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Second-Order Statistics Based Prefilter-Blind Equalization for MIMO-OFDM

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***Abstract*— In this paper, a SOS-based prefilter-blind equalizer for MIMO-OFDM system which can equalize FIR-MIMO channel at the receiver is proposed. The proposed scheme uses a set of orthogonal prefilters at the transmitter such that the transmitted signal can be colored temporally thereby satisfying the identifiability condition previously proposed by Hua and Tugnait. The scheme is shown to outperform previously proposed schemes in terms of BER and computational complexity. Simulation results have shown that the BER performance is close to that of a least-squares zero-forcing equalizer with perfect channel knowledge.**

I. INTRODUCTION

With the insatiable need to transmit more data at any time and anywhere, next generation wireless communication systems, such as IEEE 802.11n and 802.16m, are required to support higher data rate than their predecessors, while the mobile terminals are undergoing faster mobility. It is envisioned that MIMO-OFDM will be used for these systems in order to increase link reliability and capacity. However, large transmission overhead in the form preamble, signal pilot and guard interval severely hamper the performance of such systems. This has made blind channel estimation and equalization techniques for MIMO-OFDM systems an attractive alternative.

Traditionally, blind channel estimation and equalization has been based on higher-order statistics (HOS) [1], [2]. However, much of the research effort has since shifted toward using second-order statistics (SOS) after the seminal work by [3] and [4] since SOS based techniques can also estimate and equalize FIR channels at much lower latency than its HOS counterparts. This has led to the work by [5] which exploited the subspace method to estimate FIR-SIMO channels. Good performance in terms of mean squared error (MSE) can be achieved in high SNR condition. However, its performance degrades at a fast rate in low SNR condition, such as $0-10$ dB. An FIR-MIMO extension of the subspace method was proposed in [6], [7] which suffers from the same problem as its SIMO counterpart in low SNR condition. The subspace channel estimation method requires the channel transfer function matrix, $\mathbf{H}(z)$, to be *irreducible* and *column-reduced* [13], which limited the application of SOS based methods to a narrow class of communication channels.

Recently, [8] has shown that a weaker condition for the identifiability of $\mathbf{H}(z)$ exists, where $\mathbf{H}(z)$ can be identified up to scaling and permutation ambiguity if $\mathbf{H}(z)$ is *irreducible* and the power spectral density matrix of the channel input signal is a diagonal matrix with distinct diagonal functions. [9] has proposed an algorithm for estimating $\mathbf{H}(z)$ under this weaker condition, but an direct equalization algorithm was never discussed. [10] has proposed a SOS based blind equalization algorithm which implicitly uses the identifiability conditions stated in [8] for flat fading channels, where the number of transmit antennas, N_t, has to be equal to the number of receive antennas, N_r. [11] also exploited this condition by designing a novel SOS based channel estimation algorithm to estimate MIMO channels for OFDM based systems. The algorithm uses cyclic power spectral density of the received signal to decouple the MIMO channels into parallel SISO channels for estimation. The technique requires the use of a precoder to inject cyclostationarity into the input bitstream. Although not stated in [11], but the precoder actually colors the signal such that MIMO channel equalization using SOS is possible. [12] has extended the SOS algorithm in [10] such that any FIR-MIMO channel $\mathbf{H}(z)$ can be equalized up to a scaling, phase, and block delay ambiguity given that the identifiability conditions in [8] are satisfied. This was accomplished by designing the blind FIR equalizer within the space-time precoder-equalizer system where redundancy is injected into the transmitted bitstream to make FIR-MIMO channel equalization possible using an FIR equalizer. In [12], the independently distributed input signal streams were colored using a set of low complexity filters to satisfy the power spectral density condition stated in [8] such that the algorithm in [10] can be extended to be applicable to ISI channels. However, the prefilter that was proposed was not optimally designed. As shown in the sequel, this not only impacts the equalization performance, but also increases the computational complexity at the receiver.

In this paper, we proposed a new set of prefilter to perform direct channel equalization for MIMO-OFDM systems such that improved BER performance and lower receiver complexity can be achieved compared to that of [12]. In Section II, we will give a description of the system model, followed by a review of the equalization algorithm in Section III-A. We will then propose a novel prefilter design in Section III-B. Simulation results are given in Section IV followed by the conclusion in Section V.

Notation: Upper (lower) bold face letters indicate matrices (column vectors). Superscript ^{H} denotes Hermitian, ^{T} denotes transposition. $E[\cdot]$ stands for expectation. $diag(x)$ denotes a diagonal matrix with x on its main diagonal; $\mathbf{I}_N$ denotes an $N\times N$ identity matrix; $\mathbf{0}_{M\times N}$ denotes an $M\times N$ all zero matrix.

II. SYSTEM MODEL

We consider a MIMO-OFDM system with N_t transmit antennas and N_r receive antennas. Let $s_{m,\ell}^{(i)}$ denotes the complex-valued data symbol transmitted on the m^{th} tone in the ℓ^{th} OFDM symbol from the i^{th} antenna for $i=1,2,\ldots,N_t$. Also, let $K=M+v$ denote the overall OFDM symbol length, where M is the size of the FFT and v is the length of the cyclic prefix. Then the transmitted signal $u_i[n]$ can be written as [11]

$$u_i[n]=\sum_{\ell} g[n-\ell K]\sum_{m=0}^{M-1} s_{m,\ell}^{(i)} e^{j\frac{2\pi}{M}m(n-\ell K)}$$

where $g[n]$ is a rectangular function $rect_{[0,K-1]}[n]$ with

$$rect_{[T_1,T_2]}[n]=\begin{cases}1, & n=T_1,T_1+1,\ldots,T_2\\ 0, & otherwise.\end{cases}$$

Then the received signal at the k^{th} receive antenna can be written as

$$x_k[n]=\sum_{i=1}^{N_t}\left[\sum_{\ell} h_{k,i}[\ell]u_i[n-\ell]\right]+\eta_k[n] \tag{1}$$

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where $\eta_k[n]$, for $k = 1, 2, \ldots, N_r$, is the stationary additive white channel noise at the k^{th} receive antenna and $h_{k,i}[\ell]$ is the discrete-time impulse response of the channel. Defining

$$\begin{aligned}\mathbf{x}[n] &= [\, x_1[n] \;\; x_2[n] \;\; \cdots \;\; x_{N_r}[n] \,]^T,\\ \mathbf{u}[n] &= [\, u_1[n] \;\; u_2[n] \;\; \cdots \;\; u_{N_t}[n] \,]^T,\\ \boldsymbol{\eta}[n] &= [\, \eta_1[n] \;\; \eta_2[n] \;\; \cdots \;\; \eta_{N_r}[n] \,]^T\end{aligned}$$

as the receive signal vector, transmit signal vector and the channel noise vector, respectively, then (1) can be written as

$$\mathbf{x}[n] = \sum_{\ell} \mathbf{H}_\ell \mathbf{u}[n-\ell] + \boldsymbol{\eta}[n] \tag{2}$$

where $[\mathbf{H}_\ell]_{k,i} = h_{k,i}[\ell]$ is the $N_r \times N_t$ channel matrix of order q, that is,

$$\mathbf{H}(z) = \sum_{\ell=0}^{q} \mathbf{H}_\ell z^{-\ell}$$

is the channel transfer function matrix.

Assuming L OFDM symbols are transmitted. Defining the Sylvester matrix

$$\mathbf{H} \triangleq \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_q & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_q & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_q \end{bmatrix},$$

and

$$\begin{aligned}\check{\mathbf{x}}[n] &\triangleq \left[\mathbf{x}^T[n] \quad \mathbf{x}^T[n-1] \quad \cdots \quad \mathbf{x}^T[n-L+1]\right]^T,\\ \check{\mathbf{u}}[n] &\triangleq \left[\mathbf{u}^T[n] \quad \mathbf{u}^T[n-1] \quad \cdots \quad \mathbf{u}^T[n-L-q+1]\right]^T.\\ \check{\boldsymbol{\eta}}[n] &\triangleq \left[\boldsymbol{\eta}^T[n] \quad \boldsymbol{\eta}^T[n-1] \quad \cdots \quad \boldsymbol{\eta}^T[n-L+1]\right]^T.\end{aligned}$$

as the received signal vector, transmitted signal vector and noise vector, respectively, then (2) can be written as

$$\check{\mathbf{x}}[n] = \mathbf{H}\check{\mathbf{u}}[n] + \check{\boldsymbol{\eta}}[n].$$

III. SOS BASED PREFILTER-BLIND EQUALIZER DESIGN

A. Methodology of Equalization

To satisfy the blind identification conditions in [8], the transmitted signal vector $\check{\mathbf{u}}(n)$ are assumed to be spatially uncorrelated but temporally correlated with distinct power. Without loss of generality, $\check{\mathbf{u}}(n)$ can assume to have unit variance and zero mean. Define the correlation matrix of $\check{\mathbf{u}}[n]$ as $\mathbf{R}_{\check{\mathbf{u}}\check{\mathbf{u}}}(\tau) \triangleq E\left[\check{\mathbf{u}}[n]\check{\mathbf{u}}^H[n+\tau]\right]$, then $\mathbf{R}_{\check{\mathbf{u}}\check{\mathbf{u}}}(0) = \mathbf{I}_{N_t L}$. The autocorrelation matrix of $\check{\mathbf{u}}(n)$ can be expressed as

$$\begin{aligned}\mathbf{R}_{\check{\mathbf{u}}\check{\mathbf{u}}}(\tau) &= E\left[\check{\mathbf{u}}[n]\check{\mathbf{u}}^H[n+\tau]\right] = \\ &= \begin{cases} \mathbf{I}_{N_t(L+q)}, & \text{for } \tau = 0, \\ diag\left(\rho_1(\tau), \ldots, \rho_{N_t(L+q)}(\tau)\right), & \text{for } \tau \neq 0, \end{cases}\end{aligned}$$

where $\rho_1(\tau) \neq \cdots \neq \rho_{N_t(L+q)}(\tau) \neq 0$. We further assume that $\check{\boldsymbol{\eta}}[n]$ is white Gaussian distributed and is mutually uncorrelated with $\check{\mathbf{u}}[n]$. Then the autocorrelation matrix of the channel output $\check{\mathbf{x}}(n)$ can be written as

$$\mathbf{R}_{\check{\mathbf{x}}\check{\mathbf{x}}}(\tau) = \begin{cases} \mathbf{H}\mathbf{R}_{\check{\mathbf{u}}\check{\mathbf{u}}}(0)\mathbf{H}^H + \sigma^2_{\check{\eta}\check{\eta}}\mathbf{I}_{N_r L}, & \text{for } \tau = 0, \\ \mathbf{H}\mathbf{R}_{\check{\mathbf{u}}\check{\mathbf{u}}}(\tau)\mathbf{H}^H, & \text{for } \tau \neq 0, \end{cases} \tag{3}$$

where $\sigma^2_{\check{\eta}\check{\eta}}$ is the variance of the noise signal $\check{\boldsymbol{\eta}}[n]$. Defining $\check{\mathbf{v}}[n] = \mathbf{H}\check{\mathbf{u}}[n]$ as the channel output vector and

$$\mathbf{R}_{\check{\mathbf{v}}\check{\mathbf{v}}}(\tau) \triangleq E\left[\check{\mathbf{v}}[n]\check{\mathbf{v}}[n+\tau]^H\right].$$

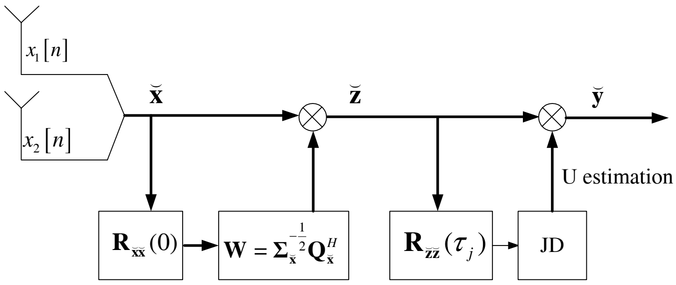

Fig. 1. Block diagram of equalization process with 2 receive antennas where JD represents joint diagonalization.

Since $\mathbf{R}_{\check{\mathbf{u}}\check{\mathbf{u}}}(0) = \mathbf{I}_{N_t(L+q)}$, therefore

$$\mathbf{R}_{\check{\mathbf{v}}\check{\mathbf{v}}}(0) = \mathbf{H}\mathbf{H}^H.$$

Let $\mathbf{W}$ be a whitening matrix that whitens $\check{\mathbf{v}}[n]$ such that

$$E\left[\mathbf{W}\check{\mathbf{v}}[n]\check{\mathbf{v}}^H[n]\mathbf{W}^H\right] = \mathbf{I}_{N_r L},$$

where

$$\mathbf{W} = \mathbf{\Sigma}_{\check{\mathbf{v}}}^{-\frac{1}{2}}\mathbf{Q}_{\check{\mathbf{v}}}^H,$$

with $\mathbf{\Sigma}_{\check{\mathbf{v}}}$ being the square root inverse of the eigenvalue matrix of $\mathbf{R}_{\check{\mathbf{v}}\check{\mathbf{v}}}(0)$, and $\mathbf{Q}_{\check{\mathbf{v}}}$ being the eigenvector matrix of $\mathbf{R}_{\check{\mathbf{v}}\check{\mathbf{v}}}(0)$. Then we can obtain

$$\begin{aligned}\mathbf{W}\mathbf{R}_{\check{\mathbf{v}}\check{\mathbf{v}}}(0)\mathbf{W}^H &= E\left[\mathbf{W}\check{\mathbf{v}}[n]\check{\mathbf{v}}^H[n]\mathbf{W}^H\right]\\ &= \mathbf{W}\mathbf{H}\mathbf{H}^H\mathbf{W}^H\\ &= \mathbf{I}_{N_r L}\end{aligned} \tag{4}$$

According to (4), the effective channel $\mathbf{U} = \mathbf{W}\mathbf{H}$ is a unitary matrix. Applying $\mathbf{W}$ to the received signal vector $\check{\mathbf{x}}[n]$, we can obtain

$$\begin{aligned}\check{\mathbf{z}}[n] &= \mathbf{W}\check{\mathbf{x}}[n]\\ &= \mathbf{W}\left[\mathbf{H}\check{\mathbf{u}}[n] + \check{\boldsymbol{\eta}}[n]\right]\\ &= \mathbf{U}\check{\mathbf{u}}[n] + \mathbf{W}\check{\boldsymbol{\eta}}[n]\end{aligned} \tag{5}$$

From (5), we see that $\mathbf{U}$ can be equalized by

$$\mathbf{U}^{-1}\check{\mathbf{z}}[n] = \mathbf{U}^H\check{\mathbf{z}}[n] = \check{\mathbf{u}}[n] + \mathbf{U}^H\mathbf{W}\check{\boldsymbol{\eta}}[n] \tag{6}$$

From (6), the problem of equalization becomes finding the unitary equalization matrix of $\mathbf{U}$. Defining the correlation matrices for $\check{\mathbf{z}}[n]$ and $\check{\boldsymbol{\eta}}[n]$ as $E\left[\check{\mathbf{z}}[n]\check{\mathbf{z}}^H[n+\tau]\right]$ and $E\left[\check{\boldsymbol{\eta}}[n]\check{\boldsymbol{\eta}}^H[n+\tau]\right]$, respectively. From (3), $E\left[\check{\boldsymbol{\eta}}[n]\check{\boldsymbol{\eta}}^H[n+\tau]\right] = 0$, for $\tau \neq 0$. Thus, the correlation matrix of $\check{\mathbf{z}}[n]$ can be written as

$$\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(\tau) = \mathbf{U}\mathbf{R}_{\check{\mathbf{u}}\check{\mathbf{u}}}(\tau)\mathbf{U}^H, \quad \text{for } \tau \neq 0. \tag{7}$$

Thus, the equalizer $\mathbf{U}$ can be obtained by diagonalizing $\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(\tau)$. According to [12], we can find $\mathbf{U}$ that equalizes frequency-selective channels if the source signal has different spectral energy. In addition, the chance of eigenvalue degeneracy can also be reduced by performing a joint diagonalization on a set of $\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(\tau)$ with various $\tau \neq 0$, i.e.

$$\begin{aligned}&\mathbf{U}^H\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(\tau_p)\mathbf{U} = diag\left(\rho_1(\tau_p), \rho_2(\tau_p), \ldots, \rho_{N_t(L+q)}(\tau_p)\right),\\ &\quad \text{for } p = 0 \leq t \leq P\end{aligned} \tag{8}$$

where $\tau_0, \tau_1, \ldots, \tau_P$ are non-zero time lags. The overall joint diagonalization (JD) equalization process is illustrated in Figure 1.

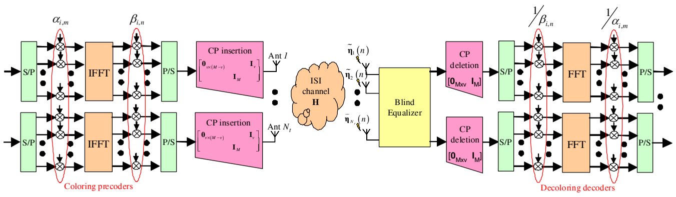

Fig. 2. MIMO-OFDM prefilter-blind equalizer system with N_t/N_r Tx/Rx antennas.

B. Prefilter Design

If all the source data streams are uncorrelated, the required temporal correlation property can be easily achieved by shaping the power spectral density of each data stream [8]. [12] proposed to use a set of low complexity filters to color the source signal stream such that FIR blind equalization is possible at the receiver to equalize FIR-MIMO channels. However, the proposed filters were chosen arbitrarily without regards on its effects on BER performance. Moreover, no investigation was carried out about how the prefilter can be used to reduce computational complexity at the receiver while sustaining equalization performance. In this paper, a new set of prefilters are proposed that will allow us to select a subset of $\{\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(\tau_p)\}$ such that it not only reduces the computational complexity at the receiver, but it also does not impact the equalization performance compare to the case when the full set of autocorrelation matrices are used. As seen in Figure 2, the prefilters are applied in the frequency domain (prior to IFFT) to all N_t transmit antennas of a MIMO-OFDM system. The set of coloring prefilters are denoted as $\{\mathbf{P}_0(z), \mathbf{P}_1(z), \ldots, \mathbf{P}_{N_t-1}(z)\}$, where

$$\mathbf{P}_i(z) = diag\left(\alpha_{i,0}, \alpha_{i,1}, \ldots, \alpha_{i,M-1}\right) \quad \text{for } i = 1, 2, \ldots, N_t.$$

$\alpha_{i,m}$ is the multiplier coefficient of the m^{th} path of IFFT for the i^{th} transmit antenna as illustrated in Figure 2. A scaling matrix is then applied in the time domain (after the IFFT), which is given as

$$\mathbf{S}_i(z) = diag\left(\beta_{i,0}, \beta_{i,1}, \ldots, \beta_{i,M-1}\right), \quad \text{for } i = 1, 2, \ldots, N_t,$$

where $\beta_{i,n}$ is a scaling factor for satisfying the condition of distinct power. At the receiver, the inverse manipulation of the transmitters is used to decolor the colored signal. The proposed real-valued multiplier $\alpha_{i,m}$ is formed with two parts. The first part generates the orthogonality among different prefilters, and the second part introduces temporal correlation to the transmitted signal. Since the performance of the joint diagonalization algorithm is based on spectral overlap of the source signals [10], this led to the use of orthogonal prefilters. $\alpha_{i,m}$ can be expressed as

$$\alpha_{i,m} = O_i(m)\left[1 - \sum_{p=0}^{P-1} C_{i,\tau_p} \cos\left(\frac{2\pi m \tau_p}{M}\right)\right] \tag{9}$$

where $O_i(m)$ is a function having only two possible values $+1$ and -1. $O_i(m)$ can be designed to generate orthogonality among different prefilters by being assigned different shape for different prefilters. C_{i,τ_p} determines the magnitude of corresponding cosine term. Distinct values of C_{i,τ_p} must be used for various values of n and p in order to satisfy the distinct power conditions in [8]. The number of cosine term can be decided arbitrarily by choosing P. Furthermore, different τ_p is used for different cosine terms with $\tau_p = 1, 2, \ldots$. The reason for using cosine is because we can completely control how many autocorrelation matrices in $\{R_{\check{\mathbf{z}}\check{\mathbf{z}}}(\tau_p)\}$ we need in (8) for the joint diagonalization. This can be seen by considering the inverse Fourier transform of $\cos\left(\frac{2\pi m \tau_p}{M}\right)$:

$$\begin{aligned}
\mathscr{F}^{-1}\left\{\cos\left(\frac{2\pi m \tau_p}{M}\right)\right\} &= \sum_{m=0}^{M-1} \cos\left(\frac{2\pi m \tau_p}{M}\right) e^{\frac{j2\pi m n}{M}} \\
&= \frac{1}{2}\left[\sum_{m=0}^{M-1} e^{\frac{j2\pi m(n+\tau_p)}{M}} + \sum_{m=0}^{M-1} e^{\frac{j2\pi m(n-\tau_p)}{M}}\right] \\
&= \frac{1}{2}\left(\delta[n+\tau_p] + \delta[n-\tau_p]\right)
\end{aligned} \tag{10}$$

Using (10), the time domain signal after IFFT can be written as

$$\begin{aligned}
&\check{u}(n) * \{1 - \tfrac{C_{i,\tau_0}}{2}(\delta[n+\tau_0] + \delta[n-\tau_0])\} \\
&= \check{u}(n) - \tfrac{C_{i,\tau_0}}{2}(\check{u}[n+\tau_0] + \check{u}[n-\tau_0])
\end{aligned} \tag{11}$$

where $*$ denotes convolution and $P = 1$. From (11), it is easy to see that temporal correlation of delay τ_0 can be generated. Therefore, only $\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(\tau_0)$ will have to be used in the joint diagonalization process at the receiver. In fact, using the rest of the $\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(\tau_p)$, $\forall p \neq 0$ will not improve the equalization performance. This will be shown in the simulation results in the next section when we compare equalization performance of our proposed algorithm using different τ_p. Besides varying the parameter τ_p, the parameter P can also be used to improve performance of the equalizer. This can be achieved by increasing the value of P such that more temporal correlation is added to the transmitted bitstream. However, as will be seen in Section IV, P cannot be increased indefinitely because the prefilter will introduce too much amplitude variation into the bitstream which degrades the BER performance, even though a better estimation of $\mathbf{U}$ can be obtained.

IV. Simulation Results

A MIMO-OFDM system is simulated to evaluate the performance of the proposed scheme. In all simulations, $N_t = 2$, $M = 64$, and $v = 16$. The channels are randomly generated. Two channels were chosen to show the efficacy of the proposed algorithm. The first one is a 3-tap channel with 4 received antennas, i.e. $N_r = 4$, with coefficients

$$\begin{aligned}
\mathbf{H}(z) = &\begin{bmatrix} 0.3487 & 0.7220 \\ 0.5121 & 0.5970 \\ -0.3651 & 0.6136 \\ 0.6202 & 0.4880 \end{bmatrix} + \begin{bmatrix} -0.4650 & 0.6189 \\ 0.7682 & -0.3980 \\ -0.9129 & 0.3835 \\ 0.2481 & 0.7807 \end{bmatrix} z^{-1} \\
&+ \begin{bmatrix} -0.8137 & -0.3094 \\ -0.3841 & 0.6965 \\ 0.1826 & 0.6903 \\ 0.7442 & -0.3904 \end{bmatrix} z^{-2}.
\end{aligned} \tag{12}$$

The second one is a 7-tap channel with 5 received antennas, i.e. $N_r = 5$, with coefficients

$$\begin{aligned}
\mathbf{H}(z) = &\begin{bmatrix} 0.5671 & -0.1796 \\ -0.2803 & 0.2466 \\ -0.4485 & 0.2949 \\ -0.4709 & 0.2253 \\ 0.1899 & 0.4222 \end{bmatrix} + \begin{bmatrix} 0.4962 & 0.2694 \\ -0.4484 & 0.3288 \\ 0.1495 & -0.2949 \\ -0.4036 & -0.1931 \\ 0.2279 & 0.3518 \end{bmatrix} z^{-1} \\
&+ \begin{bmatrix} -0.1418 & 0.2694 \\ -0.5045 & -0.4110 \\ 0.4485 & -0.5160 \\ 0.3363 & 0.4507 \\ 0.6078 & -0.2814 \end{bmatrix} z^{-2} + \begin{bmatrix} -0.3544 & -0.3592 \\ 0.3363 & -0.5754 \\ 0.2242 & -0.1474 \\ -0.4709 & 0.1931 \\ -0.3799 & 0.5629 \end{bmatrix} z^{-3}
\end{aligned}$$

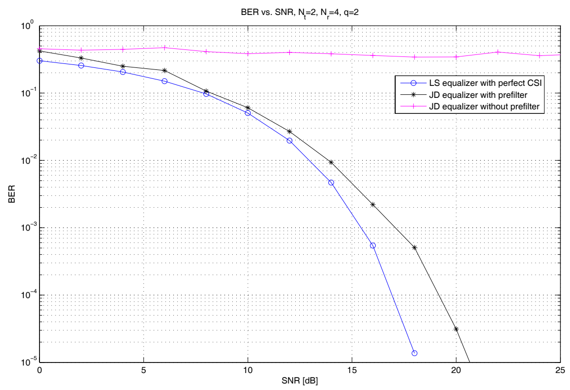

Fig. 3. Comparison of BER performance between system without coloring and with coloring for 3-tap channel.

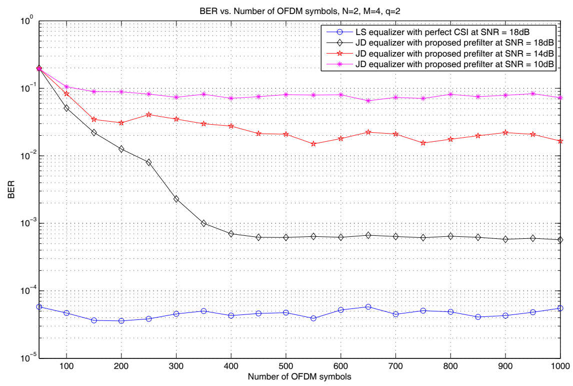

Fig. 4. BER vs. different number of received OFDM symbols for 3-tap channel at SNR = $10, 14, 18$ dB.

$$+\begin{bmatrix} 0.3544 & 0.4490 \\ 0.3363 & 0.1644 \\ -0.2242 & -0.5160 \\ 0.1345 & -0.5151 \\ -0.5318 & -0.2814 \end{bmatrix} z^{-4} + \begin{bmatrix} 0.2836 & -0.4490 \\ -0.1962 & -0.4932 \\ -0.5232 & 0.3686 \\ 0.2018 & -0.4507 \\ 0.3039 & -0.4222 \end{bmatrix} z^{-5}$$
$$+\begin{bmatrix} -0.2836 & 0.5388 \\ 0.4484 & -0.2466 \\ -0.4485 & -0.3686 \\ 0.4709 & 0.4507 \\ 0.1519 & 0.2111 \end{bmatrix} z^{-6}. \tag{13}$$

The input data stream to the IFFT at the transmitter is uniformly distributed QPSK signal with zero mean and unit variance. Except for the prefilters in Figure 8, the prefilter used in the simulations is of the form $\alpha_{i,m} = O_i(m)[1 - C_{i,1}\cos(2\pi m/64) - C_{i,2}\cos(4\pi m/64)]$, for $i = 1$ and 2. In other words, $P = 2$. $O_i(m)$ is chosen to make the inner product of the prefilter spectrum between two transmit antennas equal to 0. This, however, can easily be generalized to any number of transmit antennas. $C_{i,1}$ and $C_{i,2}$ are set to range from 0.025 to 0.25, and varies for different transmit antennas and different time indicies in order to distinctly color the signal in time. For the simulation results below, for $n = 0$, $C_{1,1} = 0.025$, $C_{1,2} = 0.25$, $C_{2,1} = 0.1$, and $C_{2,2} = 0.175$. For $n = 1$, $C_{1,1} = 0.25$, $C_{1,2} = 0.1$, $C_{2,1} = 0.175$, and $C_{2,2} = 0.025$. For $n = 2$, $C_{1,1} = 0.1$, $C_{1,2} = 0.175$, $C_{2,1} = 0.025$, and $C_{2,2} = 0.25$. For $n = 3$, $C_{1,1} = 0.175$, $C_{1,2} = 0.025$, $C_{2,1} = 0.25$, and $C_{2,2} = 0.1$. The sequence will then repeat for subsequent time index. This is done in order to satisfy the distinct power condition. Since the distinct power condition has already been satisfied by varying C_{i,τ_p}, therefore, $\beta_{i,n}$ can be assigned to have a value of 1 for all i and n.

Figure 3 compares the BER performance of the proposed prefilter with the JD equalizer with a similar system that uses the JD equalizer but without any prefilter. A least-squares (LS) zero-forcing equalizer with perfect channel state information (CSI) is used as benchmark. For the prefilter-equalizer system, the equalizer uses two correlation matrices, $\mathbf{R}_{\tilde{\mathbf{z}}\tilde{\mathbf{z}}}(\tau_1)$ and $\mathbf{R}_{\tilde{\mathbf{z}}\tilde{\mathbf{z}}}(\tau_2)$, for joint diagonalization. The 3-tap channel in (12) was used. Since the input signal $s_{m,\ell}^{(i)}$ is independently distributed, the result reaffirms the idea that if the diagonal entries of the input signal power spectral density matrix are not distinct (no prefilter is used), then it is not possible to identify and equalize FIR-MIMO channels using SOS of the received signal.

Figure 4 shows the BER results of the proposed prefilter-equalizer system as the number of OFDM symbols varies for different SNR values. Similar to previous simulations, $\mathbf{R}_{\tilde{\mathbf{z}}\tilde{\mathbf{z}}}(\tau_1)$ and $\mathbf{R}_{\tilde{\mathbf{z}}\tilde{\mathbf{z}}}(\tau_2)$ are used for joint diagonalization. As the figure shows, the BER of the proposed prefilter-equalizer system approaches that of the LS equalizer when the number of symbols increases. This is because as more symbols are used, more accurate estimation of the correlation matrix can be obtained. Furthermore, at SNR = 18 dB, the proposed algorithm is able to equalize the channel using only 350 symbols with a BER of about 10^{-3}. Compare to higher-order statistics techniques such as [1], when the number of symbols needed for equalization is in the order of 10^3, only a small amount of latency is incurred in the proposed technique in order to equalize FIR-MIMO channels.

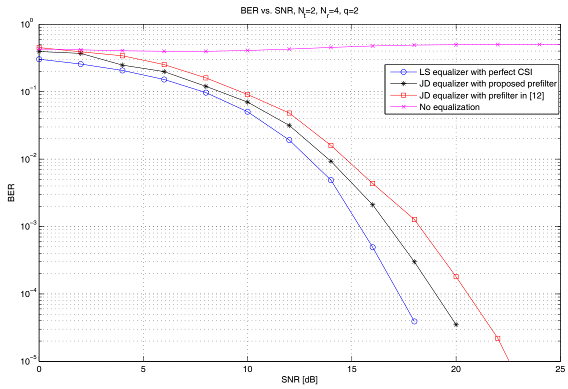

Fig. 5. Comparison of BER vs. SNR with different algorithms for 3-tap channel.

Figures 5 and 6 compare the BER performance of the proposed prefilter-equalizer system with the prefilter-equalizer scheme in [12]. Results using a LS equalizer and an identical system that uses no equalization are also shown as benchmarks. As seen in the figures, the performance gap between the LS equalizer and the proposed one remains virtually unchanged as the channel spectrum changes. This shows that the performance of the proposed scheme is insensitive to various channel responses. This can be explained by observing the equation of the prefilter in (9). Since the prefilter is composed of cosine functions, the spectrum of the prefilter will fluctuate periodically in the frequency domain. Since the amplitude of the cosine, C_{i,τ_p}, is set to a small value, even if the minimum value of the cosine term coincides with the spectral null of the channel, this will not greatly impact the BER. Compared with the prefilters in [12], the proposed prefilters perform better by at least 1 dB in all of the simulated channel conditions. Furthermore, using the proposed prefilter, the performance of the 3-tap channel outperforms that of the 7-tap because there are more coefficients in the 7-tap channel which need to be identified for equalization.

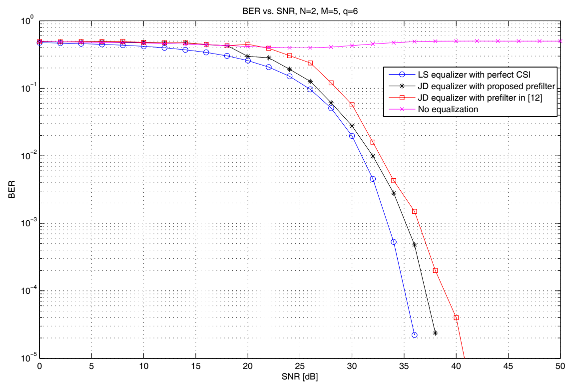

Fig. 6. Comparison of BER vs. SNR with different algorithms for 7-tap channel.

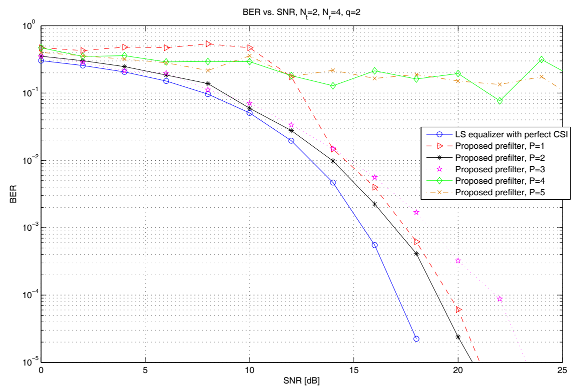

Fig. 8. BER vs. SNR for different P for 3-tap channel.

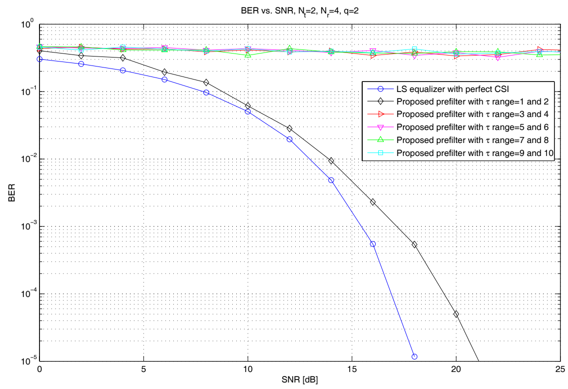

Fig. 7. BER vs. SNR with different lag for 3-tap channel.

Figure 7 shows the BER result when different lags are chosen for joint diagonalization at the receiver. As explained in Section III, if $\tau_1 = 1$ and $\tau_2 = 2$, then $\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(1)$ and $\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(2)$ become the most important correlation matrices for joint diagonalization. This is reaffirmed by the simulation results in Figure 7 when the best BER performance is attained when only $\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(1)$ and $\mathbf{R}_{\check{\mathbf{z}}\check{\mathbf{z}}}(2)$ are used for equalization. When other correlation matrices are used, the BER curves saturate to a noise floor. Since the choice for τ_p is chosen at the transmitter and it determines exactly which, as well as how many, correlation matrices should be used at the receiver for equalization, the transmitter has complete control on the computational complexity of the equalizer.

As discussed earlier, the parameter P cannot be increased indefinitely in order to enhance equalization performance at the expense of increase computational complexity. Likewise, P also cannot be made too small since it will adversely affect the equalization performance. Figure 8 shows the result of the proposed algorithm when P is allowed to vary from 1 to 5. As seen from the figure, the BER is smallest when $P = 2$. Therefore, P cannot be made arbitrary small in order to minimize computational complexity at the receiver, but it also cannot be made arbitrarily big since it will adversely impact the BER performance since this will induce too much amplitude variation into the transmitted bitstream.

V. Conclusion

A SOS-based prefilter-blind equalizer system has been proposed to equalize FIR-MIMO channels for MIMO-OFDM system. Simulation results have shown that using the proposed prefilter can not only outperform the prefilter proposed in [12] in terms of BER, but also allows the transmitter to dictate which, and how many, correlation matrices are to be used for equalization at the receiver. This decreases the amount of computational complexity at the receiver compared to the scheme in [12] since the number of correlation matrices needed for joint diagonalization can be predetermined before transmission.

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