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Non-iterative MMSE-based Channel Estimation for Mobile MIMO-OFDM Systems

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***Abstract*-In this paper a non-iterative rectangular-type pilot-aided channel estimation algorithm on mobile Multiple Input-Multiple Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) systems is proposed. After Minimum Mean Square Error (MMSE) estimation in time direction, channel interpolation must be performed in frequency direction. Recent DFT-based algorithms, in MIMO-OFDM systems, reduce Additive White Gaussian Noise (AWGN) and Inter-Channel Interference (ICI) iteratively, through time domain low-pass filtering in each iteration. Here, it is shown that extending the algorithm used for OFDM systems to be used for MIMO-OFDM systems causes some frequency shift. The proposed algorithm compensates this shift. Then, the iterations become useless and the complexity reduces considerably. The simulation results show that the proposed algorithm slightly outperforms the iterative algorithm in addition to complexity reduction by eliminating iterations.**

*Keywords:*DFT-based interpolation, MIMO-OFDM, MMSE, Non-iterative channel estimation.

I. INTRODUCTION

OFDM is an accepted modulation in wideband wireless communication due to its high data rate and various advantages in lessening the severe effects of frequency-selective fading [1]. On the other hand, MIMO improves capacity and efficiency of wireless communication systems over flat fading channels [2]. Using OFDM on MIMO converts a frequency selective channel to a set of flat fading MIMO sub-channels [3]; hence, MIMO-OFDM systems have the benefits of both MIMO and OFDM.

Coherent detection in MIMO needs channel information. This information can be obtained through channel estimation. A common approach of channel estimation in MIMO systems is to send the different training symbols referred to as 'pilots' from different transmit antennas and then estimate the channel impulse response between each transceiver pair. This requires the orthogonality of different antennas' training sequences. A straightforward method which guarantees this orthogonality is to allocate a distinct subset of sub-carriers to each transmit antenna as its training carriers; that is, when an antenna is transmitting a pilot at a specific sub-carrier, all other transmit antennas are silent as it is done in IEEE802.11a [4]. The DFT-based algorithms for MIMO-OFDM systems, based on this structure, decrease ICI and AWGN iteratively and improve initial channel estimation [5, 6]. These papers extended the previous algorithms which were used for OFDM systems to be used in MIMO-OFDM systems.

Another important issue in this method is the pilot structure. A commonly used structure is the comb-type structure in which pilots are sent in every OFDM symbol. This structure has been used in [5] and shown to be useful for even very fast fading channels, but this structure has the disadvantage of high overload which makes it unsuitable for many practical applications. A more practical and general structure is rectangular structure in which the pure data rate is much more than the first structure, but it needs an excess interpolation in time direction [6].

In this paper a non-iterative algorithm for rectangular-type pilots, based on time domain filtering, is proposed which reduces complexity considerably. Furthermore, simulation results show that it outperforms the iterative algorithm.

II. SYSTEM MODEL

Fig. 1 shows a MIMO-OFDM system model with N_s sub-carriers, M_T transmit, and M_R receive antennas. At time step n , the received signal of jth receiver antenna at kth sub-carrier can be expressed as

$$Y_j(n,k)=\sum_{i=1}^{M_T}\left(X_i(n,k)H_{i,j}(n,k)+I_{i,j}(n,k)\right) + W_j(n,k),\ k=0,\dots,N_s-1 \tag{1}$$

where $\{X_i(n,k), k=0,1,\dots,N_s-1, i=1,\dots,M_T\}$ is the transmit data block of ith transmitter antenna, $W_j(n,k)$ is the complex zero-mean AWGN for jth receiver with variance σ^2 , $I_{i,j}(n,k)$ is the Multiple Antenna Sub-Channel Interference (MASCI) which is the general form of Inter Channel Interference (ICI). In [7] it has been modeled as a zero-mean Gaussian random process with certain variance P_I. $H_{i,j}(n,k)$ is the frequency response between ith Tx antenna and jth Rx antenna and is expressed as

$$H_{i,j}(n,k)=\sum_{l=1}^{L}h_{i,j}(n,l)\exp\left(-j2\pi\frac{k\tau_{i,j}^{l}}{N_s}\right) \tag{2}$$

In fact, the channel between each transceiver pair is a multipath fading channel of which the impulse response of lth path $h_{i,j}(n,l)$ is a zero-mean Gaussian random process with autocorrelation function

$$E\left\{h_{i,j}(n+\Delta n,l)h_{i,j}^{*}(n,l)\right\}=\sigma_l^2 J_0(2\pi f_d T\Delta n) \tag{3}$$

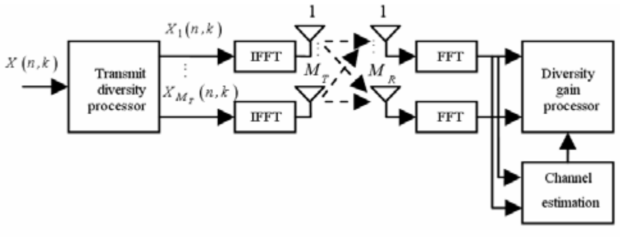


Figure 1. MIMO-OFDM system model

where $J_0(.)$ denotes the zeroth-order Bessel function of the first kind, f_d is the maximum Doppler frequency, T is the duration of an OFDM symbol and σ_l^2 is the average power of lth path. Finally, L is the number of paths and $\tau_{i,j}^l$ is the delay of lth path between ith Tx antenna and jth Rx antenna.

III. Pilot Symbol Structure

For minimizing the MSE of the LS channel estimation, the pilot sequences of different antennas must be *equipowered*, *equispaced* and *phase shift orthogonal* [8]. As mentioned, a simple way to achieve orthogonality is to assign a distinct set of pilot carriers to each antenna and keep the other antennas silent on this sub-carrier set. So, N_P pilot sub-carriers are divided into M_T groups each consisting of P pilots. On the other hand, there are both data and training OFDM symbols in rectangular structure. The pilot spacing in frequency direction and time direction is $N_f = \frac{N_s}{P}$ and N_t respectively. The transmitted signal on the ith transmit antenna for training OFDM symbols is then [6]

$$X_i(n,k) = \begin{cases} c_i & k = mN_f + (i-1)_{N_f} \\ 0 & k = mN_f + (i'-1)_{N_f} \\ d_i & o.w. \end{cases} \tag{4}$$
$$m = 0,1,\ldots,P-1 \qquad i' = 1,2,\ldots,M_T \; : \; i' \neq i$$

where, c_i is the complex pilot symbol, d_i is data symbol, and $(x)_y$ is the remainder of division of x by y. Fig. 2 shows rectangular type pilot structure for a system by two transmitter antennas and eight sub-carriers.

IV. Channel Estimation

According to (1) and the pilot structure, the received pilot sequence in an OFDM symbol from the ith transmit antenna at jth receiver antenna in vector form is as follows [6]

$$\boldsymbol{Y}_{i,j}^{(p)} = \boldsymbol{c}_i \boldsymbol{H}_{i,j}^{(p)} + \boldsymbol{I}_{i,j}^{(p)} + \boldsymbol{W}_j^{(p)} \tag{5}$$

where $\boldsymbol{c}_i$ is the transmitter pilot sequence, $\boldsymbol{H}_{i,j}^{(p)}$ is the channel frequency response, $\boldsymbol{I}_{i,j}^{(p)}$ represents the MASCI, and $\boldsymbol{W}_j^{(p)}$ is white noise vector defined as

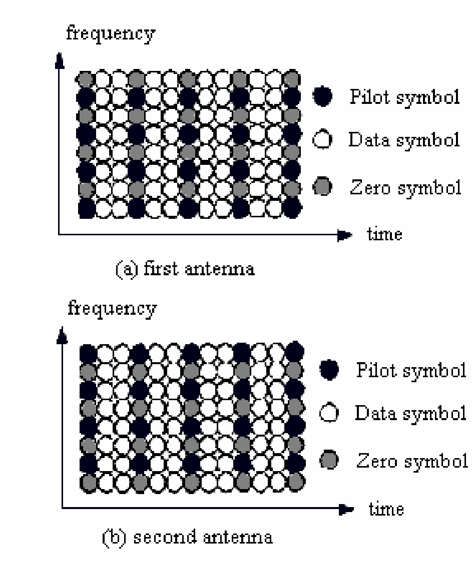


Figure 2. Typical rectangular pilot structure for a system with 2 transmitters

$$\begin{aligned} \boldsymbol{c}_i &= diag\left(c_i(1),\ldots,c_i(P)\right) \in \mathbb{C}^{P\times P} \\ \boldsymbol{H}_{i,j}^{(p)} &= \left[H_{i,j}(1),\ldots,H_{i,j}(P)\right]^T \in \mathbb{C}^{P\times 1} \\ \boldsymbol{I}_{i,j}^{(p)} &= \left[I_{i,j}(1),\ldots,I_{i,j}(P)\right]^T \in \mathbb{C}^{P\times 1} \\ \boldsymbol{W}_j^{(p)} &= \left[W_j(1),\ldots,W_j(P)\right]^T \in \mathbb{C}^{P\times 1} \end{aligned} \tag{6}$$

The LS channel estimation at P pilot sub-carriers is obtained as

$$\hat{\boldsymbol{H}}_{i,j}^{(p)} = \boldsymbol{H}_{i,j}^{(p)} + \boldsymbol{c}_i^{-1}\left(\boldsymbol{I}_{i.j}^{(p)} + \boldsymbol{W}_j^{(p)}\right) \tag{7}$$

After obtaining pilot estimations, channel response must be estimated in data locations. For estimating channel at data locations, interpolation is necessary. In time direction, it is done by MMSE estimation algorithm which is optimum in MSE criterion. This estimation also improves the initial LS estimation of pilots. Considering that the noise and channel estimation are statistically independent, the MMSE estimation of channel frequency response in pilot sub-carriers and the same sub-carriers of data OFDM symbols are

$$\begin{aligned} &\hat{\boldsymbol{H}}_{i,j}^{(d)}(m) = \boldsymbol{R}_{\boldsymbol{H}^{(d)}\boldsymbol{H}^{(p)}} \cdot \\ &\left(\boldsymbol{R}_{\boldsymbol{H}^{(p)}\boldsymbol{H}^{(p)}} + \left(\left(\sigma^2 + P_I\right)\big/P_c\right)\boldsymbol{I}_P\right)^{-1} . \hat{\boldsymbol{H}}_{i,j}^{(p)}(n) \\ &m = n,\ldots,n+N_t-1 \end{aligned} \tag{8}$$

where P_I is the power of ICI, P_c is the power of pilots, the $\boldsymbol{R}_{\boldsymbol{H}^{(d)}\boldsymbol{H}^{(p)}}$ represents the cross-correlation matrix of channel response in data locations and channel response in pilots, and $\boldsymbol{R}_{\boldsymbol{H}^{(p)}\boldsymbol{H}^{(p)}}$ is the autocorrelation matrix of channel response in pilots. Correlation function of channel frequency response depends on the time and frequency separation of different locations and has been discussed in [9] precisely.

After MMSE estimation in time direction, we have P channel response estimations at pilot sub-carriers in each OFDM symbol. Performing IDFT with size P, we get the channel impulse response during each OFDM symbol

$$\hat{h}_{i,j}(n)=\frac{1}{P}\sum_{p=0}^{P-1}\hat{H}_{i,j}^{(d)}(p)e^{j\frac{2\pi np}{P}} \tag{9}$$
$$,n=0,1,...,P-1$$

Substituting (7) in (9) gives

$$\hat{h}_{i,j}(n)=\frac{1}{P}\sum_{p=0}^{P-1}\left(H_{i,j}(p)+\frac{W_{i,j}(p)+I_{i,j}(p)}{c_i}\right)e^{j\frac{2\pi np}{P}} \tag{10}$$

This equation shows that the estimated impulse response is the IDFT of two terms: ideal frequency response and the noise component $\frac{W_{i,j}(p)+I_{i,j}(p)}{c_i}$. According to (2) the channel frequency response is a sinusoidal function of k and thus has relatively slow variations with respect to this parameter; However, the noise component is random and have fast variations; Consequently, the noise is distributed in time (transform) domain, but the signal component is located at the lower region of transform domain [5]. The channel length can be defined as $Q=\lfloor\tau_L\rfloor+1$ in which τ_L is assumed as the maximum normalized delay for all paths; thus, we can keep the first Q samples of the channel impulse response and discard the other samples which only consist of noise and interference

$$\tilde{h}_{i,j}(n)=\begin{cases}\hat{h}_{i,j}(n), & n=0,1,...,Q-1\\ 0, & n=Q,...,P-1\end{cases} \tag{11}$$

Based on this method [5,6] have presented an iterative algorithm which rejects noise and ICI by repeatedly transforming estimated samples from frequency to time domain, lowpass filtering in time domain, and transforming back the remainder to frequency again. This procedure continues until the error of estimation is below a predetermined threshold.

In the next section, we describe how the iterations can be eliminated.

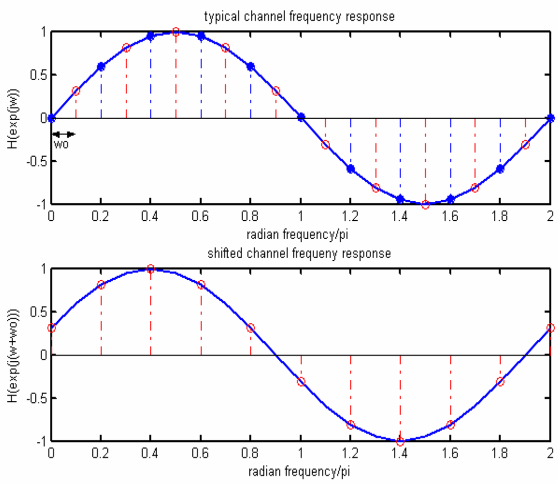


Figure 3. A simple schematic of frequency shift concept

V. PROPOSED NON-ITERATIVE ALGORITHM

With more attention in (9) we observe that IFFT has the correct mathematical form just for the first antenna, as in the used pilot construction, pilot sub-carriers for antennas except the first antenna does not begin from the first sub-carrier. That is, the channel estimation in pilot locations of these antennas are really the sampling result of the frequency-shifted channel frequency response.

This is simply illustrated in Fig. 3. The filled circles which can represent the first antenna pilot sub-carriers are the sampling result of $H\left(e^{j\omega}\right)$ in the interval of $[0,2\pi]$; however, the empty circles which can be taken as the next antenna pilot sub-carriers are, in fact, the sampling result of $H\left(e^{j(\omega+\omega_0)}\right)$ in the same interval. According to (9) this frequency shift has been ignored in IFFT and the first sample has been taken as the first sub-carrier for all antennas. Thus, IFFT of these samples (except first antenna's samples) will not result in $\hat{h}_{i,j}(n)$ but to the phase shifted version of it. This phase shift is expressed as follows [10]

$$e^{-j\omega_0 n}h[n]\xrightarrow{\Im}H\left(e^{j(\omega+\omega_0)}\right) \tag{12}$$

In pilot structure explained before, we can observe that the pilot sequence of the ith transmitter is started from ith sub-carrier. This means that the initial estimation of channel frequency response related to ith transmitter have a radian frequency shift equal to $-2\pi(i-1)/N_s$. Hence, according to (12) initial impulse response of the ith transmitter has a phase shift equal to $-2\pi(i-1)n/N_s$, which must be compensated.

Simulation results show that this compensation can simplify the channel estimation so considerably that the iterative procedure of the algorithm in [5, 6] becomes ineffective and the best result is obtained without any iteration. Hence, we can say that the error arising form outer noise is omitted before the iterative procedure; in fact, this part of iterative algorithm only compensates the error caused by the frequency shift mentioned above. The proposed algorithm is as follows:

Initial LS channel frequency response estimation at pilot locations according to (7) and MMSE estimation in time direction by (8).

Estimating the channel impulse response in each OFDM symbol according to (9) and then lowpass filtering according to (11).

Correction the phase shift in $\tilde{h}_{i,j}(n)$. This is done according to (13)

$$\tilde{h}'_{i,j}(n)=e^{\frac{j2\pi(i-1)n}{N_s}}\tilde{h}_{i,j}(n), i=1,...,M_T \tag{13}$$

Applying FFT of size N_s to $\tilde{h}'_{i,j}(n)$, i.e.

$$\hat{H}'_{i,j}(k)=\sum_{n=0}^{Q-1}\tilde{h}'_{i,j}(n)\,e^{-j\frac{2\pi nk}{N_s}} \tag{14}$$
$$,k=0,1,...,N_s-1$$

The mean square error (MSE) of the estimation is obtained by

$$MSE=\frac{\sum_{i=1}^{M_T}\sum_{j=1}^{M_R}E_k\left\{\left|\hat{H}'_{i,j}(k)-H_{i,j}(k)\right|^2\right\}}{M_T M_R} \quad (15)$$

VI. SIMULATION RESULT

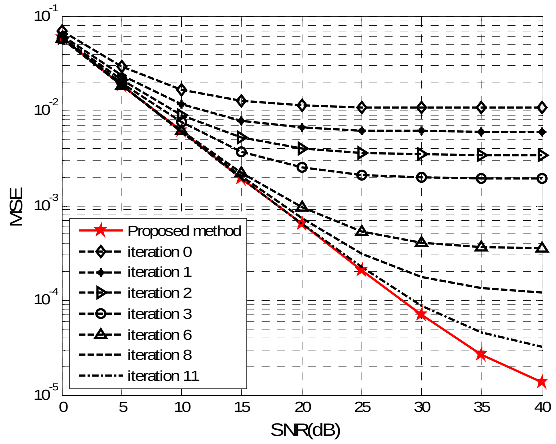
A MIMO-OFDM system with 4 transmit and 4 receive antennas is used in this section. The system has QPSK modulation of 10 MHz bandwidth with carrier frequency of 1 GHz. The number of sub-carriers is $N_s=1024$, and guard interval is $N_g=226$. The channel is a five-path Rayleigh fading channel with maximum delay (normalized to sampling time) τ_L = 136. The total number of pilot sub-carriers is N_P = 1024 which is divided into $M=4$ groups. Pilot spacing in time direction is N_t=4.

Fig. 4(a) and 4(b) show the MSE of channel estimation versus different SNR, for 5 and 120 km/h, respectively. It's clear in the results that the performance of iterative algorithm approximately reaches the proposed non-iterative algorithm after 11 iterations. Simulation results also show that iterations have absolutely no effect on the proposed algorithm, which verifies the main idea of our algorithm.

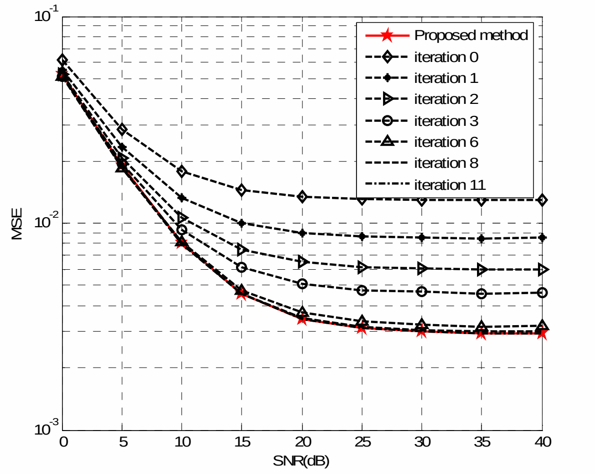
There are FFT and IFFT operations in each iteration [5, 6]. The proposed algorithm eliminates all the iterative operations but adds little complexity of $Q(M_T-1)M_R$ complex multiplication to the single step process by (13).

Fig. 4 shows that the MSE of channel estimation in high speed (120 km/h) degrades considerably. This is because of the relative vulnerability of rectangular-type pilot structure against time variations of the channel impulse response. However, with proper selection of pilot distances in time direction, this structure is a very suitable choice for slow to moderate fading channels.

The saturation in high SNR in figure 4 is a result of dominant ICI effect over noise effect. It is more obvious in figure 3(b).



(a)



(b)

Figure 4. MSE of channel estimation for a 4×4 system with N_t=4. (a) $v=5$ km/h, (b) $v=120$ km/h.

VII. CONCLUSION

In this paper, iterative pilot-aided channel estimation based on rectangular-type pilot was investigated and a new non-iterative algorithm was proposed. It was shown that the proposed algorithm highly reduces the estimation complexity by eliminating the iteration, and even outperforms the iterative algorithm slightly. The reduced complexity is obtained only at the expense of $Q(M_T-1)M_R$ multiplications instead of sequential iterations. It is obvious that the iterations have no effect in improving the new algorithm because the phase shift compensation exactly does what an infinite number of iterations did in the iterative algorithm.

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