The Restoring Casimir Force between Doped Silicon Slab and Metamaterials

Xue-Wei Li[a], Zhi-Xiang Huang[a*], Xian-Liang Wu[a,b*]
[a]Key Laboratory of Intelligent Computing and Signal Processing, Anhui University, Hefei 230039,China
[b]School of Electronic and Information Engineering, Hefei Normal University, Hefei 230061,China
*Email: zxhuang@ahu.edu.cn,xlwu@ahu.edu.cn

***Abstract*-Casimir effect is an observable macroscopic quantum effect. It has significant influence on micro-machine and nano-machine. Based on a generalization of the Lifshitz theory, we calculate Casimir force between the doped silicon slab and metamaterials. From simulating, we can see that the magnitude and the direction of the Casimir force can be changed by varying doping level, the thickness of doped silicon slab, and filling factor of metamaterials. Thus the force can be controlled by tuning these parameters. It can be restoring force in a certain range of distance, which can provide a method to deal with the stability problem, and bring much meaningful results from the practical views.**

***Key words:* Repulsive and Restoring Casimir Force; Doped Silicon; Metamaterials**

I. INTRODUCTION

Casimir effect results from the change of zero-point energy due to the existence of the boundaries and may be observed as a vacuum force. In 1948, this effect, the attractive interaction of a pair of neutral perfectly conducting parallel plates placed in the vacuum, was first proposed and theoretically derived by Casimir [1]. In recent years, the Casimir effect has attracted considerable attention. Especially, with the development of nanotechnology, Casimir force has a significant impact on the micro-electromechanical systems (MEMS) and nano-electromechanical (NEMS) [2-4]. It can be applied to dynamic devices, but also can be lead to stiction problem in MEMS and NEMS. Therefore, Casimir force has become a current research hot spot.

Boyer pointed out a repulsive Casimir force may rise between a perfect conductor plate and a material plate with infinite permeability [5]. But the material in the nature usually has not strong magnetic response in near-infrared and optical spectrum. However, with the development of artificial metamaterials, strong magnetic response material can be obtained, which provides the conditions to get a repulsive Casimir force. Various calculation techniques have been developed, and Lifshitz theory coincides well with the experimental results [6, 7]. Therefore, based on Lifshitz theory, we calculate the Casimir forces between doped silicon slab and metamaterials in vacuum, and study the changes of Casimir forces due to the changes of the doping level, the thickness of slab, and the filling factor of metamaterials.

II. THE INTERACTION BETWEEN DOPED SILICON SLAB AND METAMATERIALS

Let us consider the configuration with an infinite doped silicon slab and thick metamaterials plate separated by a distance d, in the vacuum. The thickness of doped silicon slab is D. Based on Lifshitz theory, the resulting Casimir force per unit area A is eventually expressed as [8, 9]

$$\frac{F(d)}{A}=\frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\xi\int_0^\infty k_0 k_\parallel \mathrm{d}k_\parallel\times\sum_\alpha\frac{r_\alpha^{(1)}r_\alpha^{(2)}}{e^{2k_0 d}-r_\alpha^{(1)}r_\alpha^{(2)}}\ , \quad (1)$$

where $\boldsymbol{k}_\parallel$ is the component of the wave vector parallel to the slab surface, and r_α (α = TE,TM) is the reflection coefficient for TE- or TM-polarized waves. In $r_\alpha^{(j)}$, superscripts j=1, 2 stands for the doped silicon slab and metamaterials, respectively. $k_0=\sqrt{k_\parallel^2+\xi^2/c^2}$, c is the speed of light in the vacuum.

The reflection coefficients for p-doped silicon are gave as

$$r_{\mathrm{TM}}^{(1)}=\left(\varepsilon_1 k_0-\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2}\right)\Big/\left(\varepsilon_1 k_0+\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2}\right), \quad (2a)$$

$$r_{\mathrm{TE}}^{(1)}=\left(k_0-\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2}\right)\Big/\left(k_0+\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2}\right). \quad (2b)$$

Here we characterize the electric response of doped silicon as [10]

$$\varepsilon_1(\mathrm{i}\xi)=\varepsilon_{si}(\mathrm{i}\xi)+\frac{\omega_p^2}{\xi^2+\gamma\xi}\ , \quad (3)$$

$$\varepsilon_{si}(\mathrm{i}\xi)=\varepsilon_\infty+\frac{(\varepsilon_0-\varepsilon_\infty)\omega_{si}^2}{\xi^2+\omega_{si}^2}\ , \quad (4)$$

$$\omega_p=\sqrt{\frac{Ne^2}{\varepsilon_0 m^*}}=\frac{2\pi c}{\lambda_p},\gamma=\frac{Ne^2\rho}{m^*}\ , \quad (5)$$

where ε_{si} is the dielectric function of intrinsic silicon, and $\varepsilon_\infty=1.035$, $\varepsilon_0=11.87$, $\omega_{Si}=6.6\times10^{15}\,rad/s$. $m^*=0.34m_e$ is the effective mass of the charge carrier in the silicon crystal, and m_e is the electron mass. ρ is the resistivity of silicon, and N corresponds to the doping level. The different values of the plasma frequency ω_p and the relaxation ratio γ for various densities are given in Table 1 [10].

Considering the role of the thickness D of the doped silicon slab on the Casimir interaction, the reflection coefficients $r_{\mathrm{TM}}^{(1)}$ and $r_{\mathrm{TE}}^{(1)}$ in Eq. (2a) and Eq. (2b) become [9, 11]

Table 1 the plasma frequency ω_p and relaxation ratio γ for various doping levels

N/cm^{-3}	ω_p / $\times10^{12}$ rad / s	γ / $\times10^{13}$ s^{-1}	ρ / Ωcm
1.1×10^{15}	3.1899	1.18482	13
1.3×10^{18}	110.1275	3.75193	3.5×10^{-2}
1.4×10^{19}	361.522	7.86842	6.8×10^{-3}
1.0×10^{20}	966.084	9.917551	1.2×10^{-3}

$$r_{TM}^{(1)} \to r_{TM}^{(1)} \frac{1-e^{-2\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2}D}}{1-r_{TM}^{(1)2}e^{-2\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2}D}}, \quad (6a)$$

$$r_{TE}^{(1)} \to r_{TE}^{(1)} \frac{1-e^{-2\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2}D}}{1-r_{TE}^{(1)2}e^{-2\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2}D}}. \quad (6b)$$

The reflection coefficient for metamaterials are gave as [8]

$$r_{\mathrm{TM}}^{(2)} = \left(\varepsilon_2 k_0 - \sqrt{k_\parallel^2+\varepsilon_2\xi^2/c^2}\right)\Big/\left(\varepsilon_2 k_0 + \sqrt{k_\parallel^2+\varepsilon_2\xi^2/c^2}\right), \quad (7a)$$

$$r_{\mathrm{TE}}^{(2)} = \left(\mu_2 k_0 - \sqrt{k_\parallel^2+\mu_2\varepsilon_2\xi^2/c^2}\right)\Big/\left(\mu_2 k_0 + \sqrt{k_\parallel^2+\mu_2\varepsilon_2\xi^2/c^2}\right). \quad (7b)$$

For metamaterials with partly metallic, we assume that the dielectric function has a Drude background response in addition to the resonant part. Here the optical parameters of metallic-based metamaterials are characterized by

$$\varepsilon_2(i\xi) = 1 + f\frac{\Omega_D}{\xi^2+\gamma_D\xi} + (1-f)\frac{\Omega_e}{\xi^2+\omega_e^2+\gamma_e\xi}, \quad (8a)$$

$$\mu(i\xi) = 1 + \frac{\Omega_m^2}{\omega_m^2+\xi^2+\gamma_m\xi}, \quad (8b)$$

where Ω_e and Ω_m are plasma frequency, ω_e (ω_m) is the electric (magnetic) resonance frequency, and γ_e (γ_m) is the metamaterials electric (magnetic) dissipation parameter. Ω_D and γ_D are the Drude parameters of this metallic structure. f is the filling factor that accounts for the fraction of structure contained in the metamaterials. We choose $\omega_0 = 10^{14}\,\mathrm{rad/s}$. For metamaterials with silver material, the corresponding parameters are: $\Omega_e = 0.04\Omega_D$, $\Omega_m = 0.1\Omega_D$, $\omega_e = \omega_m = 0.1\Omega_D$, $\gamma_D = 0.002\Omega_D$, $\gamma_e = \gamma_m = 0.005\Omega_D$, $\Omega_D = 137.036\omega_0$.

III. THE SIMULATION AND THE RESULT ANALYSIS

We first consider the relative Casimir force $10^3 Fd^3 / AhcK_0$ between metamaterials and doped silicon slab with different thickness. We choose the doping level $N = 1.3\times10^{18}\,\mathrm{cm}^{-3}$. As is shown in the Fig.1, we can see that there is not much change when the silicon thickness $D > 0.005\lambda_0$. The reason is that if $D > 0.005\lambda_0$, so $\exp(-2\sqrt{k_\parallel^2+\varepsilon_1\xi^2/c^2 D}) \ll 1$. The influence of thickness on this interaction disappears. It can be also explained with the skin effect of electromagnetic. The skin depth of the material is $\delta = \sqrt{2/\omega\mu\sigma}$. When $D \gg \delta$, the thickness of material almost doesn't affect the transmission and reflection of electromagnetic field. Therefore, the influence of thickness on this interaction disappears.

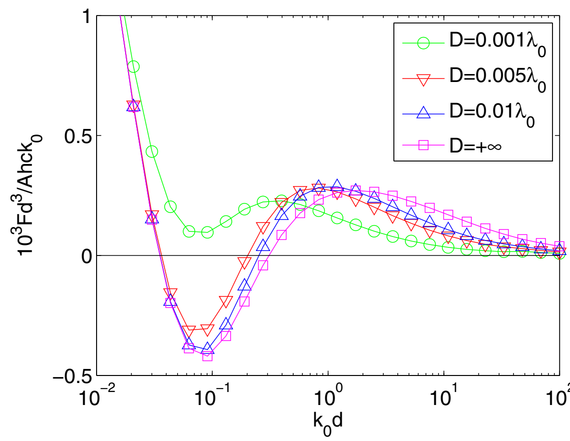

Figure 1 the Casimir interaction between doped silicon slab ($N = 1.3\times10^{18}\,\mathrm{cm}^{-3}$) and metamaterials ($f = 10^{-5}$)

The dependence of the relative force on N is illustrated in Fig.2 by varying D. We can see that the doped level has a significant impact on the Casimir effect when $D < 0.005\lambda_0$. The higher doping level, the greater the attractive force and the repulsive force is smaller; and the position of the equilibrium point that the Casimir force is zero will be different if the doping level is different. Thereby we can control the strength of the force and the equilibrium point by changing the doping level. When $D \geq 0.01\lambda_0$, the doping level almost does not affect the interaction, but the restoring force is significant in the range of 150 nm —30 μm .

The dependence of relative Casimir forces on the filling factor of metamaterials is shown in Fig.3. The filling factor affects the direction of the force. The smaller the filling factor, the greater the repulsive Casimir force. When the filling factor increases, that is, the proportion of metal structure in metamaterials increases, the Casimir interaction is only attractive.

IV. THE CONCLUSIONS

Based on the Lifshitz theory, we have studied the Casimir forces between doped silicon slab and metamaterials. It is found that the direction and the magnitude of the force are related to the doping level, the thickness and the filling factor. When the thickness of silicon slab is very small, we can control the position of equilibrium point by changing the doping level. The smaller the filling factor and the greater the repulsive force; and there exists the restoring force, which may provide a method to deal with the stiction problems.

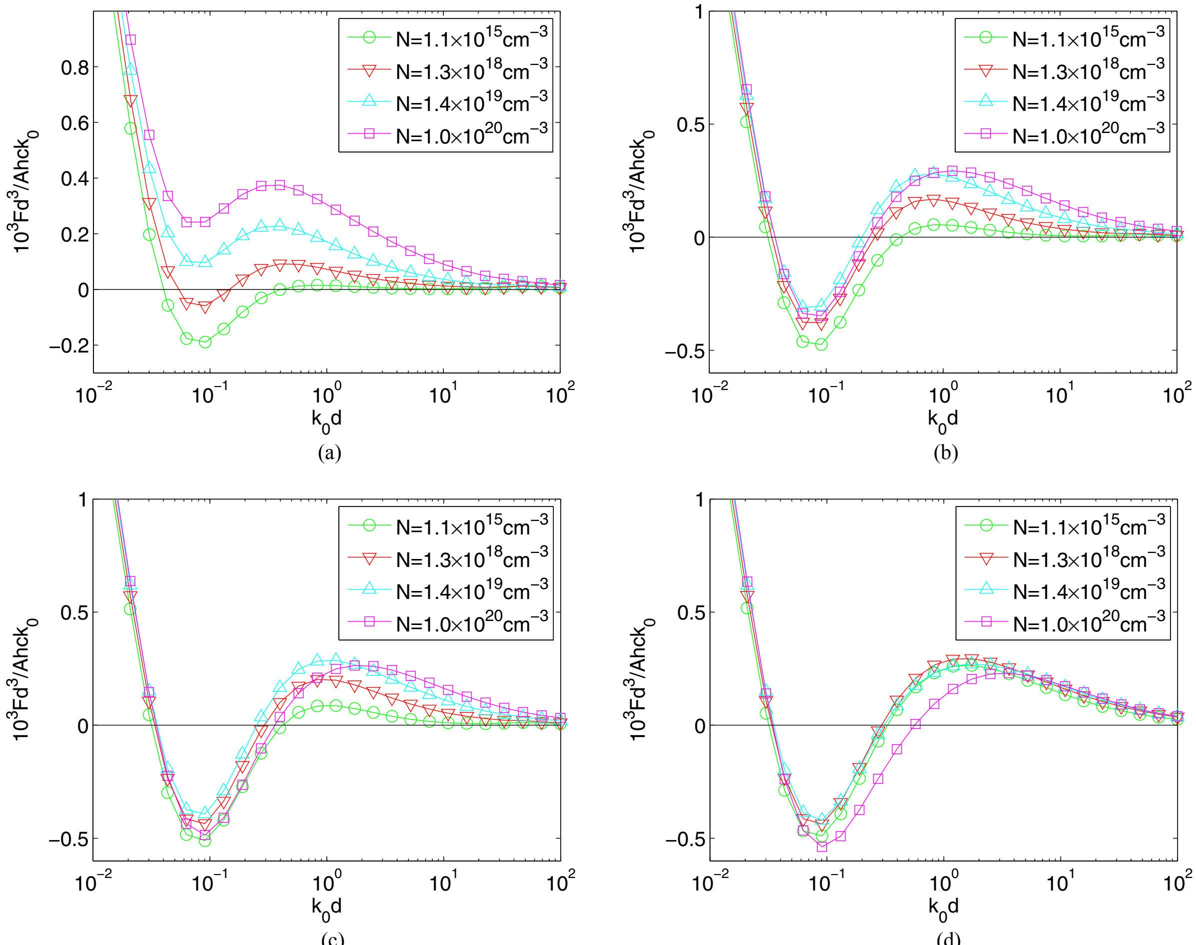

Figure 2 the relative Casimir forces between silicon slab and metamaterials ($f = 10^{-5}$)
(a) $D = 0.001\lambda_0$, (b) $D = 0.005\lambda_0$, (c) $D = 0.01\lambda_0$, (d) $D = +\infty$

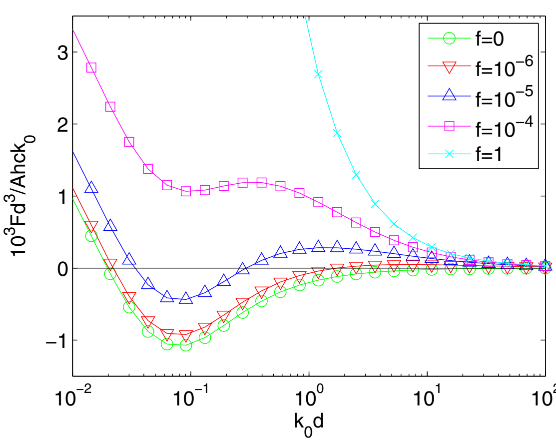

Figure 3 the relative Casimir forces between the doped silicon slab ($N = 1.3\times10^{18}$ cm^{-3} , $D = 0.1\lambda_0$) and metamaterials

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant (Nos. 60931002, 61101064, 51277001, 61201122), DFMEC (No.20123401110009) and NCET (NCET-12-0596) of China, Distinguished Natural Science Foundation (No.1108085J01), and Universities Natural Science Foundation of Anhui Province (No. KJ2011A002), and the 211 Project of Anhui University.

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