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Basic Dynamics of Elementary Cellular Automata with Mixed Rules: Periodic Patterns and Transient Phenomena

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Abstract—This paper studies the mixed rules cellular automata and its basic learning algorithm. The system can exhibit a variety of spatiotemporal patterns. The purpose of learning algorithm is storage of one periodic teacher signal and control of its stability. The genetic algorithm is used in the learning where the chromosomes correspond to local rules and the fitness corresponds to local stability. As a typical example of the teacher signal, we consider a periodic control signal of the ac-dc converter. As parameters are selected suitably, the teacher signal can be stored and can be stabilized.

1. Introduction

The cellular automata (CAs) are digital dynamical systems in which time, space and state are all discrete [1]. The time evolution of the state variable is governed by a simple rule. Depending on the rule and initial condition, the CAs can exhibit a variety of spatiotemporal phenomena [2]. Real/potential engineering applications are many, including signal processing, information compression, image processing and self-replication [3]-[6]. Analysis of the CAs are important not only as basic nonlinear problems but also for applications.

This paper studies the cellular automaton with mixed rules (MCA). The MCA includes the standard cellular automaton (SCA) and can exhibit much richer dynamics than that of the CAs. In order to visualize the dynamics of the MCA, we integrate the dynamics into the digital return map (Dmap) from a set of lattice points to itself. The Dmap can be regarded as a digital version of one-dimensional map represented by the logistic map.

As a teacher signal for the MCA, we introduce a periodic spatiotemporal pattern corresponding to control signal of a basic ac-dc converter (a typical circuit in power electronics [7][8]). We present a novel learning algorithm for storage of the teacher signal and enlargement of the domain of attraction. In the learning, we assume that the teacher signal can be stored by a subset of the mixed rules. In order to enlarge the domain of attraction, we apply the genetic algorithm (GA) in which the chromosomes correspond to undecided part of the mixed rules and the fitness corresponds to the local stability of the teacher signal. As parameters are selected suitably, the domain of attraction can be enlarged. The dynamics in the learning process is analyzed by the Dmap.

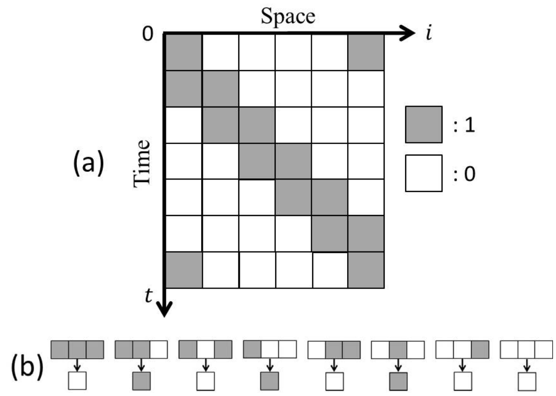

Figure 1: (a) A spatiotemporal pattern by MCA (b) An example of rule tables for $i = 2$

2. Cellular Automata with Mixed Rules

First, we defined the MCA that can exhibit huge variety of spatiotemporal patterns. MCA have binary state variable of N cells arranged on a ring-type. The dynamics is described by

$$x_i^{t+1} = F_i(x_{i-1}^t, x_i^t, x_{i+1}^t) \quad (1)$$

$i = i \sim N$, $i + N \equiv i$. The MCA is given by a mapping from the N-dimensional binary space B^N to itself:

$$\boldsymbol{x}^{t+1} = F_D(\boldsymbol{x}^t),\ \boldsymbol{x} \in \boldsymbol{B}^N \quad (2)$$

If we use decimal expression of all the elements of B^N, the B^N is equivalent to the set of rational numbers. For example, if $N = 4$, we have

$$I_D = \{C_1, C_2, \cdots, C_{2^4}\},\ C_j = j/2^4.$$

That is, F_D is equivalent to the mapping from I_D to itself. This is the digital return map (Dmap). We then give basic definition of the Dmap.

Definition 1 (Steady state): A point $p \in I_D$ is said to be a periodic point (PEP) with period k if $p = F_D^k(p)$

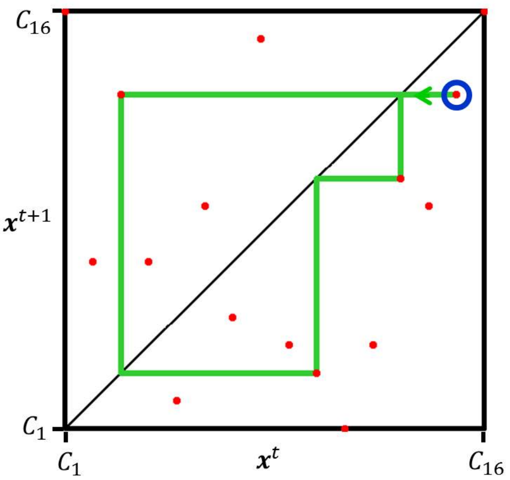

Figure 2: Digital return map (The green orbit is the PEO with period 3 and the blue circle is the DEPP to the PEO.)

and $p \neq F_D^l(p)$ for $0 < l < k$ where F_D^k is the k-fold composition of F_D. A sequence of the periodic points $\{F_D(p), F_D^2(p), \cdots, F_D^k(p)\}$ is said to be a periodic orbit (PEO). Figure 2 illustrates a PEO with period 3.

Definition 2 (Transient state): A point $q \in I_D$ is said to be a eventually periodic point (EPP) if q is not a periodic point and there exists some positive integer m such that $F_D^m(q)$ is a periodic point. In a special case, an EPP with $m = 1$ is referred to as the direct eventually periodic point (DEPP). The DEPP can characterize the domain of attraction by one step transition. Figure 2 illustrates a DEPP to the PEO with period 3.

3. Learning Algorithm

Here we define the learning algorithm based on the GA. The teacher signal is one PEO. The purpose of the learning is storage of the teacher signal PEO (TPEO) and enlargement of its domain of attraction. For simplicity, we assume that the TPEO can be stored by a subset of the mixed rules. The undecided parts of the mixed rules are referred to as the undecided rules (URs). The algorithm is defined as the following.

First, we decide the rules that can store the TPEO. For example, the spatiotemporal pattern for $i = 2$ in Fig. 1 (a), $F_2(1,1,1)$, $F_2(1,0,1)$ and $F_2(0,1,0)$ are the URs.

Step 1: Initialization

In the GA, the chromosomes correspond to the set of URs. Initial chromosomes of population size K is generated randomly. Let initial generation be $g = 0$.

Step 2: Evaluation

In order to update the chromosome, we define the fitness

$$f_c = \frac{\#\text{DEPP of TPEO}}{2^N - (\#\text{PEP of TPEO})} \quad (3)$$

It can be a measure of the local domain of attraction.

Step 3: GA operation

Preserve some chromosomes having the best fitness to the next generation. Apply the one-point crossover with probability P_c and mutation with probability P_m. Chromosomes are selected by the elite strategy.

Step 4: Termination

Let $g \to g + 1$, go to Step 2 and repeat until the maximum generation G.

4. Numerical Experiments

We propose an application of the MCA to the power electronics. A typical teacher signal is shown in Table 1. This is a periodic spatiotemporal pattern corresponding to a switching signal of a basic ac-dc converter. Figure 3 shows the circuit model of ac-dc converter and the control signals of the 6 switches. The three-phase line voltages v_a, v_b, v_c are converted into the dc voltage via the 6 switches. The switches S_1, S_3 and S_5 operate in inverse phase to the switches S_4, S_6, S_2 respectively with period $T(\omega = 2\pi/T)$. Dividing the time axis by 6 (time unit is $\omega T/6$), the switching signal can be expressed by the 6-dimensional PEO with period 6 as shown in Table 1 where $\boldsymbol{z}^t = (\boldsymbol{z}^1 \cdots \boldsymbol{z}^6)$ corresponds to the state of S_1 to S_6. For example, "$\boldsymbol{z}_1^4 = 0$" and "$\boldsymbol{z}_4^4 = 1$" respectively mean "$S_1 =$ off" and "$S_4 =$ on" at time unit 4.

The rule tables are shown in Table 2. Table 2 corresponds to the teacher signal in Table 1. We find out that #all of the URs in Table 2 is 18. The UR has "0" or "1" , so that the combination of rule tables is 2^{18}. In the GA of this paper, let the combination of the rule tables be changed.

We have fixed parameters of GA: population size $K = 20$, probability of crossover $P_c = 0.9$, probability of mutation $P_m = 0.1$ and maximum generation $G = 50$. Dmaps in Figs. 5 (a), (b) and (c) show that the teacher signal can be stored. The TPEO can be stored at $g = 0$ and the storage is preserved until the maximum generation. #DEPP increases as the generation grows.

We then show a result of basic experiments in Fig. 6. Figure 6 shows the maximum fitness and the average fitness. For $g \geq 29$, the maximum fitness is not updated. In this numerical experiment, the maximum fitness and the average fitness increases and the local domain of attraction is enlarged. That is, this algorithm can enforce the local stability.

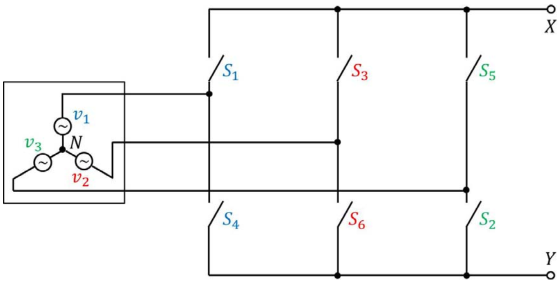

Figure 3: The three-phase ac-dc converter

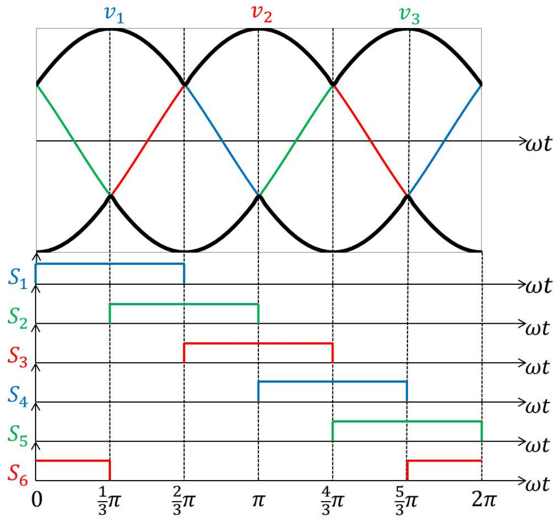

Figure 4: The input/output waveform of the ac-dc converter and the switching control signal

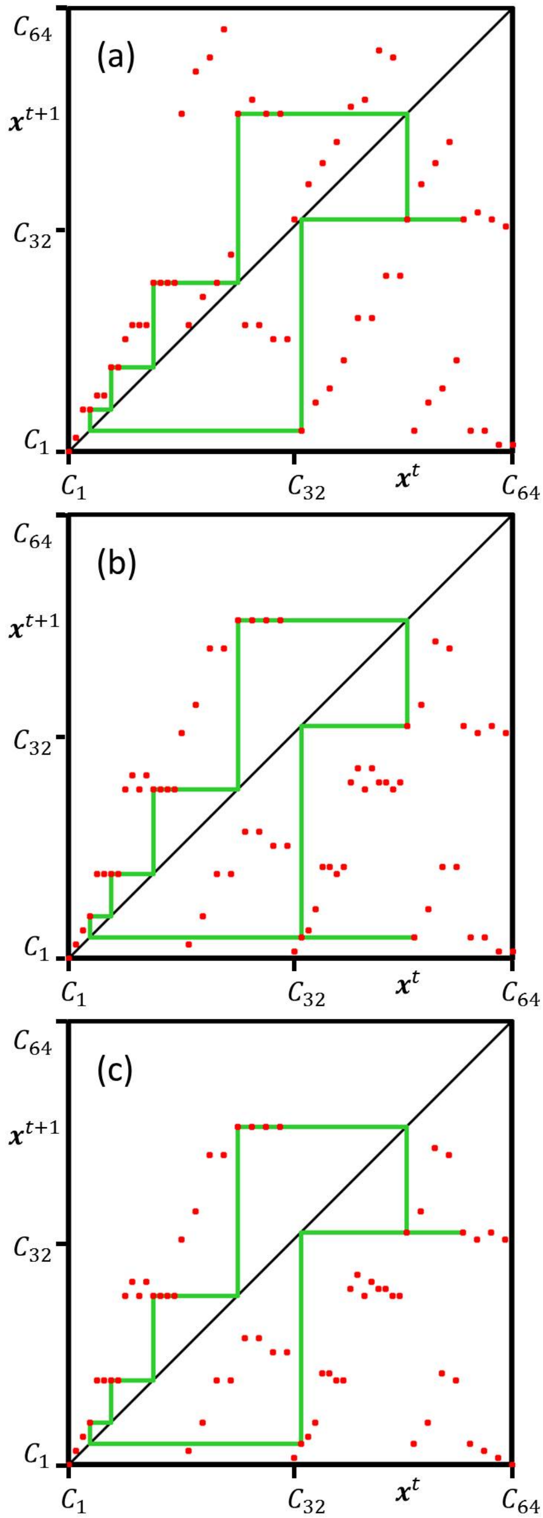

Figure 5: Dmaps corresponding to Figure 6
(a) $g = 0$; #DEPP of TPEO = 15 ($f_c = 0.26$) (b) $g = 25$; #DEPP of TPEO = 22 ($f_c = 0.38$) (c) $g = 50$; #DEPP of TPEO = 26 ($f_c = 0.45$)

Table 1: Teacher signal

$\boldsymbol{z}^1$	$(1, 0, 0, 0, 0, 1)$
$\boldsymbol{z}^2$	$(1, 1, 0, 0, 0, 0)$
$\boldsymbol{z}^3$	$(0, 1, 1, 0, 0, 0)$
$\boldsymbol{z}^4$	$(0, 0, 1, 1, 0, 0)$
$\boldsymbol{z}^5$	$(0, 0, 0, 1, 1, 0)$
$\boldsymbol{z}^6$	$(0, 0, 0, 0, 1, 1)$
$\boldsymbol{z}^7 = \boldsymbol{z}^1$	$(1, 0, 0, 0, 0, 1)$

Table 2: Rule tables corresponding to Table 1

	111	110	101	100	011	010	001	000
$i = 1$	UR	1	UR	1	0	UR	0	0
$i = 2$	UR	1	UR	1	0	UR	0	0
$i = 3$	UR	1	UR	1	0	UR	0	0
$i = 4$	UR	1	UR	1	0	UR	0	0
$i = 5$	UR	1	UR	1	0	UR	0	0
$i = 6$	UR	1	UR	1	0	UR	0	0

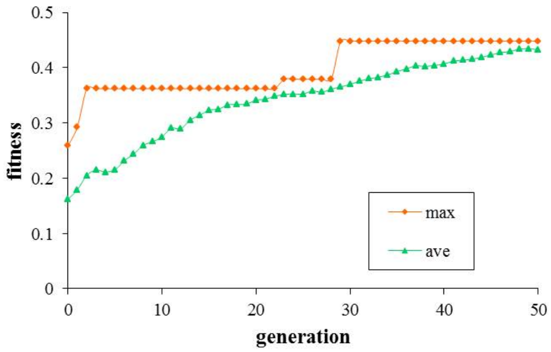

Figure 6: A result of numerical experiments

5. Conclusions

The MCA and its learning algorithm is considered in this paper. An application to the ac-dc converter is also proposed. In order to visualize the MCA dynamics systematically, the Dmap is introduced. The algorithm performance is investigated in an example of the teacher signal. The teacher signal can be stored and the local stability is enforced. Future problems include the detail analysis of the dynamics of the MCA and engineering applications.

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