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Noncoherent Ungerboeck-Type Trellis-Coded MPSK

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***Abstract*— Trellis-coded MPSK proposed by Ungerboeck is noncoherently catastrophic. In this paper, we propose a new scheme in which Ungerboeck-type trellis-coded MPSK is not a noncoherently catastrophic code. In the proposed scheme, a particularly designed differential encoder is added in front of the trellis encoder such that noncoherently-equivalent code sequences correspond to the same data bits. With this differential encoder, trellis-coded MPSK proposed by Ungerboeck is no longer noncoherently catastrophic and thus achieve better error performance. Moreover, new trellis codes which have better bit error rate than Ungerboeck's codes for the proposed scheme are found by computers.**

I. Introduction

Trellis-coded modulation (TCM) proposed by Ungerboeck [1]-[3] can provide bandwidth-efficient transmission over the additive white Gaussian noise (AWGN) channel. For the TCM in [1]-[3], coherent decoding is used and hence carrier recovery is needed.

Noncoherent detection is a detection technique that can be implemented without carrier phase tracking. Since DPSK (differential phase shift keying) cannot offer satisfactory error performance, designing coded modulation for noncoherent detection is important. In 1996, noncoherent trellis-coded MPSK (M-ary phase shift keying), for which the decoding can be implemented noncoherently, was proposed in [4]. The decoder is the Viterbi decoder using a sliding window of the observations to compute branch metrics. Each observation covers several branches of the trellis. Different decoding algorithms and metrics were proposed in [5]–[7]. The optimal decoding trellis diagram used in the Viterbi algorithm for noncoherent trellis-coded MPSK is an augmented trellis diagram for which the number of states grows exponentially with the observation length. Using the optimal decoding trellis is impractical when the observation length is not small. Hence, decoding algorithms which use the original trellis are preferred. The trivial algorithm, called basic decision feedback algorithm (BDFA) [5], is the simplest one.

Two code sequences are said to be noncoherently-equivalent if a constant phase rotation of one sequence results in the other sequence. If two code sequences are noncoherently-equivalent, the noncoherent receiver cannot distinguish one from the other. Noncoherently catastrophic codes are defined as codes in which two infinite-length code sequences are noncoherently-equivalent and correspond to different data bits [4].

A TCM is said to be rotationally invariant (RI) under a θ phase shift if rotating any code sequence by θ is always another code sequence corresponding to the same data bits [13]-[17]. Consequently, a TCM which is RI under θ is also RI under any multiple of θ. Noncoherent trellis-coded MPSK can be classified into two categories: RI and anti-RI, i.e., no two code sequences are rotated versions of each other [11]. Trellis-coded MPSK which is RI under $2\pi/M$ is not noncoherently catastrophic and hence can be used for noncoherent decoding. However, RI codes which have been proposed were designed for maximizing Euclidean distance, not noncoherent distance. Trellis-coded MPSK in [4] is anti-RI since noncoherently-equivalent code sequences do not exist. However, codes in [4] have lower transmission rate than coherent trellis-coded MPSK in [1]-[3].

Codes of trellis-coded MPSK in [1]-[3] are noncoherently catastrophic. In [8], we proposed a method which modifies trellis-coded MPSK in [1] to avoid noncoherently-equivalent code sequences. We showed that by this method the TCM is not a noncoherently catastrophic code. Then in [9], the authors extended our method on trellis-coded 16QAM to avoid noncoherently-equivalent code sequences also. However, in [10], we indicated that the code of [9] is a noncoherently catastrophic code by listing several noncoherently-equivalent code sequences in the code of [9]. Recently we realize that the modified TCM in [8] is also a noncoherently catastrophic code. The verification in [8] was erroneous in fact. In a noncoherently catastrophic code, perhaps only a few code sequences are noncoherently-equivalent. In such case, if the transmitted code sequence is long enough, the transmitted code sequence is unlikely to be the sequence which is noncoherently-equivalent to another sequence. Therefore, in [10], we indicated that a noncoherently catastrophic code does not necessarily result in catastrophic error performance. Nevertheless, the error performance can be improved if noncoherently-equivalent code sequences corresponding to different data bits can be avoided.

In this paper, we propose a novel noncoherent trellis-coded MPSK scheme. In the proposed scheme, the noncoherent trellis codes are partial-RI codes in which some, not all, code sequences rotated by a constant phase are another code sequences corresponding to the same data bits. These code are quite different to conventional noncoherent trellis codes which are either RI or anti-RI. In our scheme, Ungerboeck-type trellis-coded MPSK in [1]-[3] is used, and a particularly designed differential encoder is added in front of the trellis encoder. In stead of attempting to avoid noncoherently-equivalent code sequences as [8] and [9], we let all noncoherently-equivalent code sequences correspond to the same data bits by this differential encoder so that the code is not a noncoherently catastrophic code. Simulation results indicate that the error performance can be improved by the proposed differential encoder.

A noncoherent distance meassure for noncoherent trellis-coded MPSK was proposed in [12]. Because the distance was derived for the optimal decoding trellis, a new distance measure for BDFA which is modified from the distance in [12] is proposed in this paper. We use computers to search codes which maximize minimum noncoherent distance. For the simulations using BDFA, searched codes in general have better error performance than the original codes in [1]-[3] which were designed for coherent decoding.

II. Noncoherent Decoding and Distance

Consider trellis coded M-ary PSK signals which are transmitted over an additive white Gaussian noise (AWGN) channel. For the convenience of presentation, the number of symbols in a trellis branch is assumed to be one. The baseband transmitted symbol at time unit t is $x_t = e^{j\phi_t}$ where ϕ_t is the modulation phase. The corresponding baseband symbol at the receiver is $r_t = x_t e^{j\theta} + n_t$, where n_i is a zero-mean white complex Gaussian noise sample with variance $2\sigma^2$, and θ is an arbitrary phase shift which is uniformly distributed over $[0, 2\pi)$ and assumed to be constant for L consecutive symbols.

In [4], the decoding metric for a possible code sequence $\hat{\mathbf{x}} = \{\cdots, \hat{x}_t, \hat{x}_{t+1}, \cdots\}$ is

$$\eta^{(A)} = \sum_{t=-\infty}^{\infty} \eta_t^{(A)} = \sum_{t=-\infty}^{\infty} \left| \sum_{i=0}^{L-1} r_{t-i} \hat{x}_{t-i}^* \right|^2 . \quad (1)$$

where $\hat{x}_t^*$ is the complex conjugate of $\hat{x}_t$. The number of branches which are covered by one observation is L. In [6], two different metrics were proposed as

$$\eta^{(B)} = \sum_{t=-\infty}^{\infty} \eta_t^{(B)} = \sum_{t=-\infty}^{\infty} \Re \left\{ r_t \hat{x}_t^* \sum_{i=1}^{L-1} r_{t-i}^* \hat{x}_{t-i} \right\} \quad (2)$$

and

$$\eta^{(C)} = \sum_{t=-\infty}^{\infty} \eta_t^{(C)} = \sum_{t=-\infty}^{\infty} \left(\left| \sum_{i=0}^{L-1} r_{t-i} \hat{x}_{t-i}^* \right| - \left| \sum_{i=1}^{L-1} r_{t-i} \hat{x}_{t-i}^* \right| \right). \quad (3)$$

We describe BDFA in the following. BDFA keeps track of the associated survivor for each state. Let $J_t(v)$ be the accumulated metric for the survivor path of state v at time unit t. Let $B_t(v,l)$ represent the l-th ($l \in \{1, 2, \cdots, R\}$) branch that connect state s_l at time unit $t-1$ and state v at time unit t. Let $P_t(v,l) = \{\cdots, \hat{x}_{t-i}(v,l), \hat{x}_{t-i+1}(v,l), \cdots, \hat{x}_t(v,l)\}$ represent the path which is the union of $B_t(v,l)$ and the survivor path of state s_l at time unit $t-1$. Let $\eta_t(v,l)$ be the branch metric for $B_t(v,l)$ which can be $\eta_t^{(A)}$, $\eta_t^{(B)}$ or $\eta_t^{(C)}$. For BDFA, the metric for the path $P_k(v,l)$ is computed as

$$\hat{J}_t(v,l) = J_{t-1}(s_l) + \eta_t(v,l). \quad (4)$$

The index l that maximizes $\hat{J}_t(v,l)$, denoted as m, is chosen to update the survivor of state v at time unit t and $J_t(v) = \hat{J}_t(v,m)$.

If N_s denotes the number of states in the original trellis diagram and R denotes the number of branches diverging from a state, the number of states in the optimal trellis diagram is $N_s \times R^{L-1}$. Because the optimal trellis is complicated, decoding algorithms which use the original trellis and take future metrics into account, like modified decision feedback algorithm (MDFA) [5] and improved decision feedback algorithm (IDFA) [7], were proposed. To detect r_t, all later metrics $\eta_{t+1}, \eta_{t+2}, \cdots, \eta_{t+L-1}$ which are related to r_t should be considered in advance. Let $\tilde{J}_{t+L-1}(v,l)$ be the estimated future metric at time $t+L-1$ for $P_k(v,l)$, where

$$\tilde{J}_{t+L-1}(v,l) = \hat{J}_t(v,l) + \sum_{q=t+1}^{t+L-1} \tilde{\eta}_q \quad (5)$$

in which $\tilde{\eta}_q$ is the estimated branch metric at time q. Because $r_{q-i}\hat{x}_{q-i}^*$ for $q-i > k$ can not be obtained, MDFA and IDFA propose different methods to compute $\tilde{\eta}_q$. The index l that maximizes $\tilde{J}_{k+L-1}(v,l)$, denoted as m, is chosen to update the survivor of state v at time unit k and $J_k(v) = \hat{J}_k(v,m)$.

Consider two code sequences that diverge from the same state, say s, and after k branches remerge into a common state, say s'. Let $(2L+k-2) \times 1$ matrices $\mathbf{x} = (x_1, \cdots, x_{L-1}, x_L, \cdots, x_{L+k-1}, x_{L+k}, \cdots, x_{2L+k-2})^T$ and $\hat{\mathbf{x}} = (x_1, \cdots, x_{L-1}, \hat{x}_L, \cdots, \hat{x}_{L+k-1}, x_{L+k}, \cdots, x_{2L+k-2})^T$ represent the two code sequences where $x_1, \cdots, x_{L-1}$ are symbols of the common path before state s and $x_{L+k}, \cdots, x_{2L+k-2}$ are symbols of the same path after state s'. A $(2L+k-2) \times (2L+k-2)$ matrix $\mathbf{A}$ is defined by $A_{ij} = \hat{x}_i \hat{x}_j^* - x_i x_j^*$ where $\hat{x}_k = x_k$ for $k < L$ or $k \geq L+k$. Hence, $\mathbf{A}$ is Hermitian and can be expressed as

$$\mathbf{A} = \begin{pmatrix} \mathbf{O} & \mathbf{B} & \mathbf{O} \\ \mathbf{B^H} & \mathbf{C} & \mathbf{D^H} \\ \mathbf{O} & \mathbf{D} & \mathbf{O} \end{pmatrix} \quad (6)$$

where $\mathbf{O}$ is an $(L-1) \times (L-1)$ zero-entry matrix. In [12], it is shown that the approximate Chernoff upper bound on the pairwise error probability of using $\eta^{(B)}$ is

$$P(\mathbf{x} \to \hat{\mathbf{x}}) \leq \exp\left\{ -\frac{d^2(\mathbf{x}, \hat{\mathbf{x}})}{8\sigma^2} \right\} \quad (7)$$

where

$$d(\mathbf{x}, \hat{\mathbf{x}}) = \frac{|\mathbf{x^H A x}|}{\sqrt{\mathbf{x^H A^2 x}}} \quad (8)$$

determines the asymptotic high-SNR error probability. A code should be designed or searched for maximizing minimum noncoherent distance which is defined as the minimum value of $d(\mathbf{x}, \hat{\mathbf{x}})$ in (8) between any two code sequences $\mathbf{x}$ and $\hat{\mathbf{x}}$ corresponding to different data bits.

For BDFA, to decide the survivor of state s', $x_{L+k}, \cdots, x_{2L+k-2}$ are not taken into account. Therefore, the distance definition in (8) in not indicative for BDFA. By a derivation which is similar to the derivation in [12], we propose a distance definition for BDFA as

$$d'(\mathbf{x}, \hat{\mathbf{x}}) = \frac{|\mathbf{x}^{\mathbf{H}}\mathbf{A}'\mathbf{x}|}{\sqrt{\mathbf{x}^{\mathbf{H}}\mathbf{A}'^{2}\mathbf{x}}} \tag{9}$$

where $\mathbf{A}'$ is an $(L+k-1) \times (L+k-1)$ matrix defined by $A'_{ij} = \hat{x}_i \hat{x}_j^* - x_i x_j^*$ where $\hat{x}_k = x_k$ for $k < L$. Thus, $\mathbf{A}'$ can be expressed as

$$\mathbf{A}' = \begin{pmatrix} \mathbf{O} & \mathbf{B} \\ \mathbf{B}^{\mathbf{H}} & \mathbf{C} \end{pmatrix}. \tag{10}$$

III. Proposed Encoding Scheme

Consider DPSK first. Fig. 1 shows two descriptions of differential PSK. The common description illustrated in Fig. 1(a) is that at time t, data bits are fed into the signal mapper to obtain PSK symbol $s'_t = e^{j\Delta\phi_t}$, and then s'_t is sent into a differential encoder which results in the transmitted symbol $s_t = e^{j\phi_t}$ where $\phi_t = \phi_{t-1} + \Delta\phi_t \mod 2\pi$.

Based on this description of DPSK, conventional trellis-coded MPSK for the noncoherent receiver puts the convolutional encoder before the signal mapper and differentially encode coded symbol s'_t also. However, such differential encoding affects Euclidean distance. For example, consider two code sequences of trellis-coded 8PSK $\{0, 0, 0, 0, \cdots\}$ and $\{0, 4, 4, 0, 0, \cdots\}$. The squared Euclidean distance between them is 8. After differentially encoding, this pair becomes $\{0, 0, 0, 0, \cdots\}$ and $\{0, 4, 0, 0, \cdots\}$ whose squared Euclidean distance is only 4.

Let the modulation phase and data phase of differential QPSK be $\phi_t = b_t\pi/2$ and $\Delta\phi_t = \Delta b_t \pi/2$ respectively where $b_t, \Delta b_t \in \{0, 1, 2, 3\}$. Consequently, we have $b_t = b_{t-1} + \Delta b_t \mod 4$. The second description of DPSK is shown in Fig. 2(b). For Gray labeling, Δb_t is determined by $\Delta b_t = u_2^t \times 2 + (u_2^t \oplus u_1^t)$ where $\oplus$ denotes the XOR operator. Defining v_1^t and v_2^t by $v_2^t \times 2 + v_1^t = b_t$, we have

$$v_2^t \times 2 + v_1^t = v_2^{t-1} \times 2 + v_1^{t-1} + u_2^t \times 2 + (u_2^t \oplus u_1^t) \mod 4. \tag{11}$$

The special differential encoder in Fig. 1(b) produces v_1^t and v_2^t according to (11).

Based on the second description, we propose a novel noncoherent trellis-coded M-ary PSK as follows. For the convenience of presentation, we restrict M to be 8. Fig. 2 shows the block diagram of the proposed transmitter where the convolutional encoder is systematic and the baseband transmitted symbol is $x_t = e^{j(v_2^t \times 4 + v_1^t \times 2 + v_0^t)\pi/4}$. One property of Ungerboeck-type trellis-coded MPSK is that v_1^t and v_2^t do not affect the value of v_0^t. In other words, the value of v_0^t is determined before u_1^t and u_2^t are sent into the differential encoder.

At time t, for data bits u_1^t and u_2^t, we have

$$\Delta b_t = u_2^t \times 2 + (u_2^t \oplus u_1^t). \tag{12}$$

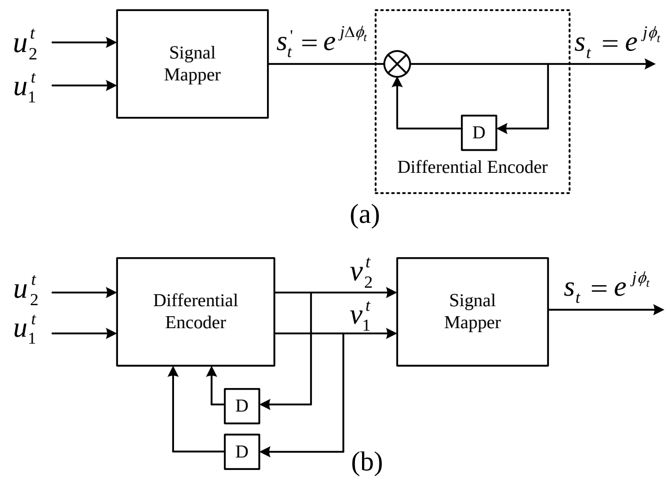

Fig. 1. Two descriptions of differential QPSK.

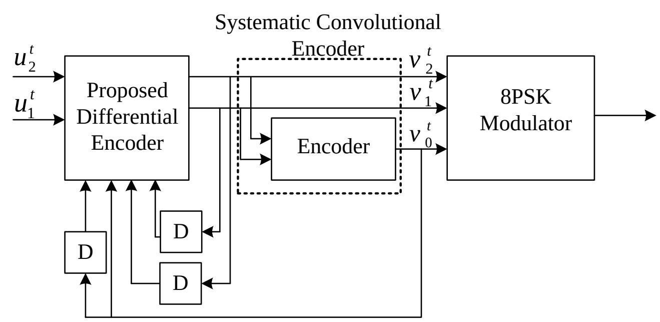

Fig. 2. Block diagram of the proposed transmitter.

The proposed differential encoder first computes b_t by

$$b_t = v_2^{t-1} \times 2 + v_1^{t-1} + \Delta b_t \mod 4. \tag{13}$$

Then the output bits of the differential encoder v_1^t and v_2^t are decided according to

$$v_2^t \times 2 + v_1^t = \begin{cases} b_t + 1 \mod 4 & \text{if } (v_0^{t-1}, v_0^t) = (1, 0) \\ b_t \mod 4 & \text{otherwise} \end{cases} \tag{14}$$

Note that if $(v_0^{t-1}, v_0^t) \neq (1, 0)$, v_1^t and v_2^t are the output symbols of conventional differential QPSK.

Concatenated with the differential encoder defined above, it can be shown that any trellis-coded MPSK that uses a systematic convolutional encoder in which v_1^t and v_2^t do not affect the value of v_0^t is not noncoherently catastrophic.

Trellis codes in [1]-[3] were not designed for noncoherent decoding, so searching new trellis codes that is appropriate for the proposed scheme is necessary. For the proposed differential encoder, convolutional encoders with 4, 8, 16 and 32 states for $L = 4$ and 8 are searched by computers. Parity-check coefficients h^2, h^1, h^0 defined in [3] are used to denote trellis codes. Table I shows searched codes for maximizing minimum noncoherent distance of (8). The minimum noncoherent distance of (8) for Ungerboeck's codes in [1]-[3] is also listed

TABLE I

UNGERBOECK CODES AND CODES SEARCHED FOR MINIMUM NONCOHERENT DISTANCE OF (8).

states	L	Ungerboeck's code		searched code	
		$h^2 - h^1 - h^0$	distance	$h^2 - h^1 - h^0$	distance
8	4	04-02-11	1.824	00-02-17	1.872
	8	04-02-11	1.971	–	–
16	4	16-04-23	1.860	00-20-35	1.907
	8	16-04-23	2.051	–	–
32	4	34-16-45	1.887	00-40-57	1.909
	8	34-16-45	2.120	–	–

TABLE II

UNGERBOECK CODES AND CODES SEARCHED FOR MINIMUM NONCOHERENT DISTANCE FROM (9).

states	L	Ungerboeck's code		searched code	
		$h^2 - h^1 - h^0$	distance	$h^2 - h^1 - h^0$	distance
8	4	04-02-11	1.670	00-02-17	1.753
	8	04-02-11	1.855	00-14-13	1.907
16	4	16-04-23	1.612	00-20-35	1.797
	8	16-04-23	1.909	–	–
32	4	34-16-45	1.717	00-40-57	1.827
	8	34-16-45	1.953	20-30-75	1.965

in Table I for comparison. Codes which have larger distance than Ungerboeck's codes can be found for 8, 16 and 32 states with $L = 4$. Table II presents searched codes for maximizing minimum noncoherent distance of (9). Codes for 8, 16 and 32 states with $L = 4$ are the same as Table I, and new codes for 8 and 32 states with $L = 8$ are obtained.

For the decoding in simulations, we use BDFA with $L = 4$ or 8 and the decoding metric is $\eta^{(B)}$. Fig. 3-6 present the simulation results for 4, 8, 16 and 32 states, respectively. For the codes in [1]-[3], adding the proposed differential encoder indeed improves the error performance. Besides, all searched codes in Table I and Table II have better error performance than the codes in [1]-[3] except for the 32-state with $L = 8$. We find that this distance measure is not well indicative of bit error rate (BER) for this scheme. Thus we search trellis codes according to their simulated BER. Table III lists some searched codes which have good BER but are different to codes in Table I and Table II. Searched codes which have the best BER obviously outperform original codes in [1]-[3] for $L = 4$.

IV. CONCLUSION

We have proposed a novel noncoherent trellis-coded MPSK scheme. In this scheme, high-rate Ungerboeck-type code is not noncoherently catastrophic because the particularly designed differential encoder let noncoeherntly-equivalent code

TABLE III

OTHER GOOD-PERFORMANCE CODES.

states	L	$h^2 - h^1 - h^0$	d_{nc} from (8)	d_{nc} from (9)
4	4	06-02-07	1.498	1.335
8	4	00-10-15	1.824	1.670
16	8	00-20-35	1.947	1.676
32	8	00-02-55	1.949	1.922

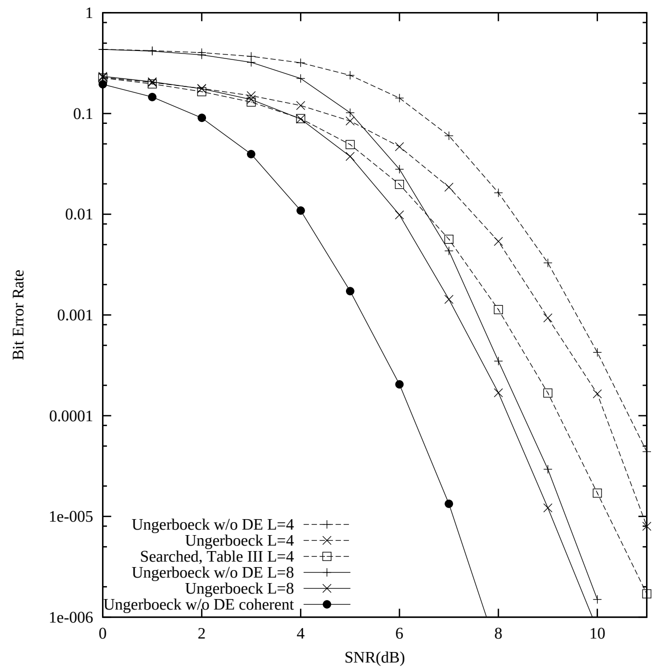

Fig. 3. Bit error rate for 4-state TCM.

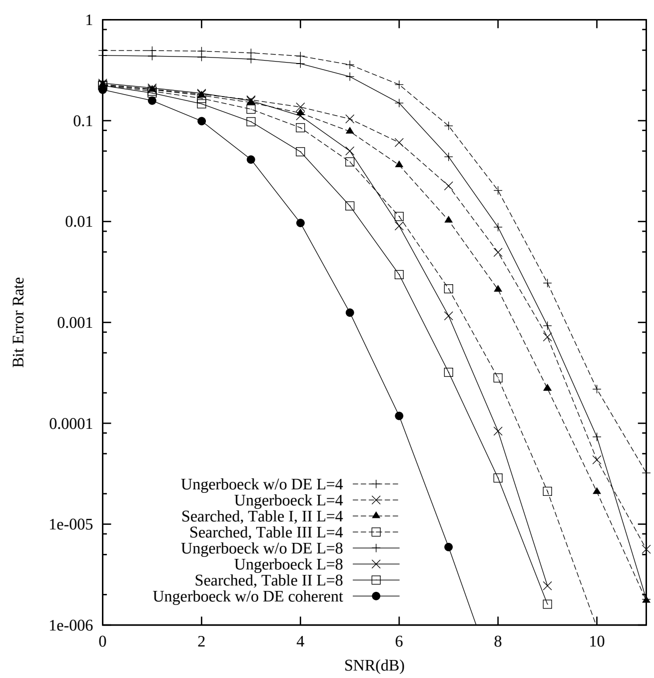

Fig. 4. Bit error rate for 8-state TCM.

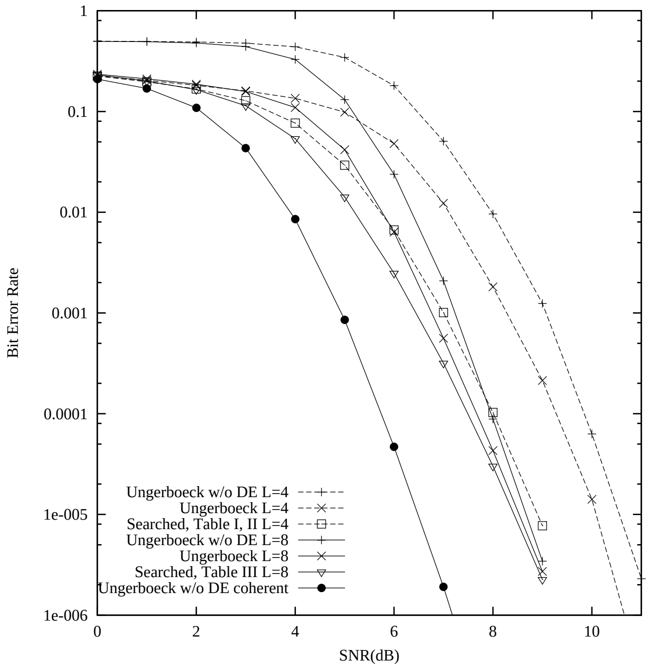

Fig. 5. Bit error rate for 16-state TCM.

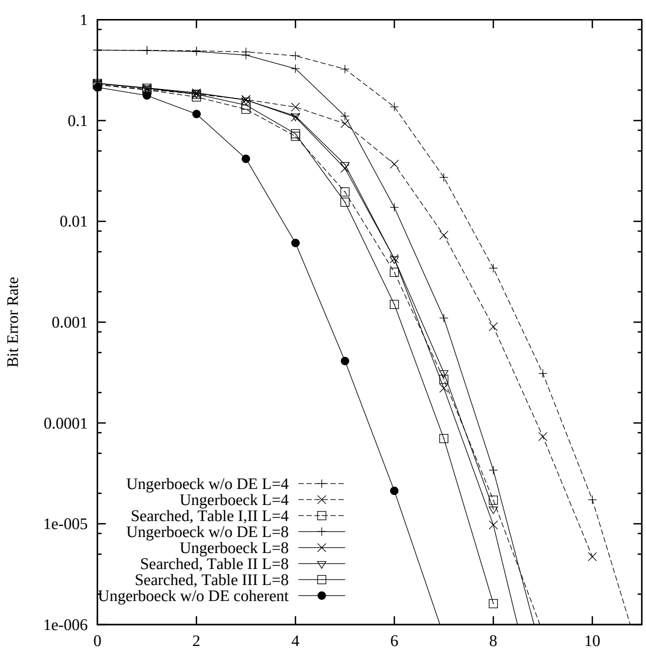

Fig. 6. Bit error rate for 32-state TCM.

sequences correspond to the same data bits. In addition, we modify the noncoherent distance definition to be suitable for BDFA. Several new codes are searched by computers, which have better error performance than the original codes in [1]-[3].

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