

# Pricing and Revenue Optimization Strategy in Macro-Femto Heterogeneous Networks

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**Abstract**—The ongoing enhancement of mobile devices capabilities and advanced technologies has largely contributed to the huge increase of wireless traffic demand. Mobile network operators around the world are struggling to manage this new era of higher generation networks by finding new means and strategies to implement in order to meet the explosion in traffic demand. One of the current and most efficient strategies to handle this issue is the femtocell technology, which now has drawn the interest of main concerned actors as for manufacturers, operators, and researchers. In this paper, our main motivation was to address the macro-femto heterogeneous networks deployment issue from the economic side. Thus, we aimed to propose a pricing strategy for macro-femto networks with a user centric vision where users would have the choice to access one of the networks based on the proposed service price when he/she is accessing it. Based on our strategy, a wireless service provider's (WSP's) revenue optimization is then performed and evaluated in order to show the efficiency of our proposed pricing scheme.

**Keywords**—macrocell, femtocell, user's utility, pricing, revenue optimization

## I. INTRODUCTION

With the ever growing gap between the amount of data demanded by users and available bandwidth, network operators are struggling to manage the data crisis within bigger networks. The witnessing of this huge growth in worldwide traffic volume and especially the big shift towards indoor mobile communications are being the major drivers behind the adoption of offloading strategies, aiming to improve global coverage and increase overall network capacity, along with providing users an efficient access way to cellular resources with specific charging mechanisms. In the last few years, the femtocell technology has gained a huge interest due to its important capacity to handle the issue of increasing in mobile traffic demand. Moreover, femto service was primary introduced to improve the quality of service within indoor environments. The main feature of femtocell concept, designed to improve wireless reception, is its capacity to overcome the problem of indoor poor coverage for end users, by being placed inside a residence or small business environment.

From a mobile network operator's perspective, deploying a femtocell network could have much more benefits, apart from improving indoor coverage. In fact, the economic incentive behind using femto service plays an important role, in the

sense of reducing overall network costs when using femto cells. Main reason for those cost saving benefits is that femto cell deployment generates much lower operational and investment costs than the macro one (regarding the small size of base stations, femto BS can be placed in any existent building, it uses the already set up wired backhaul to connect to the networks, etc.). This would lead to huge savings for mobile operators and provide cost-effective QoS for indoor networks. Assessment works for quantifying potential macro offloading benefits have mainly shown that joint deployment of macro-femto could be an effective solution for the usage improvement of macrocell network radio resources. For example, in [1], authors investigated this issue and provided deep analysis of those potential savings in macro resources, along with insights offered by the network performance assessment.

Within such a traditional heterogeneous architecture, interference characteristics are different from those of homogeneous networks. Thus, the eventual cross-tier interference, caused by an element of the femtocell tier to the macrocell one tier and vice versa, and the co-tier interference occurring between elements of the same tier, for example, between neighbouring femtocells base stations, was pointed out and has been studied now by many works [2][3], becoming a manageable issue, through some interference management techniques.

Based on the access model being used by subscribers, this kind of infrastructure could be represented under two deployment modes, according to whether the macro users are allowed to access nearby femto-BS or not. The first one is the open access mode, where all network users (both subscribed and non subscribed ones) are allowed to access the femtocell network and use its signal. This means that femto-BS can also serve nearby macro users. The second one is the closed access mode where a femto-BS just serves its indoor subscribers, as shown in Fig. 1. In this case, the femto-BS is operated, managed and secured by its owner. In some rare cases, a hybrid approach consisting of allowing a certain number of non-registered users, being nearby the femto-BS, to access and use its service, and rejecting others in the same situation, is applied in some specific situations, in order to compensate for the open and closed access drawbacks.

However, it remains an unusual situation where wireless service provider (WSP) might intend to compromise between authorized and non-authorized users, like for example in [4], where authors proposed a sophisticated access control mechanism that allows higher data rates to be guaranteed for the subscribers of the femtocells compared to other associated users in the network. A comparison between those two access modes was performed in previous works [5] and has identified the main issues and advantages for both models.

When evolving towards such heterogeneous infrastructures, the cost-efficient way for managing the network is also one of the important issues to consider, as WSP have to think about new pricing models and schemes in order to better manage their resources and perform an efficient utilization of both deployed infrastructures, along with optimizing their revenues.

In this paper, a new pricing scheme based on revenue optimization for wireless service providers is proposed within a macro-femto heterogeneous network. Our charging model follows a user-centric vision, as network subscribers would choose to connect to their desired service only if the proposed price is meeting their utilities. Based on that condition, we formulated our WSP's revenues optimization problem and proposed a mathematical model to achieve such charging strategy.

The rest of our paper is organized as follows: The related work and main interest are presented in Section 2. Our proposed charging scheme for macro-femto networks and WSP's revenues optimization model are respectively given and explained in Section 3 and Section 4. Finally, numerical results and strategy evaluation are provided in Section 5.

## II. RELATED WORK AND MAIN INTEREST

Our main motivation, while investigating the macro-femto heterogeneous network issue, is rather economic based. In fact, when considering such two-tier network model, the assessment of the technological and performance aspect that could be achieved is required, an assessment which now has been largely performed by many existent works [2], [4], [5]. However, the assessment of the economic part of the macro femto business model is also a must, especially for network operators, as one of their key challenges when considering a two-tier network is how to set up a pricing strategy reflecting the specific network architecture, by providing a certain QoS level combined with an effective charging scheme.

Several models for investigating the pricing issue in heterogeneous macro-femto networks were considered. In [6], an economic framework based on the correlation between optimal pricing and bandwidth allocation for analysis of femtocells adoption was provided. In [7], a multi-tier network with arbitrary number of services was considered, where authors formalized the joint pricing and bandwidth allocation problem as a Stackelberg game. An analytical framework was proposed in [8] to study the business and economic aspects of femtocell services based on a game theoretic model between the operator and users, where flat and partial volume pricing were considered to investigate whether it

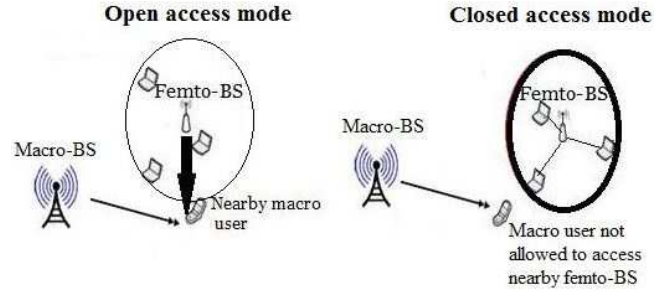


Fig. 1. Closed access mode and open access mode architectures in macro-femto heterogeneous networks.

is beneficial to make femtocells open to guest users. In [9], the framework for revenue optimization within two-tier macro-femto network was extended in order to consider not only pricing but also spectrum allocation strategy, where authors formulated theoretical results for joint optimal pricing and spectrum allocation. The issue of network revenue and resources optimization was also rigorously investigated in [10] where authors studied the economic incentive for cellular operators to introduce the femto service on top of the existing macro one. Here, other network factors were taken into consideration such as users' reservation payoffs, operational costs and femtocell frequency reuse.

Authors in [11] provided a femto business model with different flat-based proposed charging schemes related to different deployment scenarios, with a conclusion that macro-femto pricing still remains an open issue, being closely related to many other factors like network infrastructure and service provisioning.

In our work, we want to take into account a user-centric vision where mobile user will have the possibility to choose the available access network, either the macrocell or the femtocell, based on applicable service charge when he/she is accessing it. Based on that, an optimization problem will be formulated in order to maximize WSP's revenues considering such kind of specific architecture. Because this kind of two-tier infrastructure could be represented under two main deployment access models as explained before, we suppose that, in our case, the access control mode to the femto network is set up to be an open access one, where all customers of the network operator have the right to make use of any femtocell when being located in proximity.

Within our pricing strategy, a single WSP model is being considered as both tiers of the network are being charged by the same WSP. For an extended work, other deployment schemes for defining a charging policy could also be investigated. Based on that, our main interest is how we could set up a pricing strategy in such heterogeneous networks based on a user-centric vision.

## III. PROPOSED CHARGING MODEL

As we specified above, our charging process is performed based on user's decision making, since network subscriber

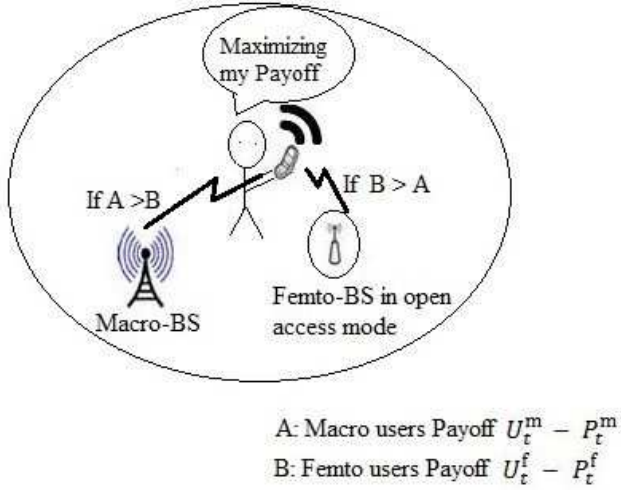


Fig. 2. Users decision making within the pricing scheme.

would choose which service to connect to, either the femto or the macro one, and have the possibility to switch between both tier, depending on the one increasing its payoff or net benefit, as shown in Fig. 2.

In order to model our pricing strategy, since we are considering a user-centric vision of charging scheme, we may consider user's utility as an indicator of their decision making. In terms of economics, utility functions are usually describing the users' level of satisfaction with the perceived quality of service.

According to that definition, the higher the utility is, the more satisfied the users would be with a given network service [12]. Some utility functions have been proposed in the literature [13], [14] in order to model the customer behaviour and evaluate pricing policies. In our strategy, we use the percentage of available system resources as indicator in order to model the user's utility among both services. In [15], authors proposed a logarithmic utility function for modelling a load balancing problem in heterogeneous networks. This logarithmic function was first proposed in [16], where the author used such kind of user's utility function related to an elastic generated traffic in the network. We propose the following utility functions for our two-tier network, respectively for femto and macro services:

$$U_t^f = \ln(b_t^f) \quad (1)$$

$$U_t^m = \ln(b_t^m) \quad (2)$$

The proposed utility functions are modelling the gradually increase of users' satisfaction with the increase of available system resources, where  $b_t^f$  and  $b_t^m$  represents femto and macro user allocated bandwidth respectively, and are set up as:  $b_t^f = \frac{B^f}{n_t^f}$  and  $b_t^m = \frac{B^m}{n_t^m}$ , where  $B^f$  and  $B^m$  are the total femto and macro-BSs allocated bandwidth,  $n_t^f$  and  $n_t^m$  are the number of femto and macro users being served at time  $t$ . Under this

scheme, a best effort service was considered, as allocated user resources directly depends on the number of users connected to each network at time slot  $t$ .

In order to simplify our scheme, we considered that all network users are being homogeneous, in the sense that they are having same kind of QoS requirements. Our main objective is how to set up an efficient pricing scheme for WSP among the two-tier network in order to optimize its total revenue.

#### IV. REVENUE OPTIMIZATION

Time is divided into multiple slots. We consider the time-of-day pricing over  $T$  time slots. Let  $P_t^f$  and  $P_t^m$  represent the prices charged by femto network and macro network at time slot  $t$ , respectively. Moreover, let  $n_t^f$  and  $n_t^m$  denote the number of users being served by femto network and macro network at time slot  $t$ , respectively. We assume that the users are rational, namely, they connect to the network if the price charged is not higher than their utility, and vice versa. Namely,

$$U_t^f = \ln\left(\frac{B^f}{n_t^f}\right) \geq P_t^f \quad (3)$$

and,

$$U_t^m = \ln\left(\frac{B^m}{n_t^m}\right) \geq P_t^m \quad (4)$$

Equation (5) represents the condition for having a stable system, where there is no incentive for users to switch between femto and macro networks.

$$U_t^f - P_t^f = U_t^m - P_t^m \quad (5)$$

Combining Eq.(3)-(5), we can obtain the relationship between the number of femto users and that of the macro users, when the system is stable.

$$n_t^f = \frac{B^f \exp(P_t^m)}{B^m \exp(P_t^f)} n_t^m \quad (6)$$

Let  $n_t$  denote the number of users that connect to either femto or macro network in slot  $t$ , we have

$$n_t = n_t^m + n_t^f \quad (7)$$

Combining Eq.(6) and (7), we have

$$n_t^m = \frac{n_t B^m \exp(P_t^f)}{B^m \exp(P_t^f) + B^f \exp(P_t^m)} \quad (8)$$

and

$$n_t^f = \frac{n_t B^f \exp(P_t^m)}{B^m \exp(P_t^f) + B^f \exp(P_t^m)} \quad (9)$$

Equation (3) and (4) could therefore be transformed to Eq.(10) and (11), respectively.

$$U_t^m = \ln\left(\frac{B^m \exp P_t^f + B^f \exp P_t^m}{n_t}\right) - P_t^f \geq P_t^m \quad (10)$$

and

$$U_t^f = \ln\left(\frac{B^m \exp P_t^f + B^f \exp P_t^m}{n_t}\right) - P_t^m \geq P_t^f \quad (11)$$

Let  $g(P_t^m, P_t^f)$  represent the net benefit when connecting to femto or macro network.

$$g(P_t^m, P_t^f) = \ln\left(\frac{B^m \exp P_t^f + B^f \exp P_t^m}{n_t}\right) - P_t^m - P_t^f \geq 0 \quad (12)$$

Our problem could be formulated to maximize the WSP's revenue as follows.

$$\begin{aligned} & \max_{\{P_t^m, P_t^f\}} \sum_{t \in T} (P_t^m \times n_t^m + P_t^f \times n_t^f) \\ & \text{subject to} \\ & g(P_t^m, P_t^f) \geq 0, \forall t \in T \end{aligned} \quad (13)$$

Let  $f(P_t^m, P_t^f)$  denote the revenue received at time slot  $t$ . We have

$$\begin{aligned} f(P_t^m, P_t^f) &= P_t^m \times n_t^m + P_t^f \times n_t^f \\ &= \frac{n_t B^m P_t^m \exp(P_t^f) + n_t B^f P_t^f \exp(P_t^m)}{B^m \exp(P_t^f) + B^f \exp(P_t^m)} \end{aligned} \quad (14)$$

We notice that Eq.(13) can be solved independently for each time slot  $t$ . Our objective is therefore to

$$\begin{aligned} & \max_{P_t^m, P_t^f} f(P_t^m, P_t^f) \\ & \text{subject to} \\ & g(P_t^m, P_t^f) \geq 0 \end{aligned} \quad (15)$$

**Proposition 1.** *The revenue received at each slot is maximized when the net benefit of femto network users and macro network users is equal to 0.*

*Proof.* Proof by contradiction is used to prove Proposition 1.

- Assume that  $g(P_t^m, P_t^f) > 0$ , then  $U_t^f > P_t^f$  and  $U_t^m > P_t^m$ . The WSP can increase the price of femto and macro network by at most  $(U_t^f - P_t^f)$  and  $(U_t^m - P_t^m)$ , respectively, without reducing the number of femto users and the number of macro users. Hence, the  $f(P_t^m, P_t^f)$  is not maximized.
- Assume that  $g(P_t^m, P_t^f) < 0$ , then  $U_t^f < P_t^f$  and  $U_t^m < P_t^m$ . This will not happen since rational users always ensure that their net benefit is non-negative before they connect to the network.

Therefore, the revenue received at each slot (i.e.,  $f(P_t^m, P_t^f)$ ) is maximized when the net benefit of femto network users and macro network users is equal to 0 (i.e.,  $g(P_t^m, P_t^f) = 0$ ).  $\square$

By using the conclusion of Proposition 1, the problem defined in Eq.(15) can be transformed to

$$\begin{aligned} & \max_{P_t^m, P_t^f} f(P_t^m, P_t^f) \\ & \text{subject to} \\ & g(P_t^m, P_t^f) = 0 \end{aligned} \quad (16)$$

The total number of users inside the femtocell and macrocell is denoted by  $N_t$ . Let  $P_t^{f*}$ ,  $P_t^{m*}$  and  $n_t^*$  denote the optimal

price for femto network, the optimal price for macro network, and the optimal number of users admitted to the femto-macro network.

**Proposition 2.** *The revenue received at slot  $t$  is maximized if and only if the following conditions are satisfied.*

$$\begin{cases} n_t^* = \min\left(\frac{B^m + B^f}{e}, N_t\right) \\ P_t^{f*} = P_t^{m*} = \max\left(1, \ln\left(\frac{B^m + B^f}{N_t}\right)\right) \end{cases}$$

*Proof.* The corresponding Lagrangian form of Eq.(17) is

$$L(P_t^m, P_t^f, \lambda) = f(P_t^m, P_t^f) + \lambda g(P_t^m, P_t^f) \quad (17)$$

where  $\lambda^m$  and  $\lambda^f$  are the Lagrange multipliers.

The optimal solution could be obtained by using the following conditions.

$$\begin{cases} \frac{\partial L(P_t^m, P_t^f, \lambda)}{\partial P_t^m} = 0 \\ \frac{\partial L(P_t^m, P_t^f, \lambda)}{\partial P_t^f} = 0 \\ \frac{\partial L(P_t^m, P_t^f, \lambda)}{\partial \lambda} = 0 \end{cases} \quad (18)$$

By solving Eq.(18), we have

$$(P_t^f - P_t^m) (B^f \exp P_t^m + B^m \exp P_t^f) = 0 \quad (19)$$

Since  $B^f \exp P_t^m + B^m \exp P_t^f > 0$ , we have

$$P_t^f = P_t^m = \ln\left(\frac{B^m + B^f}{n_t}\right) \quad (20)$$

Eq.(8) and (9) can be transformed to Eq.(21) and(22), respectively.

$$n_t^m = \frac{n_t B^m}{B^m + B^f} \quad (21)$$

and

$$n_t^f = \frac{n_t B^f}{B^m + B^f} \quad (22)$$

When the capacity of femtocell and macrocell is not enough to accommodate  $N_t$  users, the WSP can control the total number of users admitted to its femto or macro network (i.e.,  $n_t \leq N_t$ ). The revenue received at slot  $t$  that varies as a function of  $n_t$  is given as follows.

$$\begin{aligned} h(n_t) &= P_t^m \times n_t^m + P_t^f \times n_t^f \\ &= n_t \ln\left(\frac{B^m + B^f}{n_t}\right) \end{aligned} \quad (23)$$

Let  $\frac{\partial h(n_t)}{\partial n_t} = 0$ , we have

$$n_t = \frac{B^m + B^f}{e} \quad (24)$$

Furthermore, if taking the second derivative of  $h(n_t)$  with respect to  $h(n_t)$ , we get  $h''(n_t) = -\frac{1}{n} < 0$ , which suggests

that the object function  $h(n_t)$  is concave down. Thus, the optimal number of users admitted  $n_t^*$  is derived as follows.

$$n_t^* = \min\left(\frac{B^m + B^f}{e}, N_t\right) \quad (25)$$

The optimal price for femto network and macro network is

$$\begin{aligned} P_t^{f*} = P_t^{m*} &= \ln\left(\frac{B^m + B^f}{n_t^*}\right) \\ &= \max\left(1, \ln\left(\frac{B^m + B^f}{N_t}\right)\right) \end{aligned} \quad (26)$$

□

The total number of users that could be admitted to femtocell and macrocell (i.e., total network capacity in terms of number of users) is  $\frac{B^m + B^f}{e}$ . The Proposition 2 indicates that

- When the number of potential users  $N_t$  (i.e., total demand) is lower than  $\frac{B^m + B^f}{e}$  (i.e., total capacity), then all of the potential users could be admitted to connect; on the other hand, later-coming users will be rejected when the number of existing users reaches  $\frac{B^m + B^f}{e}$ .
- When the total demand is lower than the total capacity, then a fixed-rate pricing scheme (i.e.,  $p_t^m = p_t^f = 1$ ) is optimal to maximize the total revenue; on the other hand, the optimal price of time slot  $t$  varies as a convex and decreasing function of the total demand  $N_t$  (i.e.,  $p_t^m = p_t^f = \ln\left(\frac{B^m + B^f}{N_t}\right)$ ).

## V. NUMERICAL ANALYSIS

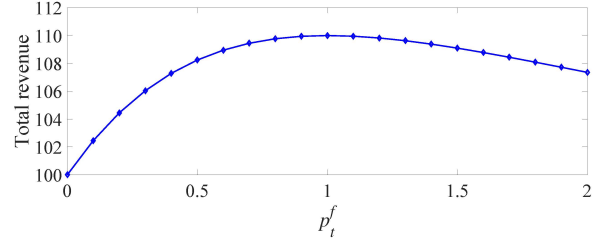
In order to show the efficiency of our proposed pricing scheme in terms of optimizing the total revenue, we set the total capacity of macro network and femto network to be  $100e$  (mbps) and  $10e$  (mbps), respectively. According to Proposition 2, the optimal number of users that could be admitted is  $\min(110, N_t)$ , and the optimal price of both macro and femto network is  $\max\left(1, \ln\left(\frac{110e}{N_t}\right)\right)$ .

We first assume that the price of macro network is charged according to the optimal price, while the price of femto network varies in the range of  $[0, 2]$ . We examine whether setting the price of femto network according to  $\max\left(1, \ln\left(\frac{110e}{N_t}\right)\right)$  is optimal to maximize the total revenue.

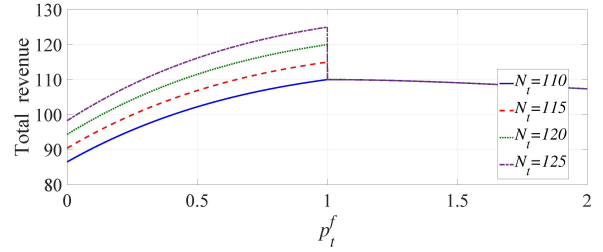
- When  $N_t \geq 100 + 10e$ , the optimal price of macro network is 1 according to Eq.(26) and the total demand is enough to let both the macro and femto network become saturated (i.e., the payoff of macro and femto users is equal to zero) even if the price of femto network is set to zero. As shown in Fig.3 (a), the total revenue stay unchanged with the total demand and the optimal total revenue is achieved when the price of femto network is equal to 1.
- When  $110 \leq N_t < 100 + 10e$ , the total demand is enough to let both the macro and femto network become saturated only if the price of femto network is higher than 1. As shown in Fig.3 (b), the total revenue varies with the total

demand and the optimal total revenue is achieved when the price of femto network is equal to 1.

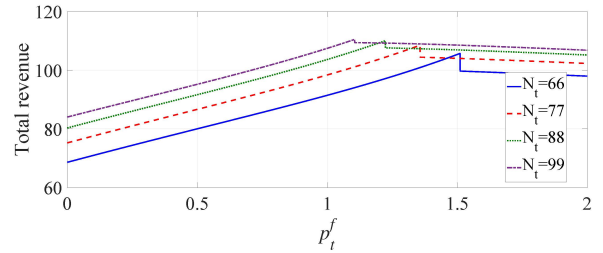
- When  $N_t < 110$ , the total demand is enough to let both the macro and femto network become saturated only if the price of femto network is higher than  $\ln\left(\frac{110e}{N_t}\right)$ . As shown in Fig.3 (c), the total revenue varies with the total demand and the optimal total revenue is achieved when the price of femto network is equal to  $\ln\left(\frac{110e}{N_t}\right)$ .



(a)  $N_t \geq 100 + 10e$



(b)  $110 \leq N_t < 100 + 10e$



(c)  $N_t < 110$

Fig. 3. Revenue vs. price charged by femto network.

Similarly, if the price of femto network is charged according to the optimal price, while the price of macro network varies in the range of  $[0, 2]$ , numerical analysis results also indicate that setting the price of macro network to  $\max\left(1, \ln\left(\frac{110e}{N_t}\right)\right)$  is optimal to maximize the total revenue. The corresponding results are omitted due to the space limitation.

Figure 4 shows the revenue varies as a function of  $p_t^m$  and  $p_t^f$  when  $N_t \geq 100 + 10e$ . Specifically, revenue is maximized when  $p_t^m = p_t^f = 1$ .

Furthermore, Fig.5 shows the relationship between the total revenue and the number of users admitted to connect when  $N_t \geq 100 + 10e$ . According to Fig.6, the revenue is maximized when the number of users admitted to connect reaches 110, which is exactly the capacity of macro and femto network.

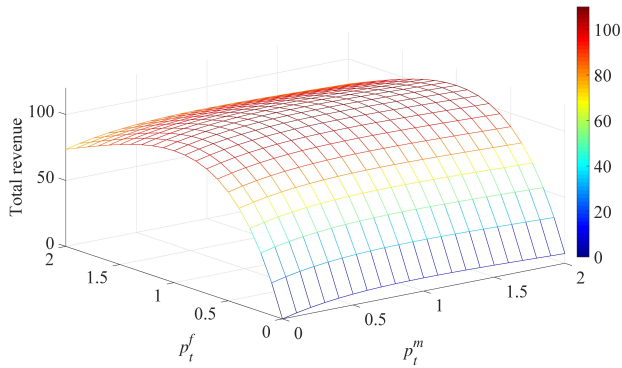


Fig. 4. Revenue vs.  $p_t^m$  vs.  $p_t^f$  ( $N_t \geq 100 + 10e$ ).

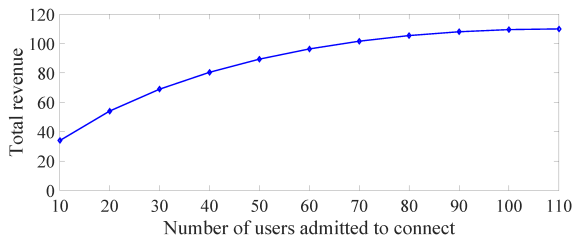


Fig. 5. Revenue vs. number of users admitted to connect ( $N_t \geq 100 + 10e$ ).

## VI. CONCLUSION

In this paper, we introduced a WSP charging scheme among two-tier macro femto heterogeneous network in order to perform a revenue optimization strategy. Our pricing mechanism is based on a user centric vision where network subscribers choose to connect to the femto or macro services if the proposed price is lower or equal to their utility. Besides, we defined a logarithmic users' utility function based on available network resources, assuming that users in our model are being homogeneous.

Based on that, we showed the efficiency of our proposed scheme in terms of optimizing total revenues. In fact, our numerical analysis results has shown that setting the price of macro or femto service according to our proposed optimal pricing model results in maximizing the WSP total revenues.

As future work, our pricing scheme could be applied on different deployed infrastructures and other network users' behaviour and charging scenarios.

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