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Searching Ability of PSO with Non-Convergent Particles

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Abstract– The number of particles affects the searching diversity of particle swarm optimization (PSO) directly. Therefore, if PSO is implemented on a useful searching conception, the increment of particles is the simplest way to improve the searching ability. However, if PSO does not have an enough control of the swarm behavior, the increment of particles cannot produce the searching diversity effectively.

In this paper, we propose a novel PSO which consists of normal particles and searching particles. The searching particles are non-convergent ones. Since they are used to control the searching direction and resolution, our PSO can produce the searching diversity effectively and maintain it. Finally, it has been confirmed by numerical simulations that our PSO has a high searching ability.

1. Introduction

PSO is an optimization method originally developed by Kennedy and Eberhart in 1995[1]. Each particle in the original PSO (O-PSO) moves, considering the personal best position (PBP) and the global best position (GBP). PBP is the position of the best solution in each particle and GBP is the position of the best solution in all the particles (i.e., swarm). As a result, all the particles can swarm to search the optimal solution. Moreover, since the algorithm is defined by only update equations of the position and velocity of each particle, it is simple and the calculation cost is very small. Therefore, PSO has been applied to various optimization problems in non-linear systems [2,3] and many PSO models based on O-PSO have been proposed [2,4-6].

The number of particles affects the searching diversity of PSO directly. Therefore, if PSO is implemented on a useful searching conception, the increment of particles is the simplest way to improve the searching ability. However, if PSO does not have an enough control of the swarm behavior, the increment of particles cannot produce the searching diversity effectively. This is because the searching direction and resolution are restricted. As a result, it is possible that PSO with a vast number of particles cannot find even a semi-optimal solution.

Most of recent PSO models improve their searching ability by not increasing particles but controlling the moves of particles. For example, the control methods are the following: the escape from the local minima by adding new particles [7,8], the restriction of information propagation between particles by the network structure [9], and the control of parameters [2,4]. However, even these models cannot maintain the searching diversity at all time and the searching direction is basically restricted so that the particles approach PBP and GBP.

In this paper, we propose a novel PSO based on the design idea that the searching diversity is effectively produced by controlling the searching direction and resolution and the searching ability is improved by maintaining the diversity. Since our PSO consists of normal particles and searching particles, we call it PSO-NSP. The normal particles approach PBP and GBP as well as O-PSO and the main role is the local search. On the other hand, the searching particles are non-convergent ones which search the solution space according to the above design idea. The main role is the global search. Especially, the searching particles give PSO-NSP a high ability to escape from local minima. Finally, it has been confirmed by numerical simulations that PSO-NSP has a high searching ability.

2. Original PSO

The dynamics of original PSO (O-PSO) [1] is given by

$$v_{j,d}^{t+1} = wv_{j,d}^{t} + c_1 r_{1\,j,d}^{\,t}(p_{j,d}^{t} - x_{j,d}^{t}) + c_2 r_{2\,j,d}^{\,t}(g_{d}^{t} - x_{j,d}^{t})\ , \quad (1)$$

$$x_{j,d}^{t+1} = x_{j,d}^{t} + v_{j,d}^{t+1}\ , \quad (2)$$

where $x^t_{j,d}$ and $v^t_{j,d}$ are the d-th dimensional position and velocity of the j-th particle at the t-th iteration respectively. $p^t_{j,d}$ and g^t_d are the d-th dimensional positions with the best evaluation value found by the j-th particle and the swarm until the t-th iteration, which are called the personal best position (PBP) and the global best position (GBP) respectively. w is an inertia weight coefficient. c_1 and c_2 are acceleration coefficients. $r_1{}^t_{j,d}$ and $r_2{}^t_{j,d}$ are uniform random numbers in the range [0,1]. Also, it is known that setting the parameters (i.e., w, c_1, c_2) to the following values is good for O-PSO [10].

$$\begin{cases} w = 0.729 \\ c_1 = c_2 = 1.49445 \end{cases} \quad (3)$$

Now let us consider the behavior of a particle using the above values. In Eq.(1), the first term generates the move by the inertia. The second and third terms generate the moves toward PBP and GBP respectively. For the sake of simplicity, we explain the effect of each term.

As the iteration proceeds, the first term is asymptotically close to zero because w is smaller than one. Moreover, a large w can produce the global search at the

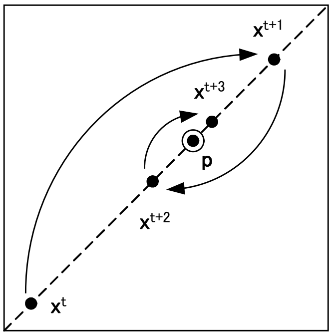

Fig.1 A particle behavior in O-PSO under the condition that there is the only second term in Eq.(1).

early iterations.

The maximum move distance by the second term is $c_1(\mathbf{p}^t_j - \mathbf{x}^t_j)$, where $\mathbf{p}^t_j \equiv [p^t_{j,d}] \in \Re^D$, $\mathbf{x}^t_j \equiv [x^t_{j,d}] \in \Re^D$, and D is the number of dimensions of the solution space. As shown in Fig.1, the particle moves across PBP every one-iteration. Even if the maximum move is continued, the particle converges to PBP finally because $1 \leq c_1 < 2$.

The third term makes the particle converge to GBP on the basis of the mechanism similar to the second term.

From these facts, we can find the following. The particles perform the global search all over the solution space at the early iterations. Afterwards PBP is close to GBP and the particles perform the local search around PBP and GBP. They converge to GBP finally. Therefore, it is clear that O-PSO produces the searching diversity from the interactions between three terms in Eq.(1).

3. PSO with Normal and Searching Particles

3.1. Proposition

The design idea of our proposed PSO (i.e., PSO-NSP) is that the searching diversity is effectively produced by controlling the searching direction and resolution and the searching ability is improved by maintaining the diversity. PSO-NSP is implemented as follows.

First, we explain the structure of PSO-NSP. It consists of normal and searching particles. A normal particle is grouped with N_{sp} searching particles. Therefore, if there are N_{np} normal particles, the number of all the particles is $N_{np}(N_{sp}+1)$. Moreover, since a normal particle has the best evaluation value in the grope, the normal particle and a searching particle may interchange during the searching process.

Next, we explain the roles of normal and searching particles. The normal particles search around PBP and GBP according to O-PSO. The main role is the local search. On the other hand, the searching particles are not guaranteed to converge and they accept the search based on the above design idea. Specifically, the parameters are controlled so that the searching particles are permitted to not only approach but also escape from GBP and they search by various resolutions as shown in Fig.2. Moreover,

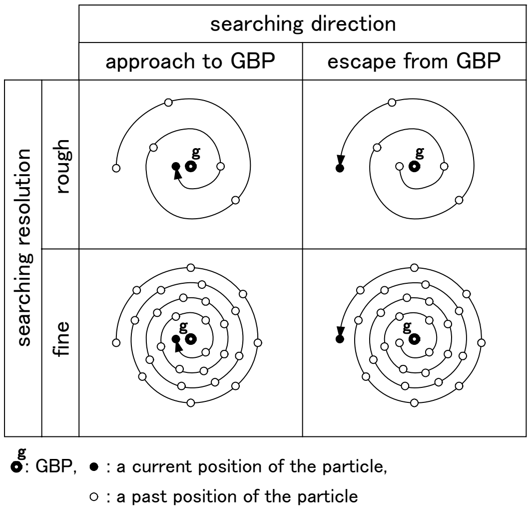

Fig.2 Conceptual illustration of searching direction and resolution for a searching particle.

the searching diversity is effectively produced by changing the parameters periodically. Therefore, the main role is the global search. Especially, the searching particles give PSO-NSP a high ability to escape from local minima.

The local search by normal particles and the global search by searching particles always coexist in PSO-NSP. Therefore, it can be expected that PSO-NSP have a high searching ability.

3.2. Searching Particles Based on O-PSO

The dynamics of a searching particle is given by

$$v^{t+1}_{n,s,d} = w^t v^t_{n,s,d} + c^{\ t}_{2n,s,d} r^{\ t}_{2n,s,d} (g^t_d - x^t_{n,s,d}) \,, \qquad (4)$$

$$x^{t+1}_{n,s,d} = x^t_{n,s,d} + v^{t+1}_{n,s,d} \,, \qquad (5)$$

where n is the index of normal particles (or groups) and s is the index of searching particles. From Eq.(4), it is clear that the base point of search is GBP. The inertia weight coefficient w^t and the acceleration coefficient $c_2{}^t_{n,s,d}$ are set as follows.

The inertia weight coefficient w^t is given by

$$w^t = w_{\max}(\cos(2\pi f t) + 1)/2 \,, \qquad (6)$$

where $w_{\max}$=0.9 and f=0.01. Therefore, the effect of the inertia is periodically changed. This means that the searching resolution is periodically changed.

The acceleration coefficient $c_2{}^t_{n,s,d}$ is set to an uniform random number in the following range:

$$c^{\ t}_{2n,s,d} \in [0,5] \,. \qquad (7)$$

$c_2{}^t_{n,s,d}$ is changed when the searching particle satisfies any one of three conditions. The first condition is that T_{sp} iterations have been executed after the last change. The second condition is that a normal particle and a searching particle interchange. The third condition is that the position $x^t_{n,s,d}$ or the velocity $v^t_{n,s,d}$ is over the range **B**.

Now, we explain the effect of the second term in Eq.(4) applying Eq.(7). Since $r_2{}^t_{n,s,d}$ is uniform random numbers

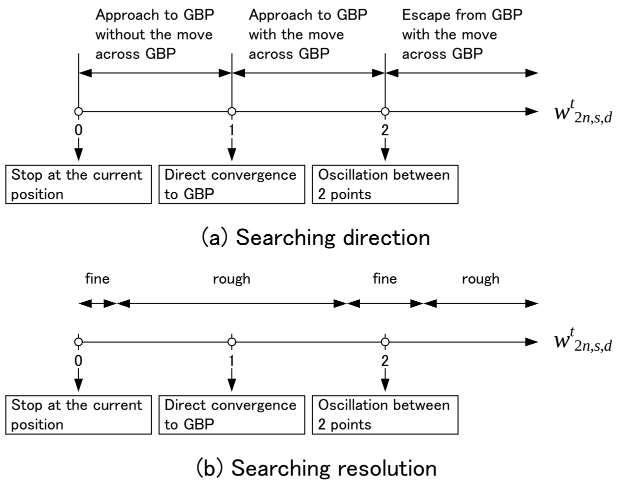

Fig.3 Conceptual illustration of searching characteristics by $w_2{}^t_{n,s,d}$ ($\equiv c_2{}^t_{n,s,d} \cdot r_2{}^t_{n,s,d}$) under the condition that there is the only second term in Eq.(4).

in the range [0,1], $c_2{}^t_{n,s,d} \cdot r_2{}^t_{n,s,d} \equiv w_2{}^t_{n,s,d} \in [0,5]$ is satisfied. Fig.3 shows the searching characteristics by $w_2{}^t_{n,s,d}$. If $w_2{}^t_{n,s,d} < 2$, the searching particle approaches GBP; otherwise it escapes from GBP. Moreover, if $w_2{}^t_{n,s,d}$ is close to and not equal to 0 or 2, the searching resolution becomes fine. Although $w_2{}^t_{n,s,d}$ cannot be directly controlled, $c_2{}^t_{n,s,d}$ can restrict the range of $w_2{}^t_{n,s,d}$. Therefore, the searching direction and resolution can be controlled by $c_2{}^t_{n,s,d}$ to some extent. For example, if $c_2{}^t_{n,s,d}$ is set to 5, there is strong possibility that the searching particle escapes from GBP and the searching resolution becomes rough.

3.3. PSO-NSP Algorithm

PSO-NSP algorithm is as follows.

1) The iteration count t is set to zero.
2) The position x and velocity v of all the particles are uniformly randomly initialized in the range **B**.
3) Calculate the evaluation value for each particle. It is kept as the personal best (PB). Also, the position is kept as the personal best position (PBP).
4) In the n-th grope, the particle with the best PB is the normal particle. Others are the searching particles.
5) Detect the global best (GB) and the global best position (GBP).
6) The update counter of each searching particle ($C_{n,s}$) is set to zero.
7) Set the parameters of each particle using Eqs.(3), (6), and (7).
8) Update x and v of each particle. The normal particles use Eqs.(1) and (2). The searching ones use Eqs.(4) and (5). Also, execute $t \leftarrow t+1$ and $C_{n,s} \leftarrow C_{n,s}+1$.
9) If the position x and the velocity v is over the range **B**, the new values of x and v are calculated by 'reflection' and 'cut off' respectively. For example,
$$x = 6 \notin \mathbf{B} \equiv [-5,5] \Rightarrow x \to 5-(6-5) = 4, \quad (8)$$
$$v = 6 \notin \mathbf{B} \equiv [-5,5] \Rightarrow v \to 5. \quad (9)$$
Moreover, if a searching particle gives rise to a bound infringement at the d-th dimension, the acceleration coefficient $c_2{}^t_{n,s,d}$ is renewed.
10) Update PB and PBP.
11) If a searching particle has the best PB in the n-th grope, the searching particle and the current normal particle interchange. Although each of them maintains its current x and v, the new searching particle renews $c_2{}^t_{n,s,d}$ at all the dimensions and sets $C_{n,s}$ to zero.
12) Update GB and GBP.
13) If $C_{n,s}=T_{sp}$, the corresponding searching particles renew $c_2{}^t_{n,s,d}$ at all the dimensions and set $C_{n,s}$ to zero. Moreover, the uniform random numbers in the range $A \times \mathbf{B}$ are added to their x and v, where A is a small positive number.
14) If $t=T_{\mathrm{PSO}}$, the search is finished; otherwise go to the step 8.

4. Numerical Simulations

Simulations have been carried out to demonstrate the effectiveness of our proposed PSO (i.e., PSO-NSP). For the function optimization problems shown in Table 1, PSO-NSP is compared with O-PSO and O-PSO-R. O-PSO-R is O-PSO with the reset function. Therefore, if all the particles have converged, O-PSO-R continues the search after reinitializing the position x and velocity v in the range **B**. However, this O-PSO-R cannot reinitialize PB, PBP, GB, and GBP.

The parameters of each PSO model are as follows. In the case of O-PSO and O-PSO-R, w=0.729, $c_1=c_2=$ 1.49445. In the case of PSO-NSP, the normal particles have the same parameters as O-PSO. On the other hand, the searching particles have $w_{\max}$=0.9, f=0.01, $c_2{}^t_{n,s,d} \in [0,5]$, T_{sp}=100, A=0.01.

The other experimental conditions are as follows. The number of all the particles N_p is set to 60 or 100. In the case of N_p=60, PSO-NSP uses (N_{np}, N_{sp})=(10,5), (15,3), and (30,1). In the case of N_p=100, PSO-NSP uses (N_{np}, N_{sp})=(10,9), (20,4), and (50,1), where N_{np} means the number of normal particles or gropes and N_{sp} is the number of searching particles with which a normal particle is grouped. The number of search trials in each system is 500 and the maximum number of iterations T_{PSO} in each trial is 10000. The search in a trial is successful if the squared error between GB and the optimal solution is smaller than 0.001.

Tables 2 and 3 show the simulation results. GB_{ave} is the average of GB. SR is the success rate. When the search has been successful, we check the iteration count. ITR_{med} is the median of the iteration count under $SR \geq 0.9$. From these results, we have found the following. Comparing with O-PSO and O-PSO-R, PSO-NSR has higher searching ability. Also, as the particles increase, the tendency becomes strong. However, the performance of PSO-NSR depends on the combination of N_{np} and N_{sp}.

Table1 Benchmark problems for numerical simulations (*D*=30).

Function name	Formula	Domain (**B**)	Minimum value
Rastrigin	$f_1(\mathbf{x}) = 10D + \sum_{d=1}^{D}(x_d^2 - 10\cos(2\pi x_d))$	$[-5,5]^D$	$f_1(\mathbf{0}) = 0$
Rosenbrock	$f_2(\mathbf{x}) = \sum_{d=1}^{D-1}(100(x_d^2 - x_{d+1})^2 + (1-x_d)^2)$	$[-5,5]^D$	$f_2(\mathbf{1}) = 0$
Schwefel	$f_3(\mathbf{x}) = \sum_{d=1}^{D} - x_d \sin(\sqrt{\lvert x_d \rvert})$	$[-512,512]^D$	$f_3(420.968750,\cdots,420.968750) = -418.98288727D$

Table 2 Performance comparisons ($N_p = 60$).

System	N_{np}, N_{sp}	Rastrigin (f_1)			Rosenbrock (f_2)			Schwefel (f_3)		
		GB_{ave}	SR	ITR_{med}	GB_{ave}	SR	ITR_{med}	GB_{ave}	SR	ITR_{med}
O-PSO	---	7.978e+1	0.0	---	2.068e+0	0.214	---	-1.050e+4	0.0	---
O-PSO-R	---	4.771e+1	0.0	---	2.122e+0	0.196	---	-1.133e+4	0.0	---
PSO-NSP	10,5	1.725e-11	1.0	2059	2.360e-2	0.428	---	-1.256e+4	0.956	5812
	15,3	2.376e-11	1.0	2294	1.578e-4	0.964	7187	-1.256e+4	0.908	6199
	30,1	7.732e-12	1.0	3167	1.608e-7	1.0	5633	-1.249e+4	0.480	---

Table 3 Performance comparisons ($N_p = 100$).

System	N_{np}, N_{sp}	Rastrigin (f_1)			Rosenbrock (f_2)			Schwefel (f_3)		
		GB_{ave}	SR	ITR_{med}	GB_{ave}	SR	ITR_{med}	GB_{ave}	SR	ITR_{med}
O-PSO	---	6.914e+1	0.0	---	1.654e+0	0.194	---	-1.073e+4	0.0	---
O-PSO-R	---	3.347e+1	0.0	---	1.771e+0	0.196	---	-1.142e+4	0.0	---
PSO-NSP	10,9	4.342e-12	1.0	1256	6.774e-4	0.842	---	-1.257e+4	1.0	3347
	20,4	1.640e-12	1.0	1529	2.590e-7	1.0	4895	-1.257e+4	0.998	3818
	50,1	5.021e-13	1.0	2167	4.060e-11	1.0	3993	-1.256e+4	0.950	6318

5. Conclusions

We have proposed PSO-NSP which consists of normal particles and searching particles. PSO-NSP uses the searching particles to control the searching direction and resolution and to maintain the searching diversity. From the results of simulations, it has been confirmed that PSO-NSP has a high searching ability.

In the future, we will consider an adaptive control of parameters and a new relationship between normal and searching particles to improve the searching ability.

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