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Pilot Power Optimization for Channel Estimation in OFDM System

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***Abstract*—For Orthogonal Frequency Division Multiplexing (OFDM) system, training preamble for channel estimation is studied when null subcarriers are embedded in each OFDM symbol. The optimal training preamble that minimize the mean squared channel estimation error is numerically found by casting the design problem into a semidefinite programming problem. A design example under the same setting as IEEE802.11a is provided to verify the efficacy of our proposal as well as to show that the preamble of IEEE802.11a is almost optimal.**

I. Introduction

Severe multipath channels often arise in high-rate transmissions. Conventional single-carrier transmissions suffer from inter-symbol interference (ISI) resulting from the multipath. Orthogonal Frequency Division Multiplexing (OFDM) is an effective high-rate transmission technique, which mitigates ISI through the insertion of cyclic prefix (CP) at the transmitter. OFDM has been adopted in wireless LAN standards, e.g., IEEE802.11a , and HIPERLAN/2, and in digital audio/video broadcasting standards.

To obtain the channel state information (CSI), training OFDM symbols or pilot symbols embedded in each OFDM symbol are utilized. Training OFDM symbols or equivalently OFDM preambles are transmitted at the beginning of the transmitted record, while pilot tones (complex exponentials in time) are embedded in each OFDM symbol, where they are separated from the information symbols in the frequency domain [1]. On the other hand, under certain conditions, pilot tones inserted into every OFDM symbol enable channel estimation of each OFDM symbol. This is known as pilot-assisted (or -aided) channel estimation [2], which allows tracking of the channel variation.

When all subcarriers are available for transmission, training OFDM preamble and pilot symbols have been well designed to enhance the channel estimation accuracy, see e.g., [3] and references therein. If all subcarriers can be utilized, then the pilot symbol sequence can be optimally designed in terms of; i) minimizing the channel mean squared estimation error [1]; ii) minimizing the bit-error rate when symbols are detected with channel estimates by pilot tones [4]; iii) maximizing the lower bound on channel capacity with channel estimates [5], [6]. It has been found that equally spaced (equi-distant) and equally powered (equi-powered) pilot symbols are optimal with respect to several performance measures. Pilot symbols are also designed for OFDM systems with multiple transmit and receive antennas [7].

In practice, not all the subcarriers are available for transmission. It is often the case that null subcarriers are set on both edges of the allocated bandwidth to mitigate the interferences from/to adjacent bands [8]. Null subcarriers render equi-distant and equi-powered pilot symbols impossible to use in practice. In [9], equi-powered pilot symbols are studied for channel estimation in multiple antenna OFDM system with null subcarriers. However, they are not always optimal even for point-to-point OFDM system. In [12], pilot symbols are numerically designed for least squares (LS) channel estimation in OFDM with null subcarriers. In this paper, we optimally allocate power to pilot symbols for minimum mean squared error (MMSE) channel estimation in OFDM with null subcarriers.

For a given pilot set, the channel MSE is readily found. However, the training preamble that minimizes the channel MSE is not available in a closed form except for some special cases. To obtain the optimal training preamble, the transmission power has to be optimally allocated to pilot tones. To find the optimal power allocation, we resort to numerical optimization. We first show that the MSE minimization problem can be cast into a semidefinite programming (SDP) problem [10] as in [12]. Then, the optimal power allocation which minimizes the channel MSE is numerically found by using an SDP solver. We present a design example under the same setting as IEEE802.11a, which verifies that our optimal preamble has smaller channel MSE than the preamble of IEEE802.11a, and reveals that the preamble of IEEE802.11a is actually comparable to the optimally designed preamble.

II. Preamble and Pilot Symbols for Channel Estimation

We consider point-to-point wireless Orthogonal Frequency Division Multiplexing (OFDM) transmissions over frequency-selective fading channels. We omit the OFDM symbol number, since we deal with one OFDM symbol.

Let the number of subcarriers be N. At the transmitter, a serial data sequence $\{s_0, s_1, \ldots, s_{N-1}\}$ undergoes serial-to-parallel (S/P) conversion to be stacked into one OFDM symbol. Then, an N-points inverse fast Fourier transform follows to produce the N dimensional data, which is parallel-to-serial (P/S) converted. A cyclic prefix (CP) of length N_{cp} is appended to mitigate the multipath effects. The discrete-time baseband equivalent transmitted time-domain signals u_n can be expressed as

$$u_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k e^{j\frac{2\pi kn}{N}}, \quad n \in [0, N-1]. \qquad (1)$$

Our discrete-time baseband equivalent FIR channel has maximum length L, and remain constant in at least one block, i.e., is quasi-static. We assume that N_{cp} is greater than the channel length L so that there is no inter-symbol interference (ISI) between OFDM symbols and denote the channel impulse response as $\{h_0, h_1, \ldots, h_{L-1}\}$.

At the receiver, we assume perfect timing synchronization. After removing CP, we apply Fourier transform to the received time-domain signal y_n for $n \in [0, N-1]$ to obtain for $k \in [0, N-1]$ that

$$Y_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y_n e^{-j\frac{2\pi kn}{N}} = H_k s_k + W_k, \qquad (2)$$

where H_k is the channel frequency response at frequency $2\pi k/N$ given by

$$H_k = \sum_{l=0}^{L-1} h_l e^{-j\frac{2\pi kl}{N}}, \qquad (3)$$

and the noise W_k is assumed to be i.i.d. circular Gaussian with zero mean and variance σ_w^2.

For the simplicity of presentation, we utilize a circular index with respect to N where the index n of a sequence corresponds to n modulo N. Let $\mathcal{K}$ be the set of active subcarriers, i.e., non-null subcarriers and $|\mathcal{K}|$ be the number of elements in $\mathcal{K}$.

For WLAN standard IEEE 802.11a, 64 slots are available in the OFDM symbol during data transmission mode, out of which 48 are assigned for information symbols, 4 for pilot tones, while the rest serves as spectral nulls to mitigate the interferences from/to OFDM symbols in adjacent bands. Thus, $\mathcal{K} = \{1, 2, \ldots, 26, 38, 39, \ldots, 63\}$ and $|\mathcal{K}| = 52$.

For channel estimation, we place $N_p(\leq |\mathcal{K}|)$ pilot symbols $\{p_1, \ldots, p_{N_p}\}$ at subcarriers $k_1, \ldots, k_{N_P} \in \mathcal{K}$, which is known to the receiver. We assume that $N_p \geq L$ so that the channel can be perfectly estimated if there is no noise and denote the index set of pilot symbols as $\mathcal{K}_p = \{k_1, \ldots, k_{N_P}\}$.

Let diag$(\boldsymbol{a})$ be a diagonal matrix with the vector $\boldsymbol{a}$ on its main diagonal. Collecting the received signals having pilot symbols as

$$\tilde{\boldsymbol{Y}} = [Y_{k_1}, \ldots, Y_{k_{N_p}}]^T, \qquad (4)$$

we obtain

$$\tilde{\boldsymbol{Y}} = \boldsymbol{D}_{H_p} \boldsymbol{p} + \tilde{\boldsymbol{W}}, \qquad (5)$$

where $\boldsymbol{D}_{H_p}$ is a diagonal matrix with its nth diagonal entry being H_{k_n} such that

$$\boldsymbol{D}_{H_p} = \operatorname{diag}\left(H_{k_1}, \ldots, H_{k_{N_p}}\right), \qquad (6)$$

and $\boldsymbol{p}$ is the pilot vector defined as $\boldsymbol{p} = \left[p_1, \ldots, p_{N_p}\right]^T$.

From $\tilde{\boldsymbol{Y}}$, we would like to estimate channel frequency responses for equalization and decoding. Let us define an index set specifying the channel frequency responses to be estimated as $\mathcal{K}_s$. In other words, H_k for $k \in \mathcal{K}_s$ have to be estimated from $\tilde{\boldsymbol{Y}}$. In a long training OFDM preamble, all subcarriers in $\mathcal{K}$ can be utilized for pilot symbols so that $\mathcal{K}_p = \mathcal{K}$. On the other hand, in pilot-assisted modulation (PSAM) [2], a few known pilot symbols are embedded in one OFDM symbol from which the channel is estimated. Thus, for PSAM, we have $\mathcal{K}_s = \mathcal{K} \setminus \mathcal{K}_p$ where $\setminus$ denotes set difference.

If we can adopt equally spaced (equi-distant) pilot symbols with equal power for channel estimation and symbol detection, then it can be analytically shown that the channel mean squared estimation error [1] as well as the bit-error rate [4] are minimized, while the lower bound on channel capacity [5], [6] is maximized. But the optimality of equi-distant and equi-powered pilots does not necessarily hold true in general. In this paper, for given sets $\mathcal{K}$, $\mathcal{K}_p$ and $\mathcal{K}_s$, we design the optimal distribution of pilot power to pilot symbols for channel estimation.

III. Mean Squared Channel Estimation Error

Let us define an $N \times N$ DFT matrix as $\boldsymbol{F}$, whose $(m+1, n+1)$th entry is $e^{-j2\pi mn/N}$. We denote an $N \times L$ matrix $\boldsymbol{F}_L = [\boldsymbol{f}_0, \ldots, \boldsymbol{f}_{N-1}]^{\mathcal{H}}$ consisting of N rows and first L columns of DFT matrix $\boldsymbol{F}$, where $\mathcal{H}$ is the complex conjugate transpose operator. We also define an $N_p \times L$ matrix $\boldsymbol{F}_p$ having $\boldsymbol{f}_{k_n}^{\mathcal{H}}$ for $k_n \in \mathcal{K}_p$ as its nth row. Then, we can express (5) as

$$\tilde{\boldsymbol{Y}} = \boldsymbol{D}_p \boldsymbol{F}_p \boldsymbol{h} + \tilde{\boldsymbol{W}}, \qquad (7)$$

where the diagonal matrix $\boldsymbol{D}_p$ and channel vector $\boldsymbol{h}$ are respectively defined as $\boldsymbol{D}_p = \operatorname{diag}\left(p_1, \ldots, p_{N_p}\right)$ and $\boldsymbol{h} = [h_0, \ldots, h_{L-1}]^T$.

Let a vector having channel responses to be estimated, i.e., H_k for $k \in \mathcal{K}_s$, be $\boldsymbol{H}_s = [H_{k_1}, \ldots, H_{k_{|\mathcal{K}_s|}}]^T$. Similar to $\boldsymbol{F}_p$, we define a $|\mathcal{K}_s| \times L$ matrix $\boldsymbol{F}_s$ having $\boldsymbol{f}_{k_n}^{\mathcal{H}}$ for $k_n \in \mathcal{K}_s$ as its nth row. Then, we obtain

$$\boldsymbol{H}_s = \boldsymbol{F}_s \boldsymbol{h}. \qquad (8)$$

We assume that the mean of the channel coefficients is zero, i.e., $E\{\boldsymbol{h}\} = 0$. The channel correlation matrix takes the form $\boldsymbol{R}_h = E\{\boldsymbol{h}\boldsymbol{h}^{\mathcal{H}}\}$, where $E\{\cdot\}$ stands for the expectation operator. Then, since (7) is linear, the minimum mean squared error (MMSE) estimate $\hat{\boldsymbol{H}}_s$ of $\boldsymbol{H}_s$ is given by [11]

$$\hat{\boldsymbol{H}}_s = E\{\boldsymbol{H}_s \tilde{\boldsymbol{Y}}^{\mathcal{H}}\} \left(E\{\tilde{\boldsymbol{Y}}\tilde{\boldsymbol{Y}}^{\mathcal{H}}\}\right)^{-1} \tilde{\boldsymbol{Y}}. \qquad (9)$$

It follows from (7) and (8) that

$$E\{\boldsymbol{H}_s \tilde{\boldsymbol{Y}}^{\mathcal{H}}\} = \boldsymbol{F}_s \boldsymbol{R}_h \boldsymbol{F}_p^{\mathcal{H}} \boldsymbol{D}_p^{\mathcal{H}}, \qquad (10)$$

and that

$$E\{\tilde{\boldsymbol{Y}}\tilde{\boldsymbol{Y}}^{\mathcal{H}}\} = \boldsymbol{D}_p \boldsymbol{F}_p \boldsymbol{R}_h \boldsymbol{F}_p^{\mathcal{H}} \boldsymbol{D}_p^{\mathcal{H}} + \sigma_w^2 \boldsymbol{I}. \qquad (11)$$

We utilize the notation $\boldsymbol{A} \succeq 0$ (or $\boldsymbol{A} \succ 0$) for a symmetric matrix $\boldsymbol{A}$ to indicate that $\boldsymbol{A}$ is positive semidefinite (or positive definite). Let us assume $\boldsymbol{R}_h \succ 0$ for the simplicity of presentation. If we define the estimation error vector $\boldsymbol{E}_s$ as $\boldsymbol{E}_s = \hat{\boldsymbol{H}}_s - \boldsymbol{H}_s$, then the correlation matrix of $\boldsymbol{E}_s$ can be expressed as [11]

$$E\{\boldsymbol{E}_s \boldsymbol{E}_s^{\mathcal{H}}\} = \boldsymbol{F}_s \left[\boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2} \boldsymbol{F}_p^{\mathcal{H}} \boldsymbol{\Lambda}_p \boldsymbol{F}_p\right]^{-1} \boldsymbol{F}_s^{\mathcal{H}}, \qquad (12)$$

where $\boldsymbol{\Lambda}_p$ is a diagonal matrix given by

$$\boldsymbol{\Lambda}_p = \boldsymbol{D}_p^{\mathcal{H}} \boldsymbol{D}_p = \mathrm{diag}\left(\lambda_1, \ldots, \lambda_{N_p}\right). \tag{13}$$

On the other hand, the Least Squares (LS) estimate of $\boldsymbol{H}_s$ is found to be $\boldsymbol{F}_s \left(\boldsymbol{D}_p \boldsymbol{F}_p\right)^{\dagger} \tilde{\boldsymbol{Y}}$, where $(\cdot)^{\dagger}$ denotes the pseudo-inverse of a matrix. The LS estimate does not require any prior knowledge on channel statistics and is thus widely applicable. In contrast, the second order channel statistics $\boldsymbol{R}_h = E\{\boldsymbol{h}\boldsymbol{h}^{\mathcal{H}}\}$ and the noise variance σ_w^2 are essential to compute the MMSE estimate. When the signal to noise ratio (SNR) gets larger, i.e., σ_w^2 gets smaller for a given signal power, the MMSE estimate converges to the LS estimate. In [12], pilot symbols are numerically designed for LS channel estimation in OFDM with null subcarriers. Here, we consider MMSE channel estimation.

Let us define $\lambda_n = |p_{k_n}|^2$ for $k_n \in \mathcal{K}_p$. For a prescribed energy to be consumed for channel estimation, we normalize the sum of pilot power such that

$$\sum_{k_n \in \mathcal{K}_p} |p_{k_n}|^2 = \sum_{k=1}^{N_p} \lambda_k = 1. \tag{14}$$

Then, our objective is to find the optimal $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_{N_p}]^T$, that minimizes a criterion function. For our criterion function, we adopt the sum of the mean squared errors, i.e.,

$$\begin{aligned} & E\{||\boldsymbol{E}_s||^2\} \\ = \; & \mathrm{tr}\left\{\boldsymbol{F}_s \left[\boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\boldsymbol{F}_p^{\mathcal{H}} \boldsymbol{\Lambda}_p \boldsymbol{F}_p\right]^{-1} \boldsymbol{F}_s^{\mathcal{H}}\right\} \\ = \; & \mathrm{tr}\left\{\left[\boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\boldsymbol{F}_p^{\mathcal{H}} \boldsymbol{\Lambda}_p \boldsymbol{F}_p\right]^{-1} \boldsymbol{R}\right\}, \end{aligned} \tag{15}$$

where $||\cdot||$ denotes the Euclidean norm and

$$\boldsymbol{R} = \boldsymbol{F}_s^{\mathcal{H}} \boldsymbol{F}_s. \tag{16}$$

Thus, our problem is to determine the optimal $\boldsymbol{\lambda}$ that minimizes the MSE in (15) under the constraint (14). It should be remarked that in place of (15), other criteria, i.e., the maximum of the mean squared errors, can also be adopted.

In the long preamble of IEEE 802.11a, equi-powered pilot symbols are utilized but is not optimal in general. Equi-powered pilot symbols are investigated for channel frequency response estimation in multiple antenna OFDM system with null subcarriers [9]. In the next section, we develop a design procedure of the optimal power distribution that minimizes $E\{||\boldsymbol{E}_s||^2\}$ in (15) even in the presence of null subcarriers.

IV. Pilot Power Distribution with SDP

The optimal power distribution can be obtained by minimizing the channel MSE with respect to $\boldsymbol{\lambda}$ under the constraints that $[1, \ldots, 1]\,\boldsymbol{\lambda} = 1, \quad \boldsymbol{\lambda} \succeq 0$, where $\boldsymbol{a} \succeq 0$ (or $\boldsymbol{a} \succ 0$) for a vector signifies that all entries of $\boldsymbol{a}$ are equal to or greater than 0 (or strictly greater than 0). As stated in the previous section, analytical solutions could not be found in general. As in [12], we will resort to a numerical design by casting our minimization problem into a semidefinite programming (SDP) problem.

The SDP covers many optimization problems [10]. The objective function of SDP is a linear function of a variable $\boldsymbol{x} \in \boldsymbol{R}^M$ subject to a linear matrix inequality (LMI) defined as $F(\boldsymbol{x}) = \boldsymbol{A}_0 + \sum_{m=1}^{M} x_m \boldsymbol{A}_m \succeq 0$, where $\boldsymbol{A}_m \in \boldsymbol{R}^{M \times M}$. The complex-valued LMIs are also possible, since any complex-valued LMI can be written by the corresponding real-valued LMI. Since the constraint defined by LMI is a convex set, the global solution can be efficiently and numerically found by the existing routines.

By re-expressing the nth row of $\boldsymbol{F}_p$ as $\tilde{\boldsymbol{f}}_n^{\mathcal{H}}$, our MSE minimization problem can be stated as

$$\min_{\boldsymbol{\lambda}} \mathrm{tr}\left[\left(\boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}}\right)^{-1} \boldsymbol{R}\right] \tag{17}$$

$$\text{subject to} \quad [1, \ldots, 1]\,\boldsymbol{\lambda} \leq 1, \quad \boldsymbol{\lambda} \succeq 0. \tag{18}$$

This problem subsumes the optimization problem in [12], which is similar to the transceiver optimization problem studied in [13]. Similar to the problem in [12], [13], our problem can be transformed into an SDP form as follows.

Now let us introduce an auxiliary matrix variable $\boldsymbol{W}$ and consider the following problem:

$$\min_{\boldsymbol{W}, \boldsymbol{\lambda}} \mathrm{tr}\left(\boldsymbol{W}\boldsymbol{R}\right) \tag{19}$$

$$\text{subject to} \quad [1, \ldots, 1]\,\boldsymbol{\lambda} \leq 1, \quad \boldsymbol{\lambda} \succeq 0$$

$$\boldsymbol{W} \succeq \left(\boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}}\right)^{-1}. \tag{20}$$

It is reasonable to assume that the number of data carriers is greater than the channel length, i.e., $|K_s| > L$, so that $\boldsymbol{R} \succ 0$. If $\boldsymbol{R} \succ 0$, then we have

$$\mathrm{tr}\left(\boldsymbol{W}\boldsymbol{R}\right) \geq \mathrm{tr}\left[\left(\boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}}\right)^{-1} \boldsymbol{R}\right], \tag{21}$$

if $\boldsymbol{W} \succeq \left(\boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}}\right)^{-1}$. It follows that minimization of $\mathrm{tr}(\boldsymbol{W}\boldsymbol{R})$ is achieved if and only if $\boldsymbol{W} = \left(\boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}}\right)^{-1}$, which proves that the minimization of $\mathrm{tr}(\boldsymbol{W}\boldsymbol{R})$ in (19) is equivalent to the original minimization in (18).

The constraint (20) can be rewritten by using Schur's complement as

$$\begin{bmatrix} \boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{W} \end{bmatrix} \succeq 0. \tag{22}$$

Using this, we finally reach the following minimization problem equivalent to the original problem:

$$\min_{\boldsymbol{W}, \boldsymbol{\lambda}} \mathrm{tr}\left(\boldsymbol{W}\boldsymbol{R}\right) \tag{23}$$

$$\text{subject to} \quad [1, \ldots, 1]\,\boldsymbol{\lambda} \leq 1, \quad \boldsymbol{\lambda} \succeq 0$$

$$\begin{bmatrix} \boldsymbol{R}_h^{-1} + \frac{1}{\sigma_w^2}\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{W} \end{bmatrix} \succeq 0. \tag{24}$$

This is exactly an SDP problem where the cost function is linear in $\boldsymbol{W}$ and $\boldsymbol{\lambda}$, and the constraints are convex, since they are in the form of LMI. Thus, the globally optimal solution can be numerically found in polynomial time [10].

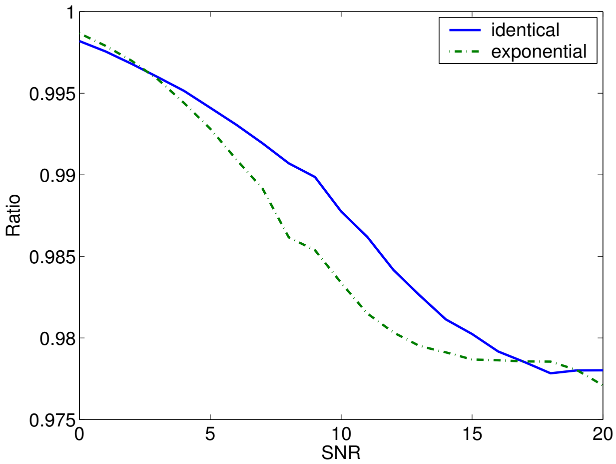

Fig. 1. Ratio of channel MSE of optimized preamble to channel MSE of conventional preamble.

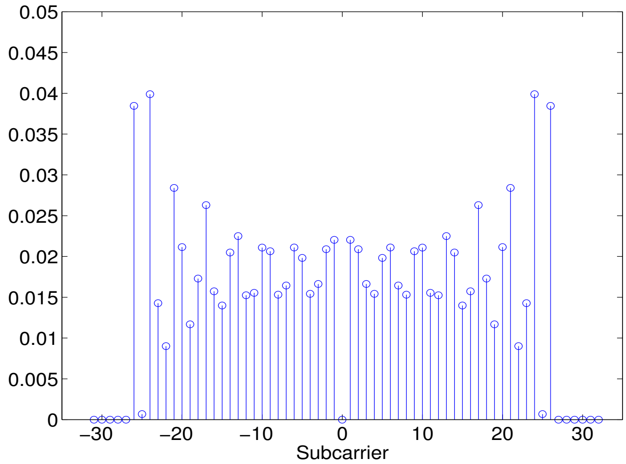

Fig. 3. Optimal $\boldsymbol{\lambda}$ for identical power profile at 20 dB.

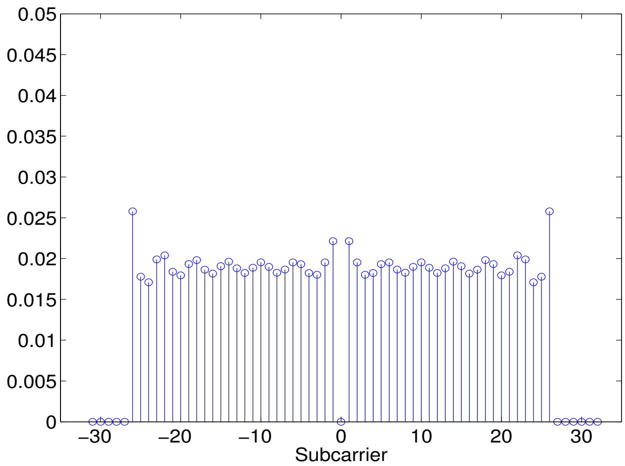

Fig. 2. Optimal $\boldsymbol{\lambda}$ for identical power profile at 0 dB.

V. Design Example

We design OFDM preamble under the same setting as IEEE 802.11a. Out of $N = 64$ subcarriers, 12 subcarriers are null (virtual subcarriers) and $\mathcal{K} = \{\pm 1, \pm 2, \dots, \pm 26\}$, where the subcarrier index $n > N/2$ is denoted as $n - N$ for convenience. The length of cyclic prefix is assigned as $N_{cp} = 16$.

We set $\mathcal{K}_s = \mathcal{K}_p = \mathcal{K}$ and define the receive SNR as

$$\frac{E\{\sum_{k \in \mathcal{K}_p} |H_k p_k|^2\}}{E\{\sum_{k \in \mathcal{K}_p} |W_k|^2\}} = \frac{\operatorname{tr} \boldsymbol{R}_h}{N_p \sigma_w^2}. \tag{25}$$

To solve our SDP problem, we utilize the LMI Control Toolbox [14] of MATLAB and numerically minimize the frequency-domain channel MSE to obtain optimal OFDM preambles for each SNR. It should be remarked that the obtained preambles are optimal within prescribed computation accuracy.

We consider Rayleigh distributed channels of length $L = 16$, having independent complex zero-mean Gaussian taps with identical power profile and with exponential power profile such as $E\{|h_l|^2\} = \exp(-l)/4$ for $l \in [0, 15]$. The channel MSE of the conventional long OFDM preamble having identical power is also computed.

The ratio of the channel MSE of the optimized preamble to the channel MSE of the conventional preamble is plotted in Fig. 1. For both channel characteristics, at low SNR, the difference between the MSEs of the optimized and the conventional preamble is small. As SNR gets higher, the advantage of the optimized preamble becomes clearer. However, the ratio is still close to 1, i.e., the difference is not so significant, which implies that the conventional preamble achieves the near-optimal channel estimation accuracy.

For the identical power profile channel, Fig. 2 and Fig. 3 depict the values of optimized λ_k on each subcarrier at 0 dB and 20 dB. At 0 dB, the λ_k's have almost identical values except at the subcarriers next to the null subcarriers, which coincides with the fact that the channel MSE of the conventional preamble is almost the same as the channel MSE of the optimized preamble as observed in Fig. 1. The subcarriers next to the null subcarriers have relatively large gains which may compensate for the null subcarriers. At 20 dB, although the difference of the channel MSEs is not so large, we notice that the optimized λ_ks have quite different values from the equi-powered pilot symbols. Some pilot symbols take large values especially near null subcarriers.

Similar results for the exponential power profile channel are obtained as shown in Fig. 4 at 0 dB and Fig. 5 at 20 dB.

VI. Conclusion

In OFDM transmissions with null subcarriers, we have formulated the minimization of channel MSE with respect to pilot symbol powers as an SDP, which enables a numerical design of pilot powers. A design example in IEEE802.11a is provided, which demonstrates that in terms of channel MSE, there exists better preambles than the conventional long OFDM preamble with equi-powered pilot symbols. We have also shown that the conventional preamble exhibits near-optimal channel estimation performance.

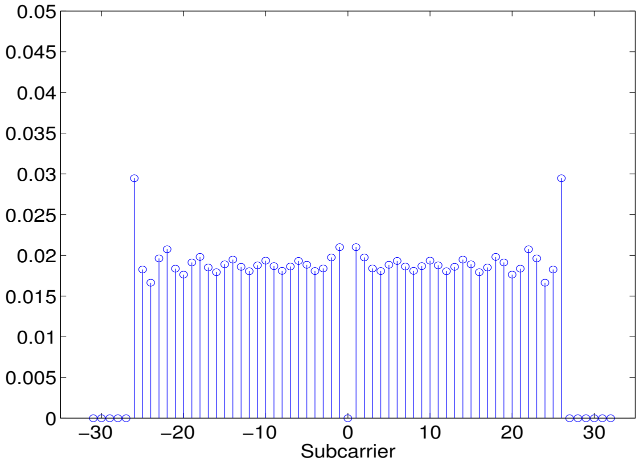

Fig. 4. Optimal $\boldsymbol{\lambda}$ for exponential power profile channel at 0 dB.

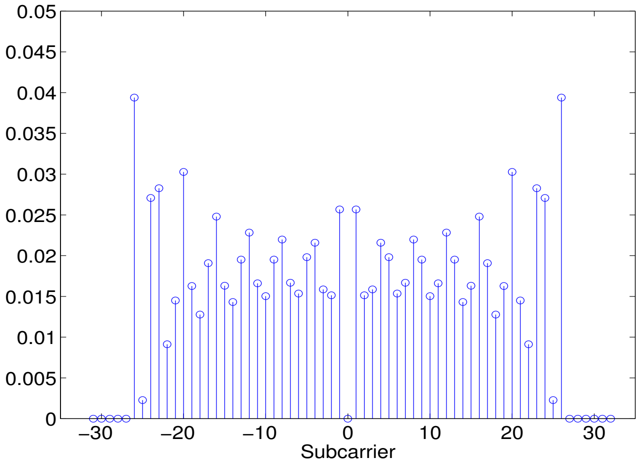

Fig. 5. Optimal $\boldsymbol{\lambda}$ for exponential power profile channel at 20 dB.

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