

# Extended Orthogonal Block Coding with Code Selection for Four Transmit Antennas and One Receive Antenna

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**Abstract**—This paper investigates closed loop extended orthogonal block coding (EO-STBC) scheme with four transmit antennas and single receive antenna under quasi-static flat Rayleigh fading. Overall system architectures and the BER performance are compared among conventional EO-STBCs through simulations and numerical analysis in the sense of the average receive SNR. Further, We proposed a new EO-STBC adopting a code selection scheme on the basis of optimal phase rotated EO-STBC for obtaining additional receive SNR gain. Finally, we confirmed that the proposed system has better BER performance than other EO-STBCs without additional feedback bits.

## I. INTRODUCTION

Diversity techniques in multiple-input multiple-output(MIMO) systems are widely used to overcome fading phenomenon and provide the reliability of transmission without additional bandwidth in wireless environment. Diversity techniques are divided according to the location of antennas, transmit diversity and receive diversity. For the downlink, it is more practical to consider transmit diversity in order to reduce the complexity of mobile stations. The space-time coding is a method to provide transmit diversity. Alamouti in [1] developed space-time coding which has full diversity and full rate in two transmit antenna case and Tarokh in [2] expanded [1] into more than two transmit antennas, which called orthogonal space-time block coding (O-STBC). However, even if O-STBC provides full diversity, it can not give full rate, and then, Jafarkhani in [3] suggested the notion of quasi orthogonal space-time block coding (QO-STBC) with full rate instead of full diversity. By [3], rotated QO-STBC that provides both full rate and full diversity by optimal rotation of signal constellation was achieved in [4]. However, both [3] and [4] have high decoding complexity than O-STBC due to quasi orthogonal nature and expanded signal constellations.

In place of the open loop system mentioned above, closed loop system with channel state information(CSI) known to transmitter side is worth to consider to elevate the BER performance. While a fully known CSI in the transmitter side is a great benefit in system in various ways (antenna selection [5] and beamforming [7]), it is not realizable due to the infinite resolutions. Partial feedback information to save bandwidth and reduce complexity in the feedback channel is commonly implemented. A class

of partial feedback system, extended orthogonal space-time coding (EO-STBC) was suggested by Akhtar in [8] with just one bit feedback information. This scheme offers full rate and full diversity, with more, simple decoding. And also, Yu in [9] proposed another scheme with two bits feedback information to increase the BER performance by improving average receive SNR and showed that it can be applied to odd number of antennas. Further, Eltayeb in [10] proposed another modified EO-STBC with optimal phase rotation which can be quantized properly according to BER performance demands of systems.

In this paper, we propose a new EO-STBC scheme with selection of two codes and advantage of optimal phase rotation scheme from [10] in four transmit antennas and single receive antenna case in particular. The results are presented not only simulations of BER in accordance with the increase of SNR, but also average receive SNR analysis mathematically, to compare the proposed scheme with [8], [9] and [10] quantitatively. We confirmed that both simulation and analysis show enhancement of the BER performance of the proposed scheme in comparison with conventional schemes, despite with the use of same number of feedback bits.

This paper is organized as follows. Section II reviews conventional EO-STBCs. Section III covers the proposed EO-STBC and Section IV and V presents performance comparison of each EO-STBC by means of analysis and simulations. Finally, we conclude in Section VI.

## II. CONVENTIONAL EO-STBCS WITH FOUR TRANSMIT ANTENNAS AND SINGLE RECEIVE ANTENNA

### A. Channel Model

Let us consider four transmit antennas and one receive antenna. In this case, channel matrix  $\mathbf{H} = [h_1 \ h_2 \ h_3 \ h_4]^T$  be the  $4 \times 1$ . The channel is assumed to be quasi-static flat Rayleigh fading which maintains constant over a frame and it changes independently every frame. Each element  $h_i$  of matrix  $\mathbf{H}$  indicates zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with variance  $\frac{1}{2}$  per real dimension. We also assume that the channel model is uncorrelated. The properties of channel  $\mathbf{H}$  are represented mathematically as follows.

$$h_i = X_i + jY_i = R_i e^{j\theta_i} \quad (1)$$

$$E[h_i] = 0, E[h_i h_j^*] = \delta_{ij} \quad (2)$$

$$E[X_i] = E[Y_i] = 0, E[X_i^2] = E[Y_i^2] = \frac{1}{2} \quad (3)$$

$X_i$  and  $Y_i$  are Gaussian distribution and  $R_i$  is Rayleigh distribution and  $\theta_i$  is uniform distribution from 0 to  $2\pi$ . Lastly, we assume perfect channel estimation and no feedback error. This channel model is applied to both conventional EO-STBCs and the proposed system in all sections of this paper.

### B. System Model

In Fig. 1, we can compare conventional EO-STBCs and the proposed EO-STBC, specially in transmitter side. In this section, we reviews conventional EO-STBCs and the proposed EO-STBC will be introduced in the next section. Firstly, we define column vectors,  $\mathbf{S}_1$  and  $\mathbf{S}_2$  in order to build EO-STBC.

$$\mathbf{S}_1 = \begin{bmatrix} x_1 \\ -x_1^* \end{bmatrix}, \quad \mathbf{S}_2 = \begin{bmatrix} x_2 \\ x_1^* \end{bmatrix} \quad (4)$$

$x_1$  and  $x_2$  are modulated symbols and  $*$  denotes complex conjugation. According to Akhtar in [8], Yu in [9], and Eltayeb in [10],  $2 \times 4$  EO-STBC is represented by as follows.

$$\mathbf{E}_A = [\mathbf{S}_1 \quad \mathbf{S}_2 \quad \mathbf{S}_1 \quad \mathbf{S}_2] \quad (5)$$

$$\mathbf{E}_Y = \mathbf{E}_E = [\mathbf{S}_1 \quad \mathbf{S}_1 \quad \mathbf{S}_2 \quad \mathbf{S}_2] \quad (6)$$

$\mathbf{E}_A$ ,  $\mathbf{E}_Y$ ,  $\mathbf{E}_E$  indicates encoder matrix of EO-STBC by Akhtar, Yu, and Eltayeb respectively. Each column corresponds to the symbols transmitted from the each antenna and each row represents each transmission period. This notation of code matrix is commonly used in this paper.

Let us consider open loop EO-STBC system. We can obtain received signal,  $\mathbf{Y} = \frac{1}{2}\mathbf{E}\mathbf{H} + \mathbf{N}$ , where  $\mathbf{E}$  is  $2 \times 4$  general EO-STBC encoder matrix,  $\mathbf{H}$  is  $4 \times 1$  channel matrix,  $\mathbf{N}$  is  $2 \times 1$  complex additive white Gaussian noise (AWGN) vector and  $\frac{1}{2}$  that is determined by  $\frac{1}{\sqrt{M_T}}$ ,  $M_T$  is the number of transmit antennas, maintain constant transmit power even if the number of antennas is changed. It is also denoted by  $\mathbf{Y}' = \frac{1}{2}\tilde{\mathbf{H}}\mathbf{X} + \mathbf{N}'$ , where  $\tilde{\mathbf{H}}$  is equivalent channel matrix including the orthogonal nature of code matrix and  $\mathbf{X}$  is uncoded transmit signal. After obtaining  $\tilde{\mathbf{H}}$  and multiplying it by its hermitian that means decoding matrix, we can obtain the matrix that is called Grammian.

$$\mathbf{G} = \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} = \begin{bmatrix} \alpha + \beta & 0 \\ 0 & \alpha + \beta \end{bmatrix} \quad (7)$$

Grammian matrix with only diagonal elements indicates that EO-STBCs has simple decoder contrary to [3] and [4]. The elements of  $\mathbf{G}$  can also be utilized to represent the receive SNR as follows.

$$\gamma = \frac{\alpha + \beta}{4} \gamma_0 \quad (8)$$

where  $\gamma_0 = \frac{E_s}{N_0}$  is the receive SNR without diversity gain,  $\alpha$  is conventional channel gain in all STBCs and  $\beta$  is additional receive SNR gain only in EO-STBCs. Here, we point out that  $\beta$  is considered as controlled interference. In open loop EO-STBC, the average of  $\beta$  is 0. It means that  $\beta$  can not be beneficial SNR gain. However,

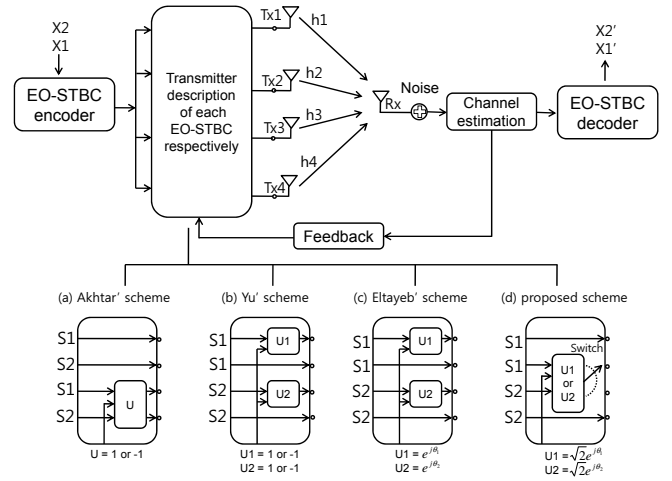


Fig. 1. EO-STBC schemes with four transmit antennas and single receive antenna based on which we can compare four schemes of (a), (b), (c) and (d)

the employment of appropriate feedback information for obtaining positive  $\beta$  at all times can be good strategy to elevate BER performance in EO-STBC systems.

In TABLE. I, we can compare receive SNR representation of each EO-STBCs with closed loop. EO-STBC in [8] utilize proper one bit feedback information using  $U$  and the idea in [9] is the modification of [8] for increasing receive SNR using  $U_1$  and  $U_2$  with 2-bits feedback. [10] propose generalization of feedback information, which is denoted as phase  $\theta$ .

### III. THE PROPOSED EO-STBC WITH FOUR TRANSMIT ANTENNAS AND SINGLE RECEIVE ANTENNA

We already defined the channel model of conventional schemes and this model is also utilized in the proposed scheme. The block diagram of the proposed scheme is depicted in Fig. 1. (d).  $U_1$  and  $U_2$  are newly defined as

$$U_m = a_m e^{j\theta_m}, \quad m = 1, 2 \quad (9)$$

$a_m$  is power allocation factor and  $\theta_m$  is phase control factor for optimal rotation. Since we already assumed perfect channel estimation and no feedback error, optimized power allocation is to assign whole power toward one of two antennas [11], which stands for the concept of antenna selection. In other words, this antenna selection makes the proposed scheme select between the two codes,  $\mathbf{E}_{P_1}, \mathbf{E}_{P_2}$

$$\begin{aligned} \mathbf{E}_P &= [\mathbf{S}_1 \quad U_1 \mathbf{S}_1 \quad U_2 \mathbf{S}_2 \quad \mathbf{S}_2] \\ \mathbf{E}_{P_1} &= [\mathbf{S}_1 \quad \sqrt{2}e^{j\theta_1} \mathbf{S}_1 \quad 0 \quad \mathbf{S}_2], \text{ if } |h_2|^2 \geq |h_3|^2 \\ \mathbf{E}_{P_2} &= [\mathbf{S}_1 \quad 0 \quad \sqrt{2}e^{j\theta_2} \mathbf{S}_2 \quad \mathbf{S}_2], \text{ if } |h_2|^2 < |h_3|^2 \end{aligned} \quad (10)$$

If  $|h_2|^2 > |h_3|^2$ , the system select antenna 2, otherwise it selects antenna 3, therefore, one feedback bit is enough to select transmit antennas. In addition, total transmit power of the proposed system should be equal to the power of  $2 \times 4$  EO-STBC systems. Accordingly, the power control factor,  $a_m$  is  $\sqrt{2}$ .

Now, let us seek the receive SNR gain term of the proposed scheme,  $\alpha_P$  and  $\beta_P$ . First, the code generation

TABLE I  
 THE COMPARISON OF RECEIVE SNR REPRESENTATION EACH EO-STBC

Scheme	Receive SNR gain representation	Feedback control factor
Akhtar [8]	$\alpha_A = \sum_{i=1}^4  h_i ^2, \quad \beta_A = 2U\text{Re}(h_1h_3^* + h_2h_4^*)$	$U = (-1)^k, \quad k = \begin{cases} 0, & \text{Re}(h_1h_3^* + h_2h_4^*) \geq 0 \\ 1, & \text{Re}(h_1h_3^* + h_2h_4^*) < 0 \end{cases}$
Yu [9]	$\alpha_Y = \sum_{i=1}^4  h_i ^2, \quad \beta_Y = 2U_1\text{Re}(h_1h_2^*) + 2U_2\text{Re}(h_3h_4^*)$	$U_1 = (-1)^l, U_2 = (-1)^m$ $(l, m) = \begin{cases} (0, 0), & \text{if } \text{Re}(h_1h_2^*) \geq 0 \text{ \& if } \text{Re}(h_3h_4^*) \geq 0 \\ (0, 1), & \text{if } \text{Re}(h_1h_2^*) \geq 0 \text{ \& if } \text{Re}(h_3h_4^*) < 0 \\ (1, 0), & \text{if } \text{Re}(h_1h_2^*) < 0 \text{ \& if } \text{Re}(h_3h_4^*) \geq 0 \\ (1, 1), & \text{if } \text{Re}(h_1h_2^*) < 0 \text{ \& if } \text{Re}(h_3h_4^*) < 0 \end{cases}$
Eltayeb [10]	$\alpha_E = \sum_{i=1}^4  h_i ^2, \quad \beta_E = 2U_1\text{Re}(h_1h_2^*) + 2U_2\text{Re}(h_3h_4^*)$	$U_1 = e^{j\theta_1}, U_2 = e^{j\theta_2}$ $\theta_1 = -\text{angle}(h_1h_2^*), \theta_2 = -\text{angle}(h_3h_4^*)$
Proposed	$\begin{pmatrix} \alpha_P \\ \beta_P \end{pmatrix} = \begin{pmatrix}  h_1 ^2 + 2 h_2 ^2 +  h_4 ^2 \\ 2\text{Re}(U_1h_2h_1^*) \end{pmatrix}, \text{ if }  h_2 ^2 \geq  h_3 ^2$ $\begin{pmatrix} \alpha_P \\ \beta_P \end{pmatrix} = \begin{pmatrix}  h_1 ^2 + 2 h_3 ^2 +  h_4 ^2 \\ 2\text{Re}(U_2h_3h_4^*) \end{pmatrix}, \text{ if }  h_2 ^2 <  h_3 ^2$	$U_1 = \sqrt{2}e^{j\theta_1}, U_2 = \sqrt{2}e^{j\theta_2}$ $\theta_1 = -\text{angle}(h_2h_1^*), \theta_2 = -\text{angle}(h_3h_4^*)$

is the same to (6). Multiplying  $U_1$  and  $U_2$  at column 2 and column 3

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \mathbf{S}_1 & U_1\mathbf{S}_1 & U_2\mathbf{S}_2 & \mathbf{S}_2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (11)$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\mathbf{Y}'} = \frac{1}{2} \underbrace{\begin{bmatrix} h_1 + U_1h_2 & U_2h_3 + h_4 \\ U_2^*h_3^* + h_4^* & -h_1^* - U_1^*h_2^* \end{bmatrix}}_{\tilde{\mathbf{H}}_P} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \end{bmatrix}}_{\mathbf{N}'} \quad (12)$$

$\tilde{\mathbf{H}}_P$  is equivalent channel matrix. Multiplying  $\tilde{\mathbf{H}}_P$  by  $\tilde{\mathbf{H}}_P^H$ , we obtain the receive SNR representation of the proposed EO-STBC depicted in TABLE. I

#### IV. THE AVERAGE SNR ANALYSIS OF EO-STBCS

In this section, we derive average receive SNR as a measure of indirect performance evaluation for each EO-STBC on the basis of the channel model in the previous section, and then, we compare the results of analysis and prove that the proposed scheme has better performance than other EO-STBCs. These derivations are our important contribution in this paper. We further investigate the quantization effect according to the quantization level.

##### A. Analysis of Conventional EO-STBCs

1) *Analysis of Akhtar's scheme:* From (2) and TABLE I,

$$E[\alpha_A] = \sum_{i=0}^4 E[|h_i|^2] = 4 \quad (13)$$

Before calculating the expectation of  $\beta_A$ , substituting  $U$ ,  $\beta_A$  can be represented simply as follows.

$$\beta_A = 2|\text{Re}(h_1h_3^* + h_2h_4^*)| \quad (14)$$

First, we should derive the pdf of random variable  $\beta_A$  to achieve expectation of  $\beta_A$ . By (1), (14) can be represented by only real terms.

$$\beta_A = 2|X_1X_3 + X_2X_4 - Y_1Y_3 - Y_2Y_4| \quad (15)$$

Let  $W_1 = X_1X_3 + X_2X_4$  and  $W_2 = Y_1Y_3 + Y_2Y_4$ . The pdf of  $W_1$  and  $W_2$  are obtained by using the characteristic function of products of Gaussian random variables

depicted in equation 6.5 in [12].

$$f_{W_i}(w_i) = e^{-2|w_i|} \quad -\infty < w_i < \infty, \quad i = 1, 2 \quad (16)$$

Through the function of random variables, the pdf of  $\beta_A$  is obtained as follows.

$$f_{\beta_A}(\beta_A) = 2(1 + 2\beta_A)e^{-2\beta_A}, \quad \beta_A > 0 \quad (17)$$

The expected value of  $\beta_A$  is  $E[\beta_A] = 1.5$ .

2) *Analysis of Yu's scheme:* From TABLE I,  $E[\alpha_Y] = 4$  and  $\beta_Y$  is more simply represented for analysis, substituting  $U_1$  and  $U_2$ .

$$\beta_Y = 2|\text{Re}(h_1h_2^*)| + 2|\text{Re}(h_3h_4^*)| \quad (18)$$

Using polar form representation in (1),  $\beta_Y$  is depicted as follows.

$$\beta_Y = 2R_1R_2|\cos(\theta_1 - \theta_2)| + 2R_3R_4|\cos(\theta_3 - \theta_4)| \quad (19)$$

We should derive the pdf of  $|\cos(\theta_1 - \theta_2)|$  and  $|\cos(\theta_3 - \theta_4)|$  respectively to obtain the expectation of  $\beta_Y$ . Referring to each term as  $Z_i$ ,  $i = 1, 2$  and using  $\theta_i$  that is uniform distribution from 0 to  $2\pi$ . And then, the pdf of  $Z_i$  is below.

$$f_{Z_i}(z_i) = \frac{2}{\pi\sqrt{1-z_i^2}}, \quad 0 < z_i < 1 \quad (20)$$

Hence,  $E[Z_i] = \frac{2}{\pi}$ . The expectation of Rayleigh distribution,  $R_i$  is known to  $\frac{\sqrt{\pi}}{2}$ , therefore,

$$E[\beta_Y] = 2E[R_1]E[R_2]E[Z_1] + 2E[R_3]E[R_4]E[Z_2] = 2 \quad (21)$$

3) *Analysis of Eltayeb's scheme:* From TABLE I,  $E[\alpha_E]$  is also 4, and substituting  $U_1$  and  $U_2$  in  $\beta_E$ .

$$\beta_E = 2\text{Re}(e^{j\theta_1}h_1h_2^*) + 2\text{Re}(e^{j\theta_2}h_3h_4^*) \quad (22)$$

Using the relation of (1) and the definition of  $\theta$ , the pdf of  $\beta_E$  is more easily represented as

$$\beta_E = 2R_1R_2 + 2R_3R_4 \quad (23)$$

And then,  $E[\beta_E] = \pi$

TABLE II

THE COMPARISON OF AVERAGE SNR GAIN OF EACH EO-STBC

Scheme	Bit	$E[\alpha]$	$E[\beta]$	$E[\alpha + \beta]$	$10\log E[\alpha + \beta]$
Akhtar [8]	1	4	1.5	5.5	7.40
Yu [9]	2	4	2	6	7.78
Eltayeb [10]	$\infty$	4	3.14	7.14	8.54
Proposed	$\infty$	5	2.87	7.87	8.96

4) *The proposed EO-STBC*: Referring to TABLE I, using order statistics in [6] and let  $N = |h_{\max}|^2$ ,  $h_{\max}$  is defined by selection between  $h_2$  and  $h_3$  based on their square magnitude.

$$f_N(n) = 2e^{-n}(1 - e^{-n}), \quad n > 0 \quad (24)$$

The expectation of this pdf is  $E[N] = 1.5$ , accordingly,  $E[\alpha_P] = 5$  from the receive SNR representation of  $\alpha_P$  in TABLE I.

Now let's calculate  $\beta_P$ . The receive SNR gain representation of  $\beta_P$  in TABLE I is modified by (1) and the definition of phase feedback  $\theta$ ,

$$\beta_P = 2\sqrt{2}\text{Re}(e^{j(\theta_{\max}-\theta_i)}R_{\max}R_i e^{j(\theta_i-\theta_{\max})}), \quad i = 1, 4 \quad (25)$$

$\theta_{\max}$  is the phase of  $h_{\max}$  and  $R_i$  is Rayleigh distribution. Then, we should know the pdf of  $R_{\max}$  denoted the maximum value between  $R_2$  and  $R_3$ . Using the order statistics, we can get the pdf.

$$f_{R_{\max}}(r_{\max}) = 4r_{\max}e^{-r_{\max}^2}(1 - e^{-r_{\max}^2}), \quad r_{\max} > 0 \quad (26)$$

We can easily find the expectation of  $R_{\max}$ .

$$E[R_{\max}] = \sqrt{\pi} - \frac{\sqrt{\pi}}{2\sqrt{2}} \quad (27)$$

$$E[\beta_P] = 2\sqrt{2}E[R_{\max}]E[R_i] = 2.87 \quad (28)$$

#### B. The effect of quantization of the proposed system

In this part, mathematical models are given in two different cases.  $\alpha_{P2}$  and  $\beta_{P2}$  are represented in 2 level quantization case,  $\alpha_{P4}$  and  $\beta_{P4}$  are denoted in four level quantization.

1) *2 level quantization*: In this case, one bit of feedback is assigned as antenna selection and another bit is assigned to represent 2 level of phase.  $E[\alpha_{P2}]$  is the same to no quantization case, and  $\beta_{P2}$  is represented as

$$\beta_{P2} = 2\sqrt{2}R_iR_{\max}|\cos(\theta_i - \theta_{\max})| \quad (29)$$

We already found the pdf of  $|\cos(\theta_i - \theta_{\max})|$  defined random variable  $Z_i$ .

$$E[\beta_{P2}] = 2\sqrt{2}E[R_i]E[R_{\max}]E[Z_i] = 1.83 \quad (30)$$

2) *4 level quantization*: In 3-bits feedback, two bits are utilized for providing 4 level quantization of phase.  $E[\alpha_{P4}]$  is also equivalent of other proposed case. To obtain  $E[\beta_{P4}]$ ,  $\beta_{P4}$  is simply represented for analysis.

$$\beta_{P4} = 2\sqrt{2}R_iR_{\max}\text{Re}(e^{j\theta}e^{(\theta_i-\theta_{\max})}) \quad (31)$$

TABLE III

THE EFFECT OF QUANTIZATION OF THE PROPOSED SCHEME

Scheme	Bit	$E[\alpha]$	$E[\beta]$	$E[\alpha + \beta]$	$10\log E[\alpha + \beta]$
2 level	2	5	1.83	6.83	8.34
4 level	3	5	2.59	7.59	8.80
no quantization	$\infty$	5	2.87	7.87	8.96

$$\theta = \begin{cases} 0, & \frac{\pi}{4} < \theta_i - \theta_{\max} \leq \frac{3\pi}{4} \\ \frac{3\pi}{2}, & \frac{3\pi}{4} < \theta_i - \theta_{\max} \leq \frac{5\pi}{4} \\ \pi, & \frac{5\pi}{4} < \theta_i - \theta_{\max} \leq \frac{7\pi}{4} \\ \frac{\pi}{2}, & \frac{7\pi}{4} < \theta_i - \theta_{\max} \leq \frac{9\pi}{4} \end{cases} \quad (32)$$

We should derive the pdf of  $\text{Re}(e^{j\theta}e^{(\theta_i-\theta_{\max})})$  and let A. the pdfs of each range in (32) is same as

$$f_A(a) = \frac{1}{\pi\sqrt{1-a^2}}, \quad \frac{1}{\sqrt{2}} \leq a < 1. \quad (33)$$

Therefore,  $E[A] = \frac{1}{\sqrt{2}\pi}$  and

$$E[\beta_{P4}] = 2\sqrt{2}E[R_i]E[R_{\max}](4E[A]) = 2.59 \quad (34)$$

The results of analysis are summarized in TABLE II and TABLE III.

#### V. SIMULATIONS

In this section, we investigate various STBCs and the proposed scheme by Monte Carlo simulation. As introduced before, the simulations are conducted in quasi-static flat Rayleigh fading channel and we assume perfect channel estimation and no feedback error. The simulations firstly focus on comparison between conventional open loop STBCs and closed loop EO-STBCs. Secondly, parallel shifts of BER curves due to the increase of average receive SNR in each EO-STBC are inspected in the simulations. We fix that the information rate of all schemes with four transmit antennas and one receive antenna is 2 bits/s/Hz in order to have fair comparison.

In Fig.2, we show QO-STBC in [3] and rotated QO-STBC in [4] with open loop. We also show EO-STBC in [8], EO-STBC in [9], EO-STBC in [10] and the proposed EO-STBC with closed loop. Especially, EO-STBC in [10] and the proposed EO-STBC has infinite resolution of feedback phase information. We can confirm that the proposed scheme is far better than open loop case. Further, the proposed EO-STBC show the improvement of BER curve in contrast with other EO-STBCs. At a bit error probability of  $10^{-3}$ , the proposed EO-STBC has the improvement of 1dB as compared with EO-STBC in [9] and the enhancement of 0.5dB against EO-STBC in [10]. These BER performance differences in this figure are caused by the average receive SNR gain and the differences are equal to the results in TABLE I.

In Fig. 3, the focus is on the BER performance according to quantization level in the proposed EO-STBC. The proposed EO-STBC with two level quantization has the improvement of 0.6dB at the point of  $10^{-3}$  in contrast with EO-STBC in [9] without additional feedback bits. In addition, the proposed EO-STBC with four level quantization of feedback phase show the BER performance adjacent to upper bound which means no quantization case. In other words, the amount of performance improvement between

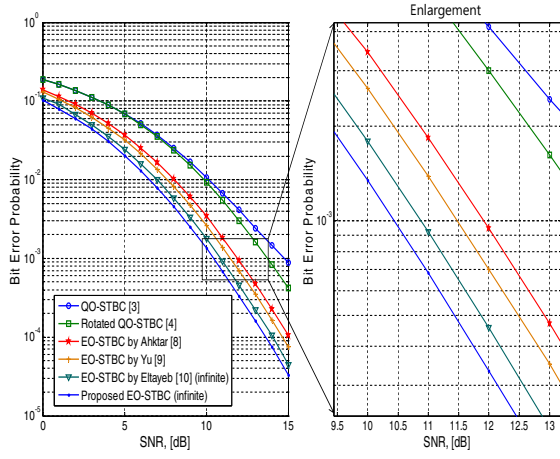


Fig. 2. The BER performance comparison among open loop STBC, conventional EO-STBC and the proposed EO-STBC with ideal CSI.

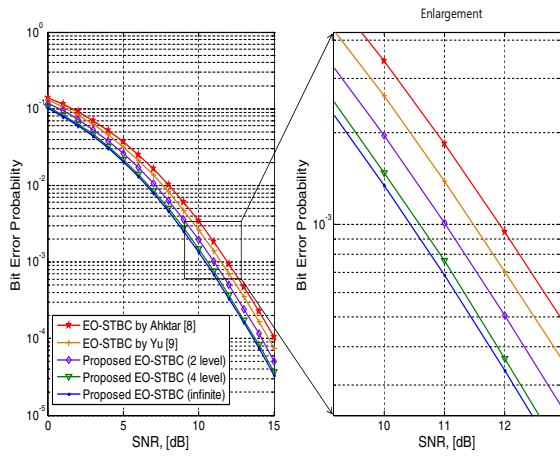


Fig. 3. The BER performance comparison of the proposed EO-STBCs according to quantization levels

the proposed EO-STBC with four level quantization and EO-STBC in [9] using additional one bit is far better than the amount of performance improvement between EO-STBC in [9] and EO-STBC in [8] using additional one bit.

These simulation results correspond to the analysis in the previous section shown in TABLE II and TABLE III. First, The relation among average BER  $\bar{P}_e$ ,  $E[\alpha + \beta]$  and diversity order  $M$  is explained as follows [13].

$$\bar{P}_e \approx c \cdot \left[ \frac{E[\alpha + \beta]}{4} \gamma_0 \right]^{-M} \quad (35)$$

where  $c$  is a constant. Since all of EO-STBC schemes in this paper have same diversity order, BER curves are determined by  $E[\alpha + \beta]$  which was already obtained by analysis. Further,  $\gamma_0$  represents the value of x axis in BER curve, hence,  $\frac{E[\alpha + \beta]}{4}$  term means parallel shift due to its log scale. In short, the results of analysis can be compared with the amount of parallel shift of BER curve as a dB scale.

## VI. CONCLUSION

This paper has shown new closed loop EO-STBC with four transmit antennas and one receive antenna, which

obtains more additional receive SNR gain than conventional EO-STBC schemes without the increase of complexity in the feedback channel. We also proved the BER performance of proposed scheme by means of simulations and numerical analysis of the average receive SNR. The proposed EO-STBC with desirable capability will be expected to be utilized in advanced wireless communication systems.

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