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Performance Degradation of Spectral Amplitude Encoding Optical Code Division Multiple Access Systems due to Group Velocity Dispersion

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Abstract—**In this paper, a modified model of Gaussian pulse propagation in optical fiber is proposed to comprehensively analyze the performance degradation of spectral amplitude encoding (SAE) optical code division multiple access (OCDMA) systems due to group velocity dispersion (GVD). The analytical results show that the number of supportable users is decreased and the maximum transmission length (i.e. the length at which BER $\leq 10^{-9}$ can be maintained) is shortened under the impact of GVD. For example, a system with 31×1 Gbit/s users and wavelength interval of 1.0 nm has a maximum transmission length of 16 km when transmitted power per bit is 0 dBm. Although, the impact of GVD is reduced when wavelength interval is decreased to 0.4 nm, it is still relatively strong. A power penalty of up to 8 dB is seen in the SAE/OCDMA system with the transmission length of 33 km. In addition, we propose a method to combat the impact of GVD, which can increase the maximum transmission length of SAE/OCDMA systems.**

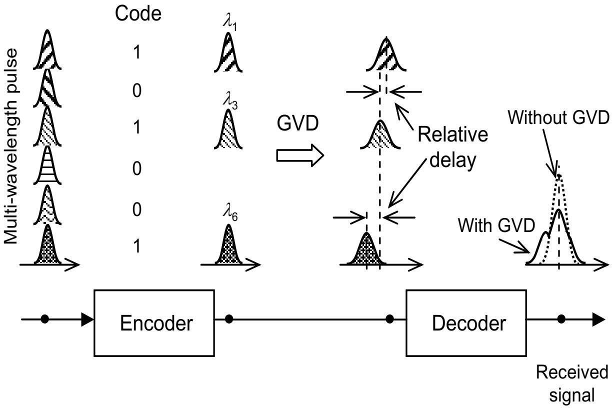

Fig. 1. Impact of GVD on the signal in the SAE/OCDMA systems.

I. INTRODUCTION

Over the past decade or so, there have been steady researches in optical code division multiple access (OCDMA) technique for fiber-optic communication, especially for local area and access networks. Several types of OCDMA systems have been proposed [1]. Based on coding schemes, OCDMA systems can be classified into time domain encoding, frequency domain encoding, and the two-dimensional (2-D) encoding, which is the combination of both [2].

Spectral amplitude encoding (SAE) OCDMA system [3], which is based on frequency domain encoding, is now received more interest because it has several advantages compared with OCDMA systems using time domain encoding. First, the electronic circuits in SAE/OCDMA system operate at the bit rate that is much lower than the chip rate therefore the expensive electronic components are not required. In addition, multiple access interference (MAI) can be canceled in SAE/OCDMA system by balanced detection receiver [3].

In SAE/OCDMA systems, data bits are encoded and transmitted by wavelength components. Under the impact of dispersion, optical pulses carried by wavelength components will be broadened and their peak powers will be reduced. Besides, relative delay between wavelength components, which is caused by the difference of their group velocities (group velocity dispersion (GVD) [4]), also affects the received signal. As shown in Fig. 1, when bit "1" is transmitted, received electric pulse is broadened, distorted and the received power per bit is reduced. As a result, bit error may happen, which degrades the performance of the system.

In order to analyze the impact of relative delay (or time skewing) caused by GVD, one can use coding theory [5] or system theory [6]. These methods, however, cannot analyze the impact of pulse broadening and peak power reduction of optical pulse. In this paper, for the comprehensive analysis of the impact of dispersion, the pulse propagation theory [4] will be used. Nevertheless, the current pulse propagation model is for investigating single-wavelength or wavelength independent system, in which time skewing effect is not included. In our work, we will thus propose a modified version of this model so that all effects of GVD including pulse broadening, peak power reduction, and relative delay can be comprehensively analyzed.

We will also include major noise and interference in the analysis, including MAI and receiver's noise. Various parameters are investigated including maximum transmission length, the number of supportable users, and required optical power. Finally, we propose a method to reduce the impact of GVD hence increase the maximum transmission length of SAE/OCDMA systems.

The rest of the paper is organized as follows. Section II presents the descriptions of the SAE/OCDMA system. The performance analysis and numerical results are presented in Section III and IV, respectively. Section V presents

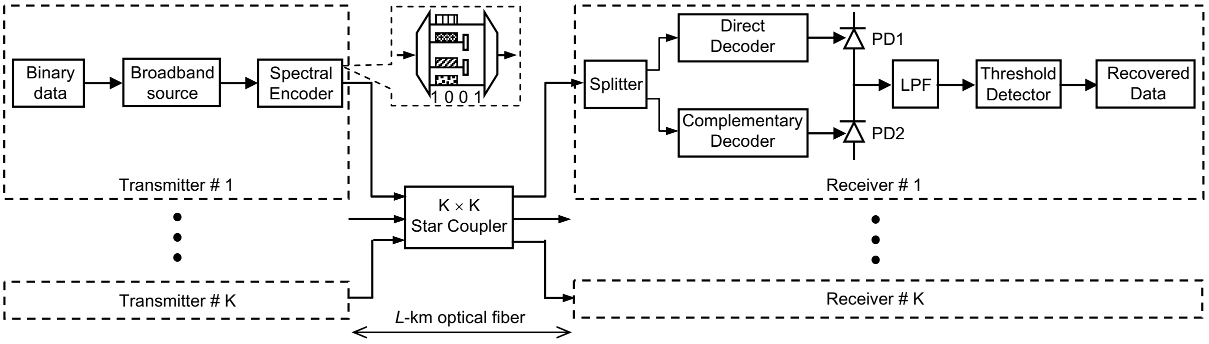

Fig. 2. Schematic diagram of a SAE/OCDMA system.

the methods for combating the impact of GVD. Finally, Section VI concludes the paper.

II. System Descriptions

The SAE/OCDMA technique was first described by Zaccarin and Kavehrad [3]. Figure 2 shows a schematic diagram of a SAE/OCDMA system. There are K pairs of transmitter and receiver corresponding to K users in the system. A star coupler is used to distribute the optical signal from one transmitter to all receivers.

At the transmitter, when bit "1" is sent, a multi-wavelength optical pulse from a broadband source launches into the spectral encoder, whereas no optical pulse is sent if the data bit is "0". Next, wavelength components of optical pulse are resolved and encoded at the spectral encoder by selectively blocking or transmitting certain wavelength components in accordance with a signature code. Encoded optical pulse is then transmitted to the receiver. Because of fiber attenuation and loss caused by the coupler, received power will be decreased. Let P_0 is transmitted power, α is fiber attenuation coefficient, and L is transmission length, the received power can be expressed as $P_s = [P_0 \exp(-\alpha L)]/K$.

At the receiver, received signal is first distributed to two decoders, direct decoders (at additive branch) and complementary decoder (at the subtractive branch), by a splitter. The direct decoder has the same spectral characteristics as that of the spectral encoder. The remaining decoder has complementary wavelength components with those of the spectral encoder. The outputs from these decoders are detected by two photodetectors (PD1 and PD2) connected in a balanced fashion. Then, low-pass filter (LPF) and threshold detector are used to recover the original data. For an interfering signal, depending on the signature code used, a part of its wavelength components will match the direct decoder, and the other part will match the complementary decoder. Since the output of the balanced receiver represents the difference between the two photodetector outputs, the interfering channels will be canceled, whereas the desired channel is demodulated.

Several signature code sets have been proposed for a SAE/OCDMA system, including m-sequence, Hadamard, and modified quadratic congruence (MQC) code sets [7]. In this paper, we use m-sequence code set, which is popular and widely studied [1]. Let c_1 be one unipolar m-sequence of length N as the code word of the 1-th user, the code word of the k-th user (c_k) can be obtained by cyclically shifting the original sequence (e.g., $c_k = T^{k-1}c_1$, where T is the shift operator). As a result, a m-sequence code set has the cardinality of N. The code weight and autocorrelation will be $(N+1)/2$. The cross-correlation between two different signature codes, c_d and c_k, as well $\overline{c}_d$ (a complementary of c_d) and c_k are $(N+1)/4$. Corresponding to a m-sequence signature code, the number of wavelength components at the spectral encoder and the direct decoder are the same, $(N+1)/2$. Whereas, for the complementary decoder, the number of wavelength components is $(N-1)/2$.

III. Performance Analysis

In this section, we will theoretically analyze the performance of the SAE/OCDMA system and derive its bit error rate (BER). First, a modified model of pulse propagation is proposed. Then, based on this model, we will analyze the performance of the SAE/OCDMA system taking into account the impact of GVD, MAI, and receiver's noise. In order to focus on the impact of GVD, optical beating interference is neglected in our analysis.

A. Pulse Propagation Model

We assume that optical pulse correspondence with each wavelength component is a Gaussian pulse. As we mentioned above, the conventional model of Gaussian pulse propagation [4] does not express the relative delay between different wavelength components.

In order to describe the relative delay between wavelength components, we first select an arbitrary wavelength component to be the reference one, e.g. λ_r. Without loss of generality, we assume the position of the reference wavelength component in the time to be stationary during propagation. The relative delay between a wavelength component λ_i and the reference one can be expressed as $(\beta_{1i} - \beta_{1r})L = \Delta\beta_{1i}L$, in which L is the transmission length, $\beta_{1r} = 1/v_{gr}$ and $\beta_{1i} = 1/v_{gi}$ with v_{gr} and v_{gi} are group velocities of the reference and wavelength component λ_i, respectively. Our proposed modified model with the new parameter ($\Delta\beta_{1i}L$) for the optical pulse at wavelength component λ_i, denoted as $c_i(t)$, then can be expressed as

$$c_i(t) = \sqrt{P_s}|A_i(t)| \exp[j(\omega_i t + \phi_i)], \tag{1}$$

where $A_i(i)$ is the normalized amplitude of Gaussian pulse, which is written by

$$A_i(t)=\frac{T_0}{(T_0^2-j\beta_{2i}L)^{1/2}}\exp\left(-\frac{(t-\Delta\beta_{1i}L)^2}{2(T_0^2-j\beta_{2i}L)}\right). \quad (2)$$

Here, P_s is received peak power of optical pulse. T_0 is the half-width of pulse (at 1/e-intensity point) and β_{2i} is GVD parameter. ω_i and ϕ_i are optical frequency and phase of optical carrier corresponding to the wavelength component λ_i.

Optical pulse remains its shape when propagating along optical fiber. Nevertheless, its width and peak power are moderated by the GVD parameter β_{2i}. In addition, the relative delay between the analyzed wavelength and the reference one is determined by the parameter $\Delta\beta_{1i}$.

B. System's Bit Error Rate

In following analysis, we denote ω_{di}, ϕ_{di} at wavelength λ_i as optical frequency and phase of the desired user. In OCDMA systems, the received optical field includes not only the desired user but also MAI. Let K be the number of active users, as well denote ω_{ki}, ϕ_{ki} as optical frequency and phase at wavelength λ_i of k-th interfering user ($1 \leq k \leq K-1$). The total optical field of the received signal including MAI then can be expressed as

$$\begin{aligned} E_s(t) &= \sum_{i=1}^{N} c_d(i)\sqrt{P_s}|A_i(t)|\exp[j(\omega_{di}t+\phi_{di})] \\ &+\sum_{k=1}^{K-1}\sum_{i=1}^{N} c_k(i)\sqrt{P_s}|A_i(t)|\exp[j(\omega_{ki}t+\phi_{ki})]. \quad (3)\end{aligned}$$

Here, to simplify the calculation, we assume that transmitted power levels of any wavelengths are the same for all users. Also, the distances between any transmitter-receiver pairs are assumed to be the same (L km).

The photocurrents on the additive branch $I^+(t)$ can be derived as

$$\begin{aligned} I^+(t) &= \Re\sum_{i=1}^{N} c_d(i)c_d(i)P_s|A_i(t)|^2 \\ &+\Re\sum_{k=1}^{K-1}\sum_{i=1}^{N} c_d(i)c_k(i)P_s|A_i(t)|^2 \\ &+n(t), \quad (4)\end{aligned}$$

where $\Re$ denotes the photodiode responsivity. The first term in the Eq. (4) represents useful data and the second one represents the photocurrent caused by MAI. The last term, $n(t)$, represents receiver's noise which includes shot noise and thermal noise. The photocurrents on the subtractive branch $I^-(t)$ can also be derived similarly as Eq. (4), where $c_d(i)$ is replaced by $\overline{c_d}(i)$, respectively.

The data current at the input of the threshold detector is expressed as

$$\begin{aligned} I_d(t) &= I_d^+(t) - I_d^-(t) \\ &= \Re\sum_{i=1}^{N} c_d(i)c_d(i)P_s|A_i(t)|^2 = \Re P_s A(t), \quad (5)\end{aligned}$$

where $A(t)$ is called the autocorrelation signal.

An example, which describes the impact of GVD on autocorrelation signal, is shown in Fig. 3. In this example, m-sequence with the sequence length of 31 ($N = 31$) is used. Without the impact of GVD, the autocorrelation peak equals to 16 (i.e. $(N+1)/2$). Under the impact of GVD, the magnitudes of distortion and broadening of data signal increase with transmission length (L). This is due to relative delays in arrival of different wavelength components.

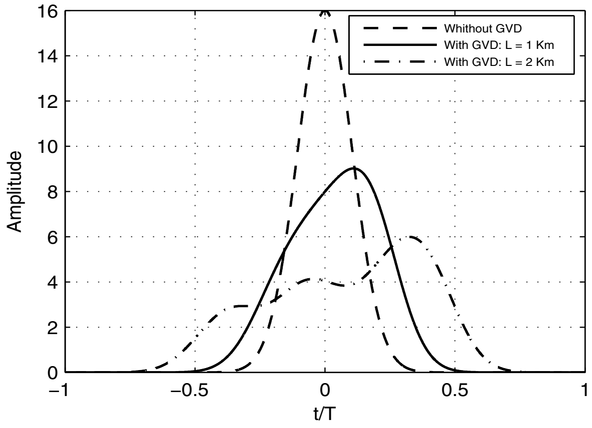

Fig. 3. Amplitude of the autocorrelation signal with $N = 31$ and $\Delta\lambda = 1.0$ nm, where T is bit duration and L is transmission length.

In order to calculate MAI currents, we first consider the average number of interfering pulses at each wavelength component. As we mentioned above, there are $K-1$ interfering users, each one has $\frac{N+1}{4}$ wavelength components matching the direct decoder and $\frac{N+1}{4}$ wavelength components matching the complementary decoder. Therefore, there are totally $\frac{(K-1)(N+1)}{4}$ interfering pulses passing through each decoder. When the number of interfering users is large enough, we can assume that interfering pulses are equally distributed over all wavelength components of each decoder. As a results, the average number of interfering pulses at each wavelength component of the direct decoder is $\frac{K-1}{2}$ and of the complementary decoder is $\frac{(K-1)(N+1)}{2(N-1)}$, respectively.

With above assumption, the MAI current at the additive and subtractive branches are calculated as

$$I_{MAI}^+(t) = \frac{K-1}{2}\Re\sum_{i=1}^{N} c_d(i)c_k(i)P_s|A_i(t)|^2 \quad (6)$$

and

$$I_{MAI}^-(t) = \frac{(K-1)(N+1)}{2(N-1)}\Re\sum_{i=1}^{N} \overline{c_d}(i)c_k(i)P_s|A_i(t)|^2. \quad (7)$$

The total of data and MAI currents for two cases, bit "1" ($b = 1$) and bit "0" ($b = 0$) are transmitted, can be express as

$$I_b = b\langle I_d\rangle + \langle I_{MAI}^+\rangle - \langle I_{MAI}^-\rangle, \quad (8)$$

where $\langle I_d\rangle$, $\langle I_{MAI}^+\rangle$, and $\langle I_{MAI}^-\rangle$ are the average currents of data and MAI signals.

Finally, we derive the value of receiver's noise including shot noise and thermal noise. Shot noise is caused by data current and MAI currents from both additive and subtractive branches, $i_{shb}^2 = 2eB(b\langle I_d\rangle + \langle I_{MAI}^+\rangle + \langle I_{MAI}^-\rangle)$

with $b \in (1, 0)$. Thermal noise can be calculated as $i_T^2 = 4K_bT_nB/R_L$. Where e is electron charge, B is receiver's electrical bandwidth, K_b is Boltzman's constant, T_n is receiver noise temperature, and R_L is the receiver load resistor. The total noise variance is thus given by $i_{nb}^2 = i_{shb}^2 + i_T^2$, where $b = 1$ or 0.

Each bit is detected by comparing the total current with a threshold current I_D. The signal to noise ratio (SNR$_b$) for two cases ($b = 0$ and $b = 1$) is calculated as

$$SNR_b = \frac{(I_b - I_D)^2}{i_{nb}^2}. \tag{9}$$

In order to minimize the BER, we consider the case that I_D is optimum [8]. Moreover, the demodulated photocurrent as well as noises can be modeled as Gaussian random variables, the total probability of error hence can be calculated as

$$\text{BER} = \frac{1}{2}\text{erfc}(\frac{1}{\sqrt{2}}\frac{I_1 - I_0}{i_{n1} + i_{n0}}), \tag{10}$$

where I_1 and I_0 are derived from Eq. (8) and

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-y^2)\,dy. \tag{11}$$

IV. NUMERICAL RESULTS

In this section, numerical results are presented to show the impact of GVD on the performance of SAE/OCDMA systems. We consider an optical access network that supports 1 Gbit/s connections. m-sequence codes are used with sequence length N = 31 corresponding to 16 wavelength components (i.e. (N+1)/2) per user. We choose λ = 1550 nm as a central wavelength and consider two values of wavelength interval, $\Delta\lambda$ = 1 nm and $\Delta\lambda$ = 0.4 nm. The transmission medium is normal single mode fiber (SMF) with attenuation coefficient α = 0.2 dB/km, dispersion coefficient $D_{1550} = 16$ ps/nm×km and dispersion slope coefficient $S_{1550} = 0.056$ ps/nm^2× km.

First, we investigate the maximum transmission length when the system is affected by GVD. Figure 4 shows the system's BER vs. the transmission length when transmitted power per bit P_b = 0 dBm and the number of active users K = 31×1 Gbit/s. In addition, the system's performance is analyzed with different values of wavelength interval.

It is seen that the maximum transmission length (i.e. the length at which BER $\leq 10^{-9}$ can be maintained) is extremely shortened under the impact of GVD. The maximum transmission length is reduced from 79 km, in case of without GVD, to 16 km for the case of $\Delta\lambda$ = 1 nm. When $\Delta\lambda$ = 0.4 nm, the maximum transmission length is longer, 33 km. This is due to the difference of group velocities among wavelength components decreases when $\Delta\lambda$ reduces.

The reduction of supportable users is depicted in Fig. 5. Because the impact of GVD is in proportion with transmission length, the number of supportable users will be decreased when transmission length increases. We keep the transmitted power per bit P_b = 0 dBm, $\Delta\lambda$ = 0.4 nm, and increase L from 40 to 50 and 60 km. The number of supportable users will reduce from 21 to 12 and 7 users, respectively.

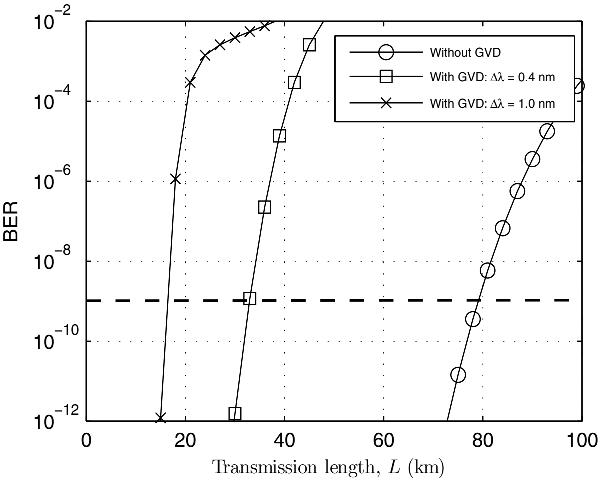

Fig. 4. BER versus transmission length L when $K = 31 \times 1$ Gbit/s users and P_b = 0 dBm.

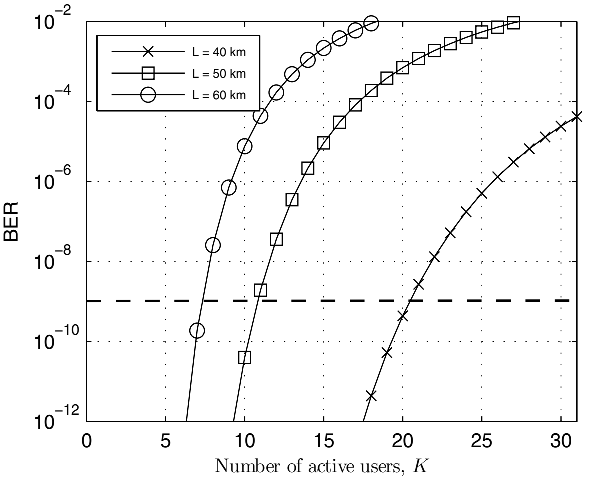

Fig. 5. BER versus number of active users when P_b = 0 dBm. The user bit rate is 1 Gbit/s and wavelength interval $\Delta\lambda$ = 0.4 nm.

Figure 6 illustrates the system's BER vs. transmitted power per bit when K = 31×1 Gbit/s and the wavelength interval $\Delta\lambda$ = 0.4 nm. From the figure, we can see that the system has to suffer a high value of power penalty, 8 dB, when transmission length is only 33 km.

According to numerical results, the performance of SAE/OCDMA system is seriously degraded due to GVD. Therefore, it is necessary to use the techniques to combat the impact of GVD, especially when the wavelength interval is large.

V. COMBATING GROUP VELOCITY DISPERSION

A. Related Works

There have been several methods for combating the impact of GVD, which are proposed to 2-D wavelength hopping/time spreading and frequency hopping OCDMA systems [5], [6]. These methods can be applied to SAE/OCDMA systems. Each method has its advantages and disadvantages, which are further analyzed as follows.

The simplest way is using dispersion-shifted fiber (DSF) instead of SMF. The difference of group velocities per

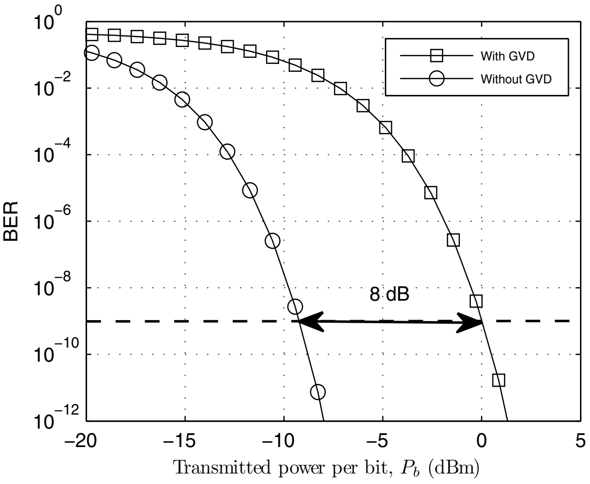

Fig. 6. BER versus transmitted power per bit P_b when $K = 31 \times 1$ Gbit/s users and L = 33 km. The wavelength interval $\Delta\lambda$ = 0.4 nm.

nanometer of DSF is much smaller, i.e. approximately 0.2 ps/km compared with 16 ps/km of SMF. Therefore, the relative delays between wavelength components will be reduced and the performance of the system is improved. Using DSF, however, has a disadvantage. The cost of DSF is higher than that of SMF. In addition, we cannot make use of available SMF in existing optical access networks.

An alternate method is to compensate for GVD directly in the encoder/decoder structures, either through pre-skewing at encoder, post-skewing at decoder, or both [5]. This is accomplished by setting the physical separation between the gratings so as to compensate for the relative delays induced by GVD. With preset transmission length, it is possible to use this technique. However, when the transmission length changes, for example when the receiver changes the connection to get the signal from one user to another, the use of pre- and post-skewing techniques are meaningless. The solution to this problem is using the same transmission length for all connections among users, regardless of the actual distance between transmitter-receiver pairs [5]. This solution is not practical when the actual distances between transmitter-receiver pairs are much different.

B. Our Proposal

In this section, we propose a new method, which use fiber Bragg gratings at encoder and in line, to compensate the relative delays of wavelength components.

From analytical results, we can get the maximum transmission length of the system under the impact of GVD (L_{GVD}). The maximum relative delay of wavelength component λ_i will be $T_i^{(max)} = \Delta\beta_{1i} L_{GVD}$. The system's BER $\leq 10^{-9}$ will be maintained if relative delay satisfies the condition, $-T_i^{(max)} \leq T_i \leq T_i^{(max)}$. By using fiber Bragg gratings at encoder to pre-compensate and in line at the distance of $2L_{GVD}$ to post- and pre-compensate the relative delays, the maximum transmission length can be increased 4 times as depicted in Fig. 7. Where $|T_i^{(encoder)}| = T_i^{(max)}$ and $|T_i^{(inline)}| = 2T_i^{(max)}$ are compensating duration at encoder and in line, respectively. Here, the relation between maximum transmission length and the number of in-line compensating points (n) will be $L_{max} = (n+1)2L_{GVD}$. Note that the distance between two adjacent compensating points is $2L_{GVD}$.

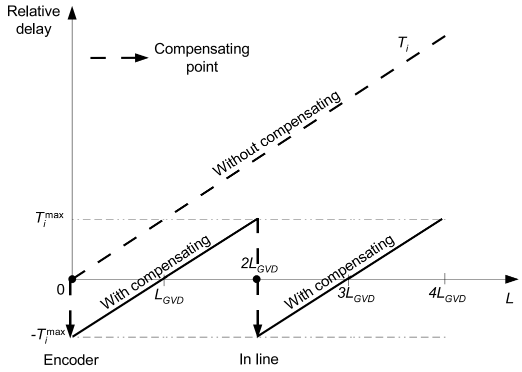

Fig. 7. Combating the impact of GVD.

Our solution is cost-effective because we can use SMF instead of DSF. Moreover, the transmission distance is more extendable by using this method.

VI. CONCLUSION

We have proposed a modified model of Gaussian pulse propagation to comprehensively study the performance degradation of the SAE/OCDMA system due to GVD. The analytical results show that, the number of supportable users is decreased and the maximum transmission length is shortened under the impact of GVD. In addition, we proposed a cost-effective method to combat the impact of GVD hence increase the maximum transmission length of the SAE/OCDMA system.

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REFERENCES

[1] Paul R. Prucnal, *Optical Code Division Multiple Access: Fundamentals and Applications*, CRC Press, 2006.

[2] K. Fouli and M. Maier, "OCDMA and Optical Coding: Principles, Applications, and Challenges," IEEE Commun. Mag. 9, IEEE, Vol. 45, pp. 27-34, Aug. 2007.

[3] D. Zaccarin and M. Kavehrad, "An optical CDMA system based on spectral encoding of LED," IEEE Photon. Technol. Lett., Vol 5, pp. 479 - 482, Apr. 1993.

[4] G. P. Agrawal, *Nonlinear Fiber Optics*, San Diego, CA: Academic, 2001.

[5] Eddie K. H. Ng, G. E. Weichenberg, and Edward H. Sargent, "Dispersion in multiwavelength optical code division multiple-access systems: Impact and Remedies," IEEE Trans. on Commu., Vol. 50, No.11, Nov. 2002.

[6] Chao Zuo, Wenhua Ma, Hongtu Pu, and Jintong Lin, "The impact of group velocity on frequency-hopping optical code division multiple access system," IEEE J. Lightwave Technol., Vol. 19, No. 10, Oct. 2001.

[7] A. T. Pham, N. Miki, and H. Yashima, "Spectral amplitude encoding optical-code-division-multiplexing system with a heterodyne detection receiver for broaband optical multiple-access networks," OSA J. Optical Networking, Vol. 4, No. 10, Sep. 2005.

[8] G. P. Agrawal, *Fiber-Optic Communication Systems*, Third Edition, A John Wiley and Sons, Inc., Publication, 2002.