagation characteristic, and thus requires prior experiments. However, these prior experiments are not favorable, because measurements take a lot of cost and time. In addition, the propagation model obtained by measurements does not hold when the environment changes.

In sensor networks, power consumption is important, because the available power is limited in sensor nodes [3]. Sensor nodes generally consume a lot of power for communication. There are two ways to process data, a centralized data processing and a distributed data processing. The centralized data processing processes data in a lump at a data center that garners all the data of sensor nodes, while the distributed data processing processes not all but a part of data at each sensor node. The distributed data processing consumes less communication cost than the centralized data processing, because data are processed in each sensor node and thus not all sensor nodes need to transmit data to a data center over relatively long distance. In addition, the distributed data processing is robust and practical for environmental changes of sensor nodes. Hence, a distributed localization algorithm is desirable in sensor networks, although the centralized algorithm can estimate a source location with high accuracy. A preceding study proposed a centralized localization algorithm that estimates a radio propagation characteristic besides a source location [4] as shown in section III.

This paper proposes a decentralized localization algorithm with unknown attenuation coefficient to prevent cumbersome prior measurements. The proposed algorithm estimates attenuation coefficient besides a source location and a signal energy at unit-distance away from the source. Since an attenuation coefficient and a signal energy at unit-distance away from a source are feature values of radio propagation characteristics, this proposed algorithm has no need of a prior measurement. The proposed algorithm has three calculation methods because of three unknown parameters: a source location, an attenuation coefficient, and a signal energy at unit-distance away from a source. Since a calculation amount is related to power consumption, we evaluate the computational complexity and localization accuracy of the proposed algorithm by computer simulation and experiment. We show that the proposed algorithm has a high estimation accuracy close to that of the centralized algorithm.

II. PROPAGATION MODEL [5]

We consider a X by Y sensor field, where N sensor nodes and one source exist. They are positioned at random. It is assumed that the received power (dBm) is inversely proportional to the square of the distance from the source. The power received by the ith sensor node ($i = 1, 2, \cdots, N$) can be expressed as

$$P_i = P_0 - 10\kappa \log \frac{d_i}{d_0} + X_i \tag{1}$$

where $d_i = \| \boldsymbol{\rho} - \mathbf{r}_i \|$ is the distance between the ith sensor node and the source, $\mathbf{r}_i$ is the known location of the ith sensor node, and $\boldsymbol{\rho}$ is the location of the source. P_0 is the signal energy at unit-distance (d_0) away from the source, and κ is the attenuation coefficient. X_i is the energy of the background noise at the ith sensor node that denotes a zero mean Gaussian random variable.

III. Centralized Localization Algorithm

Define that $\boldsymbol{\theta} = [\boldsymbol{\rho}^T P_0\ \kappa]^T$ is an unknown parameter. '$[\ ]^T$' is a transpose matrix. The centralized algorithm updates $\boldsymbol{\rho}$ and $[P_0\ \kappa]$ alternately to maximize the likelihood function $\ell(\boldsymbol{\theta})$. This algorithm can be separated into two stages, namely two separate ML estimation problems. $\ell(\boldsymbol{\theta})$ can be expressed as

$$\ell(\hat{\boldsymbol{\theta}}) = \sum_{i=1}^{N} \left\{ P_i - P_0 + 10\kappa \log\left(\frac{d_i}{d_0}\right) \right\}^2 \tag{2}$$

where $\ell(\hat{\boldsymbol{\theta}})$ is a negative likelihood function. In the negative likelihood function, ML estimation minimizes the likelihood to estimate a location. Two stages are iterated as follows. In stage one, $\boldsymbol{\rho} = [x\ y]$ is estimated by minimizing $\ell(\hat{\boldsymbol{\theta}})$ with respect to x and y. In general, the estimated value $\hat{\boldsymbol{\rho}}$ that minimizes $\ell(\hat{\boldsymbol{\theta}})$ satisfies eqs. (3) and (4). Thus, $\hat{\boldsymbol{\rho}}$ can be obtained by solving eqs. (3) and (4).

$$\left[\frac{\partial \ell(\boldsymbol{\rho}, \hat{P_0}^{n-1}, \hat{\kappa}^{n-1})}{\partial x}\right] = 0 \tag{3}$$

$$\left[\frac{\partial \ell(\boldsymbol{\rho}, \hat{P_0}^{n-1}, \hat{\kappa}^{n-1})}{\partial y}\right] = 0 \tag{4}$$

where a value with '$\hat{\ }$' means that it is an estimated value and n is the iteration index. In the first iteration, P_0^0 and κ^0 are given arbitrarily. In the nth (≥ 2) iteration, $\hat{P_0}^{n-1}$ and $\hat{\kappa}^{n-1}$ are used as the values of P_0 and κ, respectively.

In stage two, P_0 and κ are estimated. The estimated values of $\hat{P_0}^n$ and $\hat{\kappa}^n$ are obtained by solving eqs. (5) and (6).

$$\left[\frac{\partial \ell(\hat{\boldsymbol{\rho}}^n, P_0, \kappa)}{\partial P_0}\right] = 0 \tag{5}$$

$$\left[\frac{\partial \ell(\hat{\boldsymbol{\rho}}^n, P_0, \kappa)}{\partial \kappa}\right] = 0 \tag{6}$$

This algorithm repeats above the processes, stage one and stage two n times.

IV. Proposed Distributed Algorithm

Assume that the sensor nodes constitute a cyclic network where each sensor node is connected with the nearest sensor nodes. The sensor nodes are labeled a number $1, 2, \cdots, N$ sequentially. Messages are passed between nodes in the order of sensor number $1, 2, \cdots, N, 1, 2, \cdots, N, 1, 2, \cdots$. One cycle denotes a transmission of a set of data from sensor node 1 to sensor node N.

Define that $\boldsymbol{\theta} = [\boldsymbol{\rho}^T P_0\ \kappa]^T$ is an unknown parameter. This algorithm estimates $\boldsymbol{\rho}$, P_0, and κ alternately in each sensor node and updates $\boldsymbol{\theta}^n$ that maximizes the likelihood function. $\boldsymbol{\theta}^n$ denotes the parameter at the nth iteration of this algorithm. "Iteration" represents an update of $\boldsymbol{\theta}$ using information from all the sensor nodes in the centralized algorithm, while "iteration" represents an update at each sensor node in the proposed distributed algorithm. In the proposed algorithm, the ith sensor node uses data of three sensor nodes, namely the $(i-2)$th, $(i-1)$, and ith sensor nodes to estimate $\boldsymbol{\theta}$. The $(i-1)$th sensor node sends $\{P_{i-2}, \mathbf{r}_{i-2}\}$, $\{P_{i-1}, \mathbf{r}_{i-1}\}$, and $\{\hat{\boldsymbol{\rho}}^{i-1}, \hat{P_0}^{i-1}, \hat{\kappa}^{i-1}\}$ to the ith sensor node. Then the ith sensor node updates $\hat{\boldsymbol{\rho}}$, $\hat{P_0}$, and $\hat{\kappa}$ using them. After that, the ith sensor node sends the data to the $(i+1)$th sensor node similarly.

The negative likelihood function at the ith sensor node $\ell(\boldsymbol{\theta})_i$ can be expressed as

$$\ell(\hat{\boldsymbol{\theta}})_i = \sum_{k=i-2}^{i} \left\{ P_i - P_0 + 10\kappa \log\left(\frac{d_i}{d_0}\right) \right\}^2. \tag{7}$$

The proposed distributed algorithm has three calculation methods to estimate $\hat{\boldsymbol{\theta}}$ because of three unknown parameters. They are expressed as follows.

In the method 1, $[P_0\ \kappa]$ and $\boldsymbol{\rho}$ are estimated alternately. First, $[\hat{P_0}^i\ \hat{\kappa}^i]$ are estimated by minimizing $\ell(\boldsymbol{\theta})_i$ with respect to P_0 and κ, and then $\hat{\boldsymbol{\rho}}^i$ is calculated geometrically by using $[\hat{P_0}^i\ \hat{\kappa}^i]$ and eq. (1).

In the method 2, $[\boldsymbol{\rho}\ \kappa]$ and P_0 are estimated alternately. First, $[\hat{\boldsymbol{\rho}}^i\ \hat{\kappa}^i]$ are estimated by minimizing $\ell(\boldsymbol{\theta})_i$ with respect to $\boldsymbol{\rho}$ and κ, and then $\hat{P_0}^i$ is calculated by using $[\hat{\boldsymbol{\rho}}^i\ \hat{\kappa}^i]$ and eq. (8).

$$\hat{P_0}^i = \frac{1}{3} \sum_{k=i-2}^{i} \left\{ P_k + 10\hat{\kappa}^i \log\left(\frac{\hat{d_k}^i}{d_0}\right) \right\} \tag{8}$$

In the method 3, $[\boldsymbol{\rho}\ P_0]$ and κ are estimated alternately. First, $[\hat{\boldsymbol{\rho}}^i\ \hat{P_0}^i]$ are estimated by minimizing $\ell(\boldsymbol{\theta})_i$ with respect to $\boldsymbol{\rho}$ and P_0, and then $\hat{\kappa}^i$ is calculated by using $[\hat{\boldsymbol{\rho}}^i\ \hat{P_0}^i]$ and eq. (9).

$$\hat{\kappa}^i = \frac{1}{3} \sum_{k=i-2}^{i} \left\{ \frac{\hat{P_0}^i - P_k}{\log\left(\frac{\hat{d_k}^i}{d_0}\right)} \right\} \tag{9}$$

The proposed algorithm performs above the processes at each sensor node, NK times as a whole, where K is the number of cycles. Minimum likelihood is searched by comparing all the temporary likelihoods that can be obtained by changing $\boldsymbol{\theta}$ by $\Delta\boldsymbol{\theta}$, where $\Delta\boldsymbol{\theta}$ is a step size arbitrarily decided. $[\hat{\boldsymbol{\rho}}^0\ \hat{\kappa}^0]$ are used as initial values. $\hat{\boldsymbol{\rho}}^0$ is adjusted to near the location of the sensor node that received RSSI most strongly, and $\hat{\kappa}^0$ is set to 3.0.

In the proposed algorithm, $\hat{\boldsymbol{\rho}}$ does not completely converge, because only a small amount of information received at sensor nodes is used for each update, and the information includes noise. To reduce the effect of noise, the proposed algorithm uses "the average estimated location" [6]. The estimated locations in the first cycle are not used for the average estimated location, because they are possibly far away from the actual location of the source. We define j as the number of updates in the second cycle or later. In the second cycle or later, the $(i-1)$th sensor node sends an average estimated location $\hat{\boldsymbol{\rho}}_{\mathrm{ave}}^{j-1}$ and the value '$j-1$' to the ith sensor node. The update of $\boldsymbol{\rho}_{\mathrm{ave}}$ at the ith sensor node can be expressed as

$$\hat{\boldsymbol{\rho}}_{\mathrm{ave}}^{j} = \frac{\hat{\boldsymbol{\rho}}_{\mathrm{ave}}^{j-1} \times (j-1) + \hat{\boldsymbol{\rho}}^t}{j}. \tag{10}$$

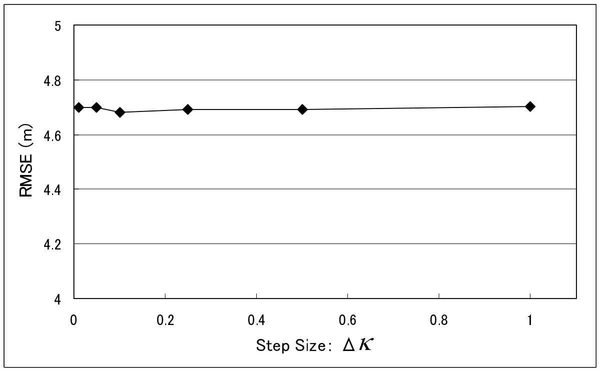

Fig. 1. RMSE versus a step size $\Delta\kappa$ in method 1

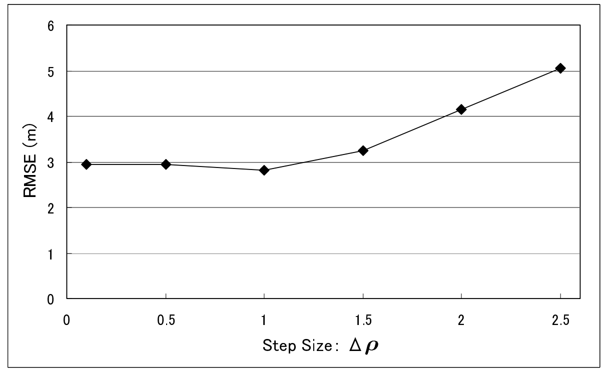

Fig. 2. RMSE versus a step size $\Delta\boldsymbol{\rho}$ in method 3

After K cycles, we use $\hat{\boldsymbol{\rho}}^{j}_{\text{ave}}$ for the final estimated location.

V. Simulation Results

We compare the proposed algorithm to the centralized algorithm by computer simulation. The size of the sensor field is $10\ \text{m} \times 10\ \text{m}$. The number of sensor nodes is $N = 10$. Sensor nodes and a source are positioned at random each trial. The background noise has $\mathcal{N}(0, 0.5)$ distribution. The number of trials is 10000, and the number of cycles is 10.

A. RMSE vs Step Size

To investigate the influence of a step size in the proposed algorithm, we evaluate the RMSE (Root Mean Square Error) and a step size in the method 1 and method 3. The method 1 has only $\Delta\kappa$ as a step size, and the method 3 has only $\Delta\boldsymbol{\rho}$ similarly. We do not consider the method 2, because it has two parameters: $\Delta\boldsymbol{\rho}$ and $\Delta\kappa$. Figs. 1 and 2 show the RMSE versus a step size in the method 1 and method 3, respectively. From Fig. 1, we can see that the RMSE is almost constant in the step size $\Delta\kappa = \{0.01, 0.05, 0.1, 0, 25, 0.5, 1.0\}$. This is because the source location $\boldsymbol{\rho}$ is calculated from κ and P_0 estimated in the likelihood calculation, where the estimated κ and P_0 have some errors regardless of the value of $\Delta\kappa$. From Fig. 2, we can see that there exists an appropriate step size in the method 3. When the step size is large, the RMSE is large because the number of searched coordinates is small. When the step size is small, the RMSE is not improved so much because the likelihood function has errors in the received power and κ.

From these results, we use $\Delta\kappa = 0.1$ and $\Delta\boldsymbol{\rho} = 0.5$ as step sizes unless noted in the following simulation. The reason why we use a step size of $\Delta\boldsymbol{\rho} = 0.5$ not 1.0 is because we consider a localization of human whose shoulder length is about 0.5 m.

B. RMSE vs Number of Cycles in Methods 1, 2, 3

To investigate a relation between the estimation accuracy and power consumption, we evaluate the RMSE of each method at several calculation amounts. We consider the number of times that we compare the temporary likelihood obtained by changing $\boldsymbol{\theta}$ by $\Delta\boldsymbol{\theta}$ to search minimum likelihood in one sensor node as "Calculation Amount", because it takes a lot of calculation in the proposed algorithm. In the centralized algorithm, "Calculation Amount" means the number of times to search maximum likelihood in one iteration of stage one and stage two. Calculation Amount depends on the step sizes. With $\Delta\kappa = 0.1$ and $\Delta\boldsymbol{\rho} = 0.5$, the methods 1, 2, 3 need about 30, 8000, and 300 times to search minimum likelihood, respectively. We compare each method at the same numbers of search times: 30, 300, and 8000 times by adjusting a step size according to these numbers of times, respectively. As a comparison, the performance of the centralized data processing is shown in the results.

Figs. 3 - 5 show RMSE versus the number of cycles at 30, 300, and 8000 times to search minimum likelihood, respectively. In each figure, the RMSE performance of each method converges about 5 cycles. Thus, we evaluate the average of RMSE from cycle number 5 to 10.

TABLE I
RMSE (m) of methods 1, 2, 3 at several numbers of times to search minimum likelihood

	30 times	300 times	8000 times
method 1	4.68 (m) ($\Delta\kappa$=0.1)	4.70 (m) ($\Delta\kappa$=0.0067)	4.69 (m) ($\Delta\kappa$=0.00025)
method 2	1.60 (m) ($\Delta\kappa$=1.0, $\Delta\rho$=5.0)	1.12 (m) ($\Delta\kappa$=1.0, $\Delta\rho$=1.0)	1.11 (m) ($\Delta\kappa$=0.1, $\Delta\rho$=0.5)
method 3	5.05 (m) ($\Delta\boldsymbol{\rho}$=2.5)	2.90 (m) ($\Delta\boldsymbol{\rho}$=0.58)	3.07 (m) ($\Delta\boldsymbol{\rho}$=0.11)
centralized	2.56 (m) ($\Delta\kappa$=1.0, $\Delta\rho$=5.0)	0.95 (m) ($\Delta\kappa$=0.1, $\Delta\rho$=0.58)	0.70 (m) ($\Delta\kappa$=0.1, $\Delta\rho$=0.11)

TABLE I shows the average of RMSE from cycle number 5 to 10 of each method at several numbers of times to search minimum likelihood. From TABLE I, we can see that the method 2 has a high estimation accuracy at every number of times. This is because the method 2 has two parameters to estimate in the likelihood calculation, namely $\boldsymbol{\rho}$ and κ, so that the likelihood function can be made smaller by setting two parameters appropriately; the methods 1 and 3 have only one parameter, κ and $\boldsymbol{\rho}$, respectively. The reason why the method 2 at 30 times achieves the high estimation accuracy in spite of $\Delta\boldsymbol{\rho} = 5.0$ m is because of "the average estimated location". Although the method 2 with $\Delta\boldsymbol{\rho} = 5.0$ has a small number of searched coordinates, "the average estimated location"

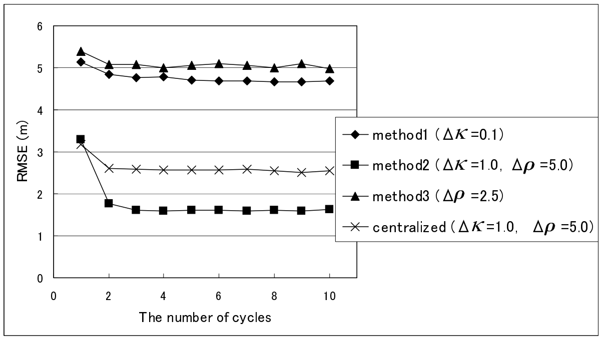

Fig. 3. Comparison of RMSE of methods 1, 2, 3 at 30 times to search minimum likelihood

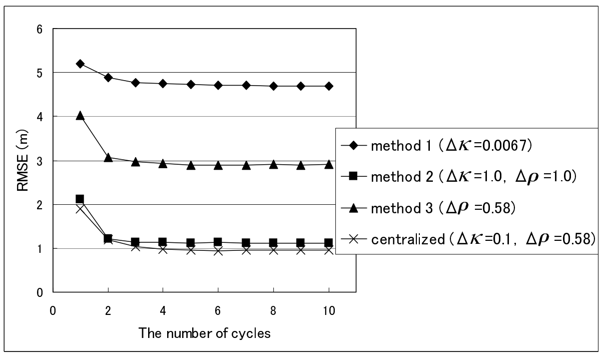

Fig. 4. Comparison of RMSE of methods 1, 2, 3 at 300 times to search minimum likelihood

originates a high estimation accuracy by averaging the estimation results of all the sensor nodes. These results show that we can obtain a high estimation accuracy with a small amount of calculation in the method 2. When we compare the method 1 to the method 3, the method 3 gets high estimation accuracy. This is because the method 3 directly searches the source location $\boldsymbol{\rho}$, while the method 1 calculates $\boldsymbol{\rho}$ from the estimated κ and P_0 with error. The centralized algorithm has the highest estimation accuracy at 300 and 8000 times, but it achieves the second best RMSE at 30 times. This is because the step size at 30 times is large and because the centralized algorithm does not use "the average estimated location".

The reason why RMSE is improved by the increase of the number of cycles from 1 to 2 is because all sensor data contributes to the calculation of "the average estimated location" in the second cycle or later, while only three sensor nodes' data are used to estimate $\boldsymbol{\theta}$ in the first cycle.

VI. Experimental Result

We confirmed our proposed algorithm through experiments in a room. The room is 8.0 meter wide, 8.0 meter long, and 3.0 meter high. The floor is covered by a carpet, the wall at one side has windows, and the other sides of the room are made of concrete. A 5 meter by 5 meter field in the center of the room is used as a sensor field, where there were no people and no obstacles. Sensor nodes are MICAz of Crossbow company. We set eight sensor nodes in the sensor field as shown in Fig. 6. Sensor nodes are placed on the ground. A target node is located at 32 places except four corners in each trial: the coordinate (0, 1), (0,

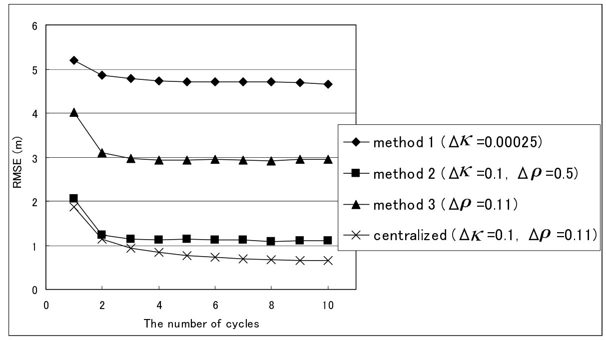

Fig. 5. Comparison of RMSE of methods 1, 2, 3 at 8000 times to search minimum likelihood

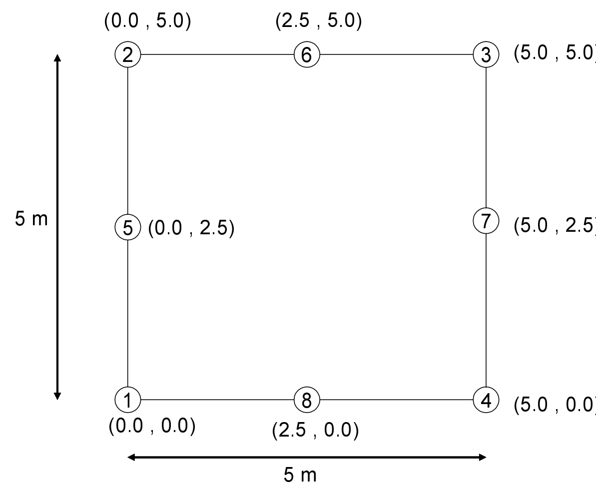

Fig. 6. Sensor field and sensor node position

2), ···, (5, 3), (5, 4). We measured the RSSI from the target node and estimated the location of the target node in 32 cases. The step sizes in each method were set to $\Delta\kappa$ = 0.1 and $\Delta\boldsymbol{\rho}$ = 0.5 according to the simulation results. The number of times to search minimum likelihood in the methods 1, 2, 3 and the centralized algorithm is 20, 8000, 400, and 8000 times, respectively.

Fig. 7 shows the RMSE of the methods 1, 2, 3 versus the number of cycles in the experiment. The RMSE is the average of the RMSE in 32 cases. From Fig. 7, we can see that the experiments follow a similar trend as the simulations: the RMSE improves by the increase of the number of cycles, and the method 2 obtains the highest estimation accuracy among the three proposed methods.

VII. Conclution

In this paper, we proposed a decentralized localization algorithm with unknown attenuation coefficient in wireless sensor networks. In the proposed algorithm, each sensor node estimates attenuation coefficient and the signal energy at unit distance away from the source besides the source location to prevent troublesome prior measurements. Our algorithm has three calculation methods to estimate them because of three unknown parameters. The simulation and experimental results show that the method 2 that searches directly the source location and attenuation coefficient in the likelihood calculation achieves

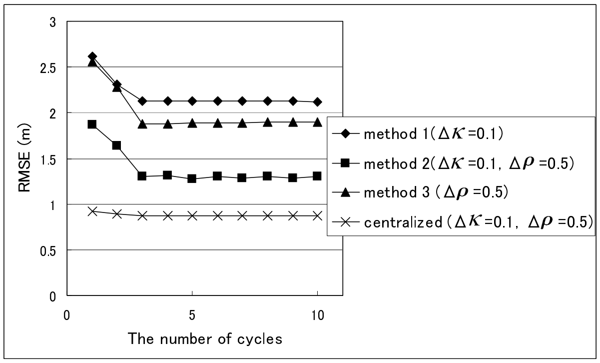

Fig. 7. Comparison of RMSE of methods 1, 2, 3 in experiment

the highest estimation accuracy among the three proposed methods.

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