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Path-Averaged Peak Power Considering Dispersion Effect

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***Abstract*- We derive the path-averaged peak power considering dispersion effect of optical fiber for the first time in our knowledge, in order to attain the results of the theoretical analysis of optical fiber transmission more accurately than the conventional path-averaged power which considers only the fiber attenuation. The ultimate accuracy of the newly developed path-averaged peak power is verified by computer simulations based on the transmission of differential phase-shift keying signal with data rates of 5 Gbps and 40 Gbps, in both transmission systems using the dispersion management and the optical phase conjugation. The numerical simulation results are in better agreement with the frequency response analysis of the phase error using the path-averaged peak power considering dispersion effect comparing to the phase error analysis obtained by using the conventional path-averaged peak power that considers only the fiber attenuation.**

Keywords: Optical fiber communication, optical fiber amplifiers, optical fiber dispersion, optical Kerr effect, optical phase conjugation

I. INTRODUCTION

Several recent high-speed long-haul optical differential phase shift-keying (DPSK) transmission experiments [1]-[3] have demonstrated that the optical phase conjugation (OPC) shows greater performance than the periodical dispersion management (DM) in reducing the nonlinear phase noise induced by the Kerr effect [4]. The numerical simulation results, shown in [5], also agree well with the experiments [4], however, with the ambiguity of frequency response analysis results [5], which cannot mention clearly that the performance of OPC system is greater than that of DM system.

So far, the frequency response analysis [5]-[7] of optical signal transmission in an optical fiber has been applying the approximation of signal power value, which is averaged over a path considering only the fiber attenuation (path-averaged peak power) [8]. Nevertheless, especially for OPC systems and DM systems using relatively long DM period, the dispersion will accumulate over the relatively long fiber path, which may be several hundred kilometers to several thousand kilometers, before being compensated. Such large accumulated dispersion causes the signal peak power to reduce more rapidly than the effect of fiber attenuation. Thus, for obtaining more accurate result of the frequency response analysis, the path-averaged peak power approximation must also take into account the fiber dispersion.

In this paper, the path-averaged peak power including the fiber dispersion is presented. We demonstrate that, by using our path-averaged peak power including fiber dispersion, the frequency response analysis results for the DPSK transmission in OPC and DM systems can clear the ambiguity of the results in [5], and are in better agreement with the results of simulations than using the conventional path-averaged peak power considering only the fiber attenuation.

II. PATH-AVERAGED PEAK POWER CONSIDERING DISPERSION EFFECT FOR SINGLE-BIT TRANSMISSION

Due to fiber loss and periodical amplification, signal power varies along transmission length. For obtaining the frequency response of a system, the power variation has been so far averaged over the amplification span l_a to be

$$\overline{P}_L = P_0\left[\frac{1-\exp(-\alpha l_a)}{\alpha l_a}\right], \qquad (1)$$

where P_0 is the input signal power and α is the fiber loss coefficient.

In fact, the signal peak power is also reduced by the fiber dispersion. Assuming that the signal exhibits the Gaussian shape, the exact envelope of a single Gaussian pulse $A(z,T)$, which evolves along the distance z under the effects of both fiber loss and dispersion becomes [8]

$$A(z,T) = \sqrt{P_0}\exp(-\alpha z/2)\cdot \frac{T_0}{\sqrt{T_0^2 - j\beta_2 z}}\exp\left(-\frac{T^2}{2\left(T_0^2 - j\beta_2 z\right)}\right), \qquad (2)$$

where β_2 denotes the group-velocity dispersion (GVD) and T_0 denotes the pulse width. The power of the single pulse at distance z ($P(z)$) can be obtain as

$$P(z) = P_0\exp(-\alpha z)\left(T_0^2\Big/\sqrt{\left(T_0^4+\beta_2^2 z^2\right)}\right). \qquad (3)$$

Then, we can calculate the average value ($\overline{P}_{LD}$) of $P(z)$ by using

$$\overline{P}_{LD} = \left(\sum_{n=1}^{N}\int_0^{l_a} P_0\exp(-\alpha z)\left(T_0^2\Big/\sqrt{\left(T_0^4+\beta_2^2 z^2\right)}\right)^n dz\right)\Big/\left(N\cdot L_a\right)$$

, (4) where N is the number of amplification span in DM period before being compensate, or the number of amplification span before OPC. Since a numerical integration method is necessary to obtain $\overline{P}_{LD}$, we use Legendre-Gauss Quadrature [9] with 24th order, that can give sufficient accuracy of the solution of the integration.

According to the 24th order Legendre-Gauss Quadrature algorithm, the integral of $\int_0^{l_a} P(z)\,dz$ can be obtained as

$$\int_0^{l_a} P(z)\,dz \approx \frac{l_a}{2}\sum_{i=1}^{n} w_i P\left(\frac{l_a}{2}x_i + \frac{l_a}{2}\right) \quad , \qquad (5)$$

where

$$w_i = \frac{2\left(1-x_i^2\right)}{(n+1)^2\left[P_{n+1}(x_i)\right]^2}, \qquad (6)$$

and x_i is obtained by solving $P_n(x_i)=0$ when

$$P_n(x_i) = \frac{1}{2^n n!}\frac{d^n}{dx_i^n}\left[\left(x_i^2-1\right)^n\right]. \qquad (7)$$

III. Path-Averaged Peak Power Considering Dispersion Effect for Multiple-Bit Transmission

Next, since the signal is usually a continuous bit stream, $\overline{P}_{LD}$ from section II, must be further modified to account for multiple-bit transmission. Without fiber nonlinearity, the peak power of a single pulse evolution in a fiber follows $P(z)$ derived above. For DPSK signal consisted of multiple bits, the fiber dispersion causes the power between two adjacent bits both to enhance and to cancel each other according to their phase relation. Therefore, if the signal is a pseudo-random sequence, the average peak power value considering multiple bits ($\overline{P}_{LD,n-bit}$) should be smaller than $\overline{P}_{LD}$.

Since it is quite difficult to exactly calculate $\overline{P}_{LD,n-bit}$, we perform a simulation of signal transmission in a 50-km fiber with increasing the number of bit from 1 to 1024, and neglecting the fiber nonlinearity. Then, we estimate the ratio between the average value of the peak power of multiple-bit signal ($\overline{P}_{n-bit}(l)$) and the peak power of 1-bit signal ($P(l)$), at the output of fiber. In the simulation, the standard single-mode fiber (SMF) is used for signal transmission with α = 0.2 dB/km, β_2 = -20.78 ps^2/km (at signal wavelength of 1,550 nm). For P_0 = 3 mW, the results in terms of output signal peak power and the ratio $\overline{P}_{n-bit}(l)/P(l)$, both as a function of number of bit are shown in Fig. 1 as an example.

According to Fig.1 for the number of bit larger than 8, $\overline{P}_{n-bit}(l)$ starts to converge to a value of 0.87 mW, resulting in the convergence of the ratio $\overline{P}_{n-bit}(l)/P(l)$ to a value of 0.57. Using this ratio value, $\overline{P}_{LD,n-bit}$ can be obtained as $\overline{P}_{LD,n-bit} \approx \left(\overline{P}_{n-bit}(l)/P(l)\right)\overline{P}_{LD} \approx 0.57\overline{P}_{LD}$. Noted that, with the assumption that the signal power evolution in a fiber obeys $P(z)$, the ratio $\overline{P}_{n-bit}(l)/P(l)$ is constant independently on the fiber length used for simulation.

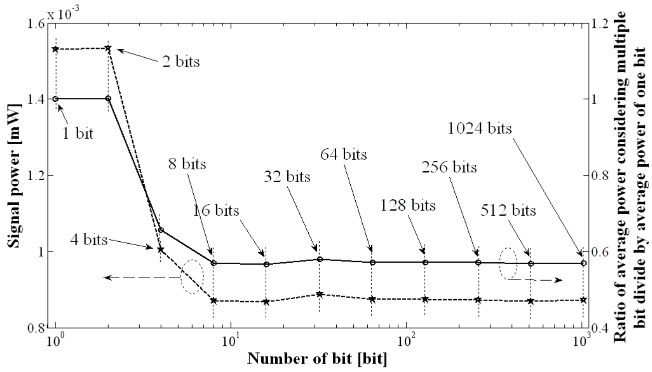

Figure 1. Output signal peak power from 50-km SMF, and the ratio $\overline{P}_{n-bit}(l)/P(l)$, both as a function of number of bit, when P_0 = 3mW.

IV. Frequency Responses and Computer Simulations

In long-haul DPSK transmission, the nonlinear phase error induced from Kerr effect is an important limitation. Using both $\overline{P}_L$ and $\overline{P}_{LD,n-bit}$, we calculate the frequency response of the nonlinear phase error in both OPC and DM systems by modulating the optical carrier in both inphase component with small signal a_m and quadrature-phase component with small signal b_m [5], [6]. The phase shift $\Delta\phi_m$, or phase error, according the small signal modulation is obtained as

$$\Delta\phi_{\mathrm{m}} = \tan^{-1}\left(\frac{b_m}{\sqrt{\overline{P}}+a_m}\right), \qquad (8)$$

where $\overline{P}$ is the average power of the carrier before being modulated. At given distance, $\Delta\phi_m$ which will be enhanced by the Kerr effect and fiber dispersion can be estimated by substituting a_m and b_m into the modified nonlinear Schrodinger equation [8]. Assuming the optical lump amplifiers are placed periodically at an interval of l_a, for the DM and OPC transmission, the covariance matrix, as a function of modulated frequency, of optical amplifier noise $B_{DM}(\omega_m)$ and $B_{OPC}(\omega_m)$ at receiver can be calculated by using Eq. (9) and Eq. (10), respectively,

$$B_{DM}(\omega_m) = \frac{S_0}{2}\sum_{k=1}^{N}\left[M(\omega_m,l_a)\right]^{N-k}\left[M^T(\omega_m,l_a)\right]^{N-k}, \qquad (9)$$

$$B_{OPC}(\omega_m) = \left(S_0\sum_{k=1}^{N/2}\left[M(\omega_m,l_a)\right]^{\frac{N}{2}-k}\left[M^T(\omega_m,l_a)\right]^{\frac{N}{2}-k}\right) + \left(\frac{S_0}{2}\left[M(\omega_m,l_a)\right]^{\frac{N}{2}}\left[M^T(\omega_m,l_a)\right]^{\frac{N}{2}}\right) - \frac{S_0}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \qquad (10)$$

where

$$M(\omega_m, z) = \begin{bmatrix} \cos(\kappa z) & -\Gamma\sin(\kappa z) \\ \Gamma^{-1}\sin(\kappa z) & \cos(\kappa z) \end{bmatrix}. \quad (11)$$

In Eq. (11),

$$\Gamma = \sqrt{\frac{\beta_2\omega_m^2}{(\beta_2\omega_m^2 + 4\gamma\bar{P})}}, \quad (12)$$

and

$$\kappa = \frac{\sqrt{\beta_2\omega_m^2(\beta_2\omega_m^2 + 4\gamma\bar{P})}}{2}, \quad (13)$$

where S_0, N ,and γ are the power spectral density of optical amplifier noise, the number of optical lump amplifier, and the nonlinear coefficient of optical fiber, respectively.

For demonstrating, we choose l_a = 50 km, each lump amplifier is assumed to have a noise figure of 5 dB, the SMF is used for signal transmission with α = 0.2 dB/km, β_2 = -20.78 ps^2/km (at signal wavelength of 1,550 nm), and γ = 1.06 W^{-1}km^{-1}. For the DM system, the DM period is 50 km, and we assume the complete compensation at the end of each DM period. The transmission distance is 5,000 km.

Fig. 2(a), (b), and (c) compare the calculated frequency response of the nonlinear phase error $\Delta\phi_m$, shown by frequency shift from carrier frequency, of the OPC system and the DM system, obtained by both $\bar{P}_L$ and $\bar{P}_{LD,n-bit}$, when P_0 are 1 mW, 3 mW, and 5 mW, respectively.

The frequency responses shown in Fig. 2 all appear in similar characteristic. That is, the phase error becomes relatively high, and nearly constant, for a bandwidth around the carrier frequency, and finally converges to a minimum value when frequency shift from the carrier increases. We can examine the performance of DPSK transmission from the frequency response as follows. In case that the phase error bandwidth is wider than the signal bandwidth, and the phase error is constant over the entire bandwidth of signal (as an example, a signal bandwidth smaller than 5 GHz in DM system in the case of Fig. 2(a)), although the phase error is relative high, the performance of the transmission will not degrade because all signal components experience almost equivalent amount of phase shift. However, if the phase error bandwidth is narrower than the signal bandwidth, low difference between the maximum and minimum values of the phase error will give good transmission result.

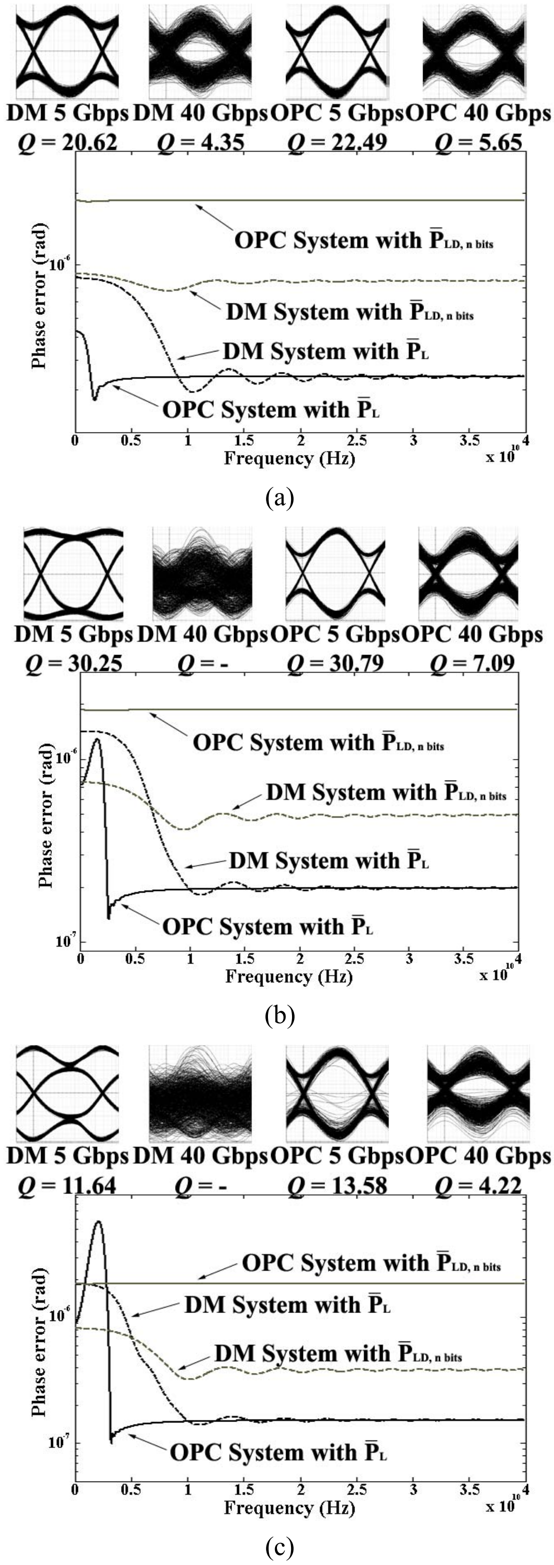

Figure 2. Frequency response of nonlinear phase error, shown by frequency shift from carrier frequency when (a) P_0 = 1 mW (b) P_0 = 3 mW and (c) P_0 = 5 mW. The frequency response is calculated by both $\bar{P}_L$ and $\bar{P}_{LD,n-bit}$. The insets show the simulated eye patterns of both OPC and DM systems at data rates of 5 Gbps and 40 Gbps.

When $\bar{P}_L$ is used for calculating the frequency response, at P_0 = 1 mW (Fig. 2(a)) and 3 mW (Fig. 2(b)), the OPC appears to give better transmission performance than the DM for relatively high data rate (> 10 Gbps).

However, for such data rates lower than 5 Gbps, there is an ambiguity to determine whether the OPC or the DM is more suitable. Moreover, this ambiguity can be seen more obviously at P_0 = 5 mW (Fig. 2(c)) for all data rates even higher than 10 Gbps. On the other hand, when we use $\overline{P}_{LD,n-bit}$ to calculate the frequency response instead of $\overline{P}_L$, all frequency responses in Fig. 2(a)-(c) indicate clearly that the OPC gives greater performance than the DM.

To examine whether the conventional $\overline{P}_L$ or our new $\overline{P}_{LD,n-bit}$ can accurately analyze the nonlinear optical signal transmission, we perform some numerical simulations using a pseudo-random 1024-bit 66%-RZ-DPSK signal whose data rates are 5 Gbps and 40 Gbps. The OPC process is assumed to be ideal. The quality of detected signal is evaluated in term of *Q*-factor. The SMF and other system parameters are the same as used for obtaining the frequency responses in Fig. 2. The simulated results in terms of detected eye patterns and the corresponding *Q*-factor values are shown in the insets in Fig. 2. For the data rate of 5 Gbps, the simulated *Q*-factor obtained from OPC system using P_0 = 1 mW, 3 mW, and 5 mW are 22.49, 30.79, and 13.58, respectively, while those obtained from DM system are 20.62, 30.25, and 11.64, respectively. On the other hand, for the data rate of 40 Gbps, the *Q*-factor of DPSK signal transmitted in OPC system with P_0 = 1 mW, 3 mW, and 5 mW are 5.65, 7.09, and 4.22, respectively, while the *Q*-factor resulted from DM system using P_0 = 1 mW is 4.35. For P_0 = 3 mW and 5 mW, the *Q*-factor cannot be extracted from the transmitted signal due to very severe signal distortion. Since all simulation results have clearly indicated that the OPC yields greater performance for DPSK signal transmission than the DM for both data rates of 5 Gbps and 40 Gbps, the results agree well with what has been predicted by the frequency response analysis using the path-averaged signal peak power considering the fiber dispersion effect.

V. Conclusion

The path-averaged peak power which takes into account the peak power reduction due to fiber dispersion effect was presented. We demonstrated by computer simulations that this new path-averaged peak power can give much accuracy than the path-averaged peak power considering only fiber loss in analyzing for such long-haul OPC transmission, as well as the DM transmission that uses a DM period that occupies multiple amplification spans.

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