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Attractor-preserving control to avoid saddle-node bifurcation

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Abstract—In nonlinear dynamical systems, periodic orbits meeting a saddle-node bifurcation may generally disappear, and they are going to be a chaotic orbit, other periodic solutions, and divergence or equilibrium points. However, right after the bifurcation, some orbits transitionally wonder around the trace of the saddle and node periodic orbits, i.e., the orbit stays long around the trace, then eventually movies to the other stable attractor. We direct our attention to this phenomenon, a controller keeping a periodic solution regardless of the saddle-node bifurcation. To realize this, the external force control technique has been applied. Some numerical simulation results are given.

1. Introduction

Various methods have been available for controlling chaos. As one of the most famous technique, a delayed feedback control (DFC) method has been developed [1]. This is a method of controlling chaos in continuous dynamical systems and it stabilizes the desired unstable periodic orbit (UPO) by providing the feedback composed of the difference between the current state and the delayed state. Advantages of the DFC are that (1) the parameter, which is necessary for control, is only period τ of the UPO, (2) no stability analysis of the UPO is required, and (3) it is relatively easy to implement the controller by using the memory hardware.

As the other control technique, the OGY method [2] has been proposed. Utilizing the recurrence properties of chaotic dynamics can stabilize UPOs. Since the trajectory may approach neighborhood of any UPO in the transient state of chaos a controller based on linear control theory works well, that is, and the distance (error) between the current position of the chaos and the UPO is applied into the system as a control input. By choosing appropriate feedback gain, the UPO can be stabilized with reasonably small amplitude of the control input. Various methods including delayed feedback control (DFC) [1], external force control (EFC) [3], occasional proportional feedback (OPF) [4], and so on [5–9] have been proposed.

These control techniques are intended to stabilize UPOs. They are embedded into the chaotic attractor originally. Above control methods absolutely require this existence of UPOs. However, no one has investigated the case that UPOs are disappeared by the parameter variations.

The saddle-node bifurcation is phenomena that two orbits are collided by variation of parameters. When happened this, these orbits are disappeared. However just after the bifurcation, attraction regions of the trace of the periodic orbit are still remained. In fact, even as the periodic solution occurred the bifurcation does not exist in theory, the trajectory tends to wrap around the orbit near one.

In this paper, we propose the controller on compensation of attractors vanished by the saddle-node bifurcation. By using the features of the trace of periodic orbits and applying the EFC method, we try to stabilize the periodic solution occurred saddle-node bifurcation. In general, when the saddle-node bifurcation occurs, the other attractor captures the orbit after a transitional state. By applying this method, it may be possible to reduce such damage.

2. Saddle-node bifurcation and the subsequent response

In nonlinear dynamical systems, there is a saddle-node bifurcation that is one of the most representative of the local bifurcations. When a real multiplier of the limit cycle exceeds unity, this bifurcation is occurred. The condition of this is expressed by $\mu_i = 1,\ \exists i = 1,\ \ldots,\ n$. If a saddle-node bifurcation occurs, a couple of periodic orbits are disappeared, and many cases are happened, i.e., the orbit is captured by

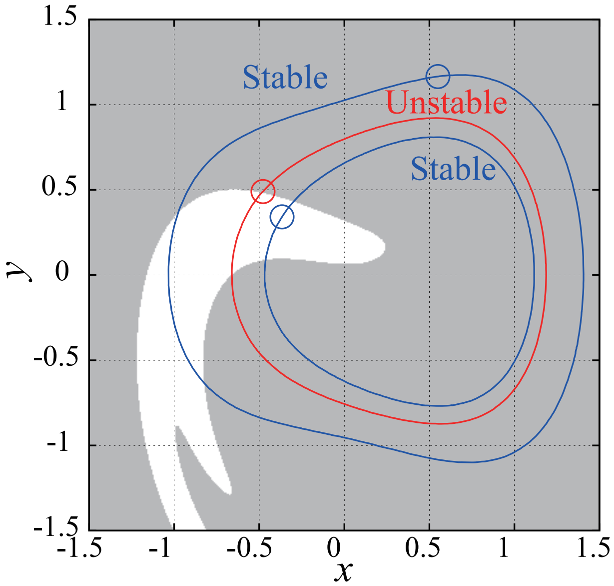


Figure 1: The phase portrait ($k = 0.2$, $B_0 = 0.27$, $B = 0.28$).

other periodic orbit, chaotic attractor or equilibrium points, or divergence. We show the saddle-node bifurcation in the Duffing equation [10] as an example. The Duffing equation is a two dimensional non-autonomous system as follows:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -ky - x^3 + B_0 + B\cos(t) \end{aligned} \quad . \tag{1}$$

Here, $\boldsymbol{x} = (x,\ y)^\top$ is the state variable, k, B_0, B represents a parameter. The bifurcation analysis has been already investigated, and the bifurcation structure was clarified in detail [11]. Now, we assume parameters as $k = 0.2$, $B_0 = 0.27$, and $B = 0.28$, the Duffing equation shows three periodic orbits shown in Fig. 1. In the figure, a stable periodic orbit of blue, red indicates the unstable periodic orbit. Each colored circle represents the fixed point of the Poincaré map. The background depicts a basin boundary in each of the initial value, and orbits in the white initial points converge to the inner periodic orbit. When increase the parameter B_0, inner two periodic orbits gradually approach, and overlap. At this time, these periodic orbits disappear by a saddle-node bifurcation. Detail information of the saddle-node bifurcation is shown in Tab. 1.

The periodic orbit is qualitatively disappeared when saddle-node bifurcation is occurred. However, in the parameter immediately after the occurrence, periodic orbit does not exist, but any attraction area of trace of periodic orbits remains. Figures 3 show trajectories after the saddle-node bifurcation. From these figures, it is confirmed that the trajectory is wrapped around the periodic orbit that caused the saddle-node bifurcation for a finite period of time. However, the trajectory away from it gradually, and finally the trajectory converges the outer large stable periodic orbit. We want to try the stabilization of the periodic orbit that occurred the saddle-node bifurcation by adding the control input to the system.

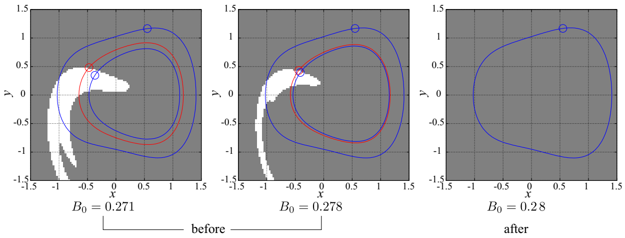


Figure 2: The phase portrait before and after the saddle-node bifurcation ($K = 0.2$, $B = 0.28$).

Table 1: Saddle-node bifurcation information (parameter generation, coordinate, characteristic multiplier).

Parameters	$k = 0.2$, $B_0 = 0.279$, $B = 0.28$
Coordinate	$\boldsymbol{x}_0 = (-0.425,\ 0.413)$
Multiplier	$\mu_0 = 0.081$, $\mu_1 = 1$

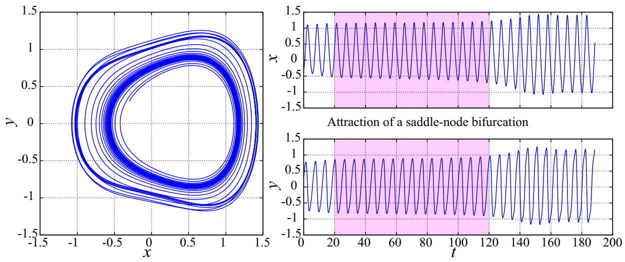
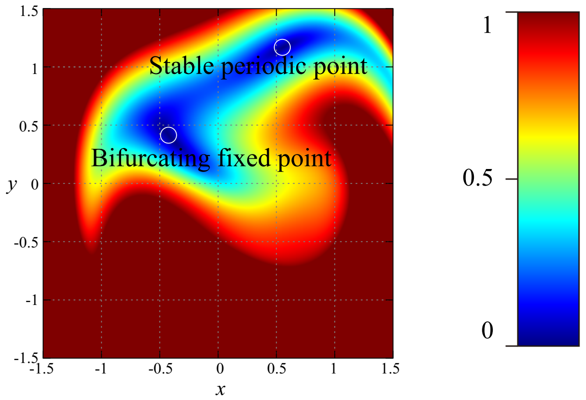


Figure 3: The trajectory after the saddle-node bifurcation ($K = 0.2$, $B_0 = B = 0.28$, $\boldsymbol{x}(0) = (-0.3,\ 0.3)^\top$). It is confirmed that the trajectory is wrapped around the periodic orbit that does not actually exist for a finite period of time.

Figures 4 are distance coloring diagrams and the color map diagram. Where, in Fig 4(a), each axis shows initial position $\boldsymbol{x}(0)$, and, circles mean the stable periodic point and the bifurcating fixed point. In the distance coloring, the pixel is drawn with color depending on the distance between the initial position and Poincaré map of it. Figure 4(b) shows the color map of the distance, and, the distance of the map is short in the blue region. By these figures, the feature point that does not move really exists around the bifurcating fixed point. Therefore, it also seems that any attraction area of trace of periodic orbits remains.



(a) (b)

Figure 4: (a):Distance coloring diagram, (b): color map diagram.

3. EFC method

In this paper, using the external force control (EFC) method as a control technique. One of the most famous techniques is the delayed feedback control (DFC) method. The DFC method feed back the state in delay time T. However, EFC method feed back the arbitrary orbit. Thus, the target trajectory is determined. By doing this, the system is stabilized to objective orbits, and the pseudo periodic solution is constant, so accuracy of controller can be improved. However, it is necessary that the target trajectory is a clear in preliminary. Analytic information is required, but it can be computed by using Poincareé map and Newton's method. One can get analytic information easily. Equations of controller are expressed as follows:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{u}(t),\ \boldsymbol{u}(t) = K(\boldsymbol{x}^*(t) - \boldsymbol{x}(t)), \quad (2)$$

where, $K \in \boldsymbol{R}^{n\times n}$ is the control gain, $\boldsymbol{x}^*(t) \in \boldsymbol{R}^n$ is the target orbit, $\boldsymbol{u}(t) \in \boldsymbol{R}^n$ is control input. Note that the time series data of the pseudo periodic orbit is the trajectory data just before the saddle-node bifurcation. It does not exist in the parameter, but the error $\boldsymbol{\xi}(t)$ is expected to be small when the orbit stays around the trace.

4. Results

We show some results of the attractor-preserving controlling. The target system is the Duffing equation (Eq. 1), and parameter are $k = 0.2$, $B_0 = B = 0.28$. In this parameter, the system shows large stable periodic orbit. By adding the control input that is shown Eq. 2, the trace of periodic orbit is stabilized. The pseudo periodic solution $\boldsymbol{x}^*$ is shown by Fig. 5.

Figure 6 shows the phase portrait of orbits that is controlled. Initial values of each trajectory are

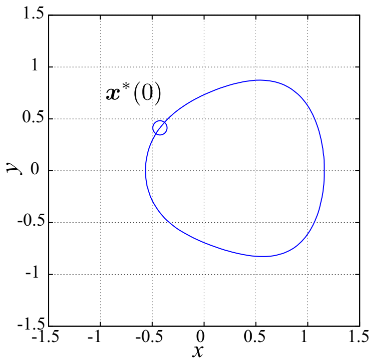


Figure 5: The pseudo periodic solution $\boldsymbol{x}^*$ ($B_0 = 0.279$, $\boldsymbol{x}^* = (0.425,\ 0.413)^\top$).

$(\pm 1.2,\ \pm 1.2)$. It is confirmed that trajectories are converged to the pseudo periodic solution $\boldsymbol{x}^*$ in each initial value. Note, the feedback gain is $K = 0.3I$, where I is an 2×2 identity matrix.

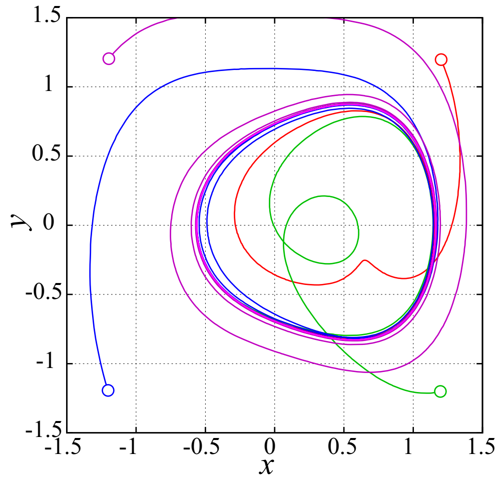


Figure 6: Phase portrait of the trajectory with control ($K = 0.3I$, I means a unit matrix.).

Figures 7 shows the time response of the controlled system. The amount of the control input $\boldsymbol{u}(t)$ is quickly decreased in the large, and $\boldsymbol{x}(t)$ is converged into the referenced periodic solution $\boldsymbol{x}^*(t)$. Figures 8(b) is an enlargement of the stationary state in Fig. 8(a) and it depicts a very small amplitude periodic ripple. The system in this state is equivalent to the original equation with a very small periodic perturbation. It is noteworthy the pseudo periodic solution is preserved by this tiny control energy.

We numerically confirm that $\boldsymbol{u}(t)$ is kept small if the system parameter is changed little bit. This may suggest the controller is robust. From Fig. 6, the EFC technique can be control to $\boldsymbol{x}^*(t)$ at any initial coordi-

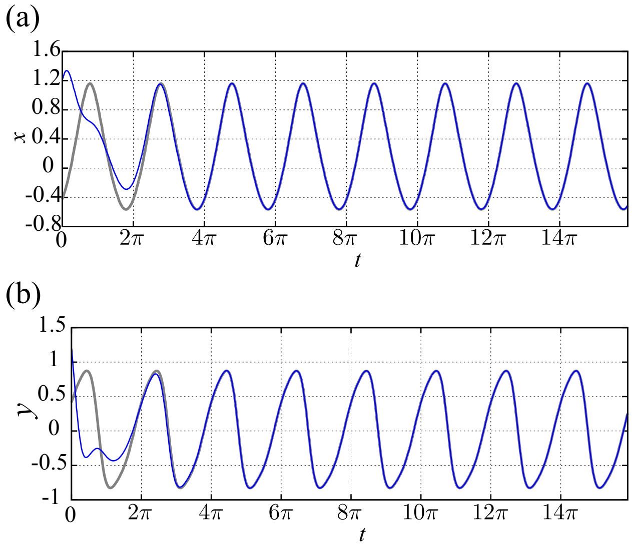


Figure 7: Time pulse trains of orbits ($\boldsymbol{x}(0) = (1.2,\ 1.2)^\top$). Gray lines show the pseudo periodic solution $\boldsymbol{x}^*$.

nate. Thus, the controller provides good performance. However, the system has a large-scale periodic orbit originally, but it is hidden because the controller profoundly affects one. In our feature works, we want to design the controller that the referenced periodic solution coexists with the original solution.

5. Conclusion

In this study, we discuss the compensation of attractors vanished by the saddle-node bifurcation.

Firstly, we show the saddle-node bifurcation and disappearance of the periodic orbit. Just after the bifurcation, the attraction region with the trace of periodic orbit remains, and so we try the stabilization of the trace of the periodic orbit. We use the EFC method, and the periodic orbit that is point of the saddle-node bifurcation as the state feedback. It is necessary the analytic approach to feed back the periodic orbit, but you can compute by using the Poincaré map and Newton's method. Next, we show simulation results. Using the EFC method can stabilize the trace of periodic orbit. By this way, we can show the compensation of attractors vanished by the saddle-node bifurcation. The EFC method is forceful controller, and it can be converge to the pseudo periodic solution from any initial points. By these results, the EFC method is helpful technique on compensation of attractors vanished by the saddle-node bifurcation.

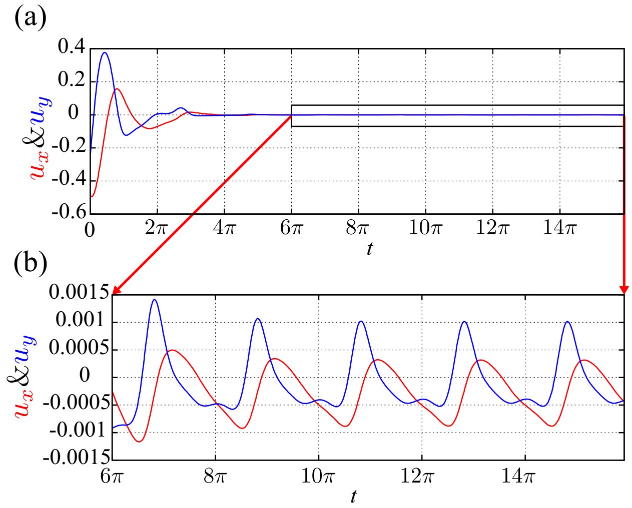


Figure 8: Control input ($\boldsymbol{x}(0) = (1.2,\ 1.2)^\top$).

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References

[1] K. Pyragas, Phil. Trans. R. Soc. A, vol.364, pp.2309–2334, 2006.

[2] E. Ott, C. Grebogi, and J.A. Yorke, Phys. Lett., Vol. 64, No. 11, pp. 1196–1199, 1990.

[3] K. Pyragas, Phys. Lett., Vol. A, No. 170, pp. 421-428, 1992.

[4] Rajarshi Roy, T. W. Murphy, Jr., T. D. Maier, Z. Gills, and E. R. Hunt, Phys. Rev. Lett. 68, pp.1259–1262, 1992.

[5] T. Kapitaniak, Academic Press, 1996.

[6] G. Chen, and X. Dong, IEEE Trans., Vol. CAS-40, No. 9, pp. 591-601, 1993.

[7] Filipe J. Romeiras, Celso Grebogi, Edward Ott, and W. P. Dayawansa, Physica, Vol. D, No. 58, pp. 165–192, 1992.

[8] T. Kousaka, T. Ueta, H. Kawakami, Proc. NOLTA'97, pp. 471-474, 19981129 Nov. 1997.

[9] T. Ueta and H. Kawakami, IECE Trans., Vol. E77-A, No. 6, pp. 709–714, 1995.

[10] I. Kovacic, (Wiley, New York, 1992), p. 1-12.

[11] H. Kawakami, IEEE Trans. Circuits & Syst., CAS-31, 3, pp.248-260, 1984.