IEICE Proceeding Series

Parameter Setting Procedure by using Golden Angle for Generation of Diversity

Takuya Shindo, Kenya jin'no

Vol. 1 pp. 146-149
Publication Date: 2014/03/17
Online ISSN: 2188-5079

2012 International Symposium on Nonlinear Theory and its Applications
NOLTA2012, Palma, Majorca, Spain, October 22-26, 2012

Parameter Setting Procedure by using Golden Angle for Generation of Diversity

Takuya Shindo† and Kenya jin'no‡

†Graduate School of Electrical Engineering, Nippon Institute of Technology
4-1 Gakuendai, Miyashiro-machi, Minamisaitama-gun, Saitama, 345-8501 Japan
Email: e2092009@estu.nit.ac.jp, jinno@nit.ac.jp

Abstract—In our previous studies, we confirmed that the deterministic PSO which was removed the stochastic factors from the conventional PSO to analyze its dynamics, has bad search performance comparing with the conventional PSO. The cause is that the parameters in the deterministic PSO is time invariant, namely, each particle doesn't have diversity. Based on a golden angle property, we propose a parameter setting procedure for the deterministic PSO to generate diversity. In this article, we confirm the optimal solution search performance of proposed parameter setting procedure using plural benchmark functions.

1. Introduction

Searching for an optimal value of a given evaluation function is very important. In order to solve such optimization problems speedily, various meta-heuristic optimization algorithms have been proposed. Particle swarm optimization (PSO), which was originally proposed by J. Kennedy and R. Eberhart [1], [2], is one of such meta-heuristic algorithm. The PSO algorithm is a useful tool for optimization problems [3]-[6].

The original PSO is described as

$$\boldsymbol{v}_j^{t+1} = w\boldsymbol{v}_j^t + c_1\boldsymbol{r}_1(\boldsymbol{pbest}_j^t - \boldsymbol{x}_j^t) + c_2\boldsymbol{r}_2(\boldsymbol{gbest}^t - \boldsymbol{x}_j^t) \quad (1)$$

$$\boldsymbol{x}_j^{t+1} = \boldsymbol{x}_j^t + \boldsymbol{v}_j^{t+1} \quad (2)$$

where $w \geq 0$ is an inertia weight coefficient, $c_1 \geq 0$, and $c_2 \geq 0$ are acceleration coefficients, and $\boldsymbol{r}_1 \in [0, 1]^N$ and $\boldsymbol{r}_2 \in [0, 1]^N$ are two separately generated uniformly distributed random number vectors. $\boldsymbol{x}_j^t \in \mathbb{R}^N$ denotes the location vector of the j-th particle on the t-th iteration in the N-dimensional parameter space, and $\boldsymbol{v}_j^t \in \mathbb{R}^N$ denotes the velocity vector of the j-th particle on the t-th iteration. $\boldsymbol{pbest}_j^t \in \mathbb{R}^N$ represents the location that gives the best value of the evaluation function of the j-th particle on the t-th iteration. $\boldsymbol{gbest}^t \in \mathbb{R}^N$ is the location that gives the best value of the evaluation function on the t-th iteration in the swarm.

The particles in the swarm fly through the N-dimensional space according with Eqs. (1) and (2). Each particle shares information of a current optimal value of the evaluation function and its corresponding location of the best particle. Also, each particle memorizes its record of the best evaluation value and its best location. On the basis of such information, the moving direction and velocity are calculated by Eq. (1). Namely, all particles will move toward a coordinate that gives the current best value of the evaluation function.

The dynamics of the PSO systems is very complicated. In order to analyze the dynamics of such PSO, Clerc, and Kennedy proposed a simple deterministic PSO system, and analyzed its dynamics theoretically [4]. The simple deterministic PSO system does not contain stochastic factors, namely, the random coefficients have been omitted from the original PSO system. The analysis of such a deterministic PSO is very important for determining the effective parameters of the standard PSO [4], [7].

Moreover, we are trying to implement the proposed system by an electronic circuit [8]. Considering the implementation, it is desirable that the system does not contain any stochastic factors. Therefore, we pay attention to a deterministic system. The simplicity acceleration coefficients of the deterministic PSO system can be described as

$$\begin{cases} \boldsymbol{p}_j^t = \dfrac{c_1\boldsymbol{pbest}_j^t + c_2\boldsymbol{gbest}^t}{\psi} \\ \psi = c_1 + c_2 \end{cases} \quad (3)$$

where $\boldsymbol{p}_j^t$ can be regarded as a desired fixed point.

In this case, Eqs. (1) and (2) can be transformed into the following matrix form:

$$\begin{bmatrix} \boldsymbol{v}_j^{t+1} \\ \boldsymbol{y}_j^{t+1} \end{bmatrix} = \begin{bmatrix} w & -\psi \\ w & 1-\psi \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_j^t \\ \boldsymbol{y}_j^t \end{bmatrix} \quad (4)$$

where $\boldsymbol{y}_j^t = \boldsymbol{x}_j^t - \boldsymbol{p}_j^t$.

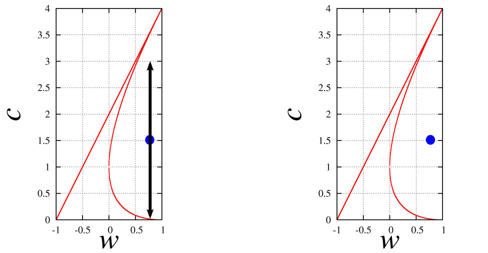
The dynamics of the particles of the deterministic PSO is characterized by the eigenvalues λ as shown in Eq. (4).

$$\lambda = \frac{(1 + w - c) \pm \sqrt{(1 + w - c)^2 - 4w}}{2} \quad (5)$$

The damping factor Δ and the rotation angle θ are given by the eigenvalues.

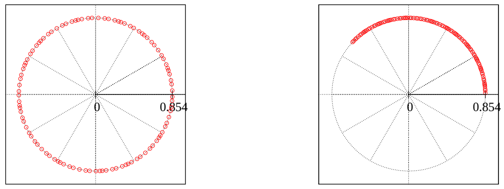
$$\Delta = \sqrt{w} \quad (6)$$

$$\theta = \arctan \frac{\sqrt{4w - (1 + w - c)^2}}{(1 + w - c)} \quad (7)$$



(a) Conventional PSO (b) Deterministic PSO

Figure 1: Diversity of the parameter in each PSO



(a) Diversity of the parameter by a golden angle (b) Range of golden angle

Figure 2: The rotation angle θ acquired from a golden angle

The damping factor and the rotation angle are regarded as the parameters of the deterministic PSO. Note that this system does not contain stochastic factors, therefore, this system can be regarded as a deterministic system.

2. Influence of a Random Number

Since the deterministic PSO does not contain stochastic factors, the searching ability is deteriorated comparing with the conventional PSO.

In deterministic PSO, random numbers are assumed to be $r_1 = r_2 = 1$. On the other hand, since the value of r_1 and r_2 is uniform random numbers in the conventional PSO, an average is set to 0.5. Figure 1 shows the relationship of these parameters; w and $c = c_1 r_1 + c_2 r_2$. Note that the parameters r_1 and r_2 of the deterministic PSO are set as $r_1 = r_2 = 0.5$ to compare the performance with the conventional stochastic PSO. The parameters set within the triangular area in Fig. 1 to guarantee the stability of the system. In the case of the deterministic PSO, the parameters is given as a certain point. The parameters of the conventional stochastic PSO are varies with the iteration, however, the parameters of the deterministic PSO are time-invariant.

3. Deterministic PSO using a Golden Angle

We have proposed the parameters setting procedure which the golden angle is applied to the rotation angle to improve the performance of the searching ability. The golden angle is the smaller of the two angles created by sectioning the circumference of a circle according to the golden ratio $\phi = 2\pi/(1 + \frac{1+\sqrt{5}}{2})$. In our proposed procedure, the rotation angle of each dimension of the particle is determined by the golden angle.

Applying the golden angle, the rotation angle does not have an overlap, namely, the system has the diversity as shown in Fig. 2(a). The radius corresponds to the damping factor. Considering the parameter range of the conventional stochastic PSO, the range of the rotation angle is normalized as illustrated in Fig. 2(b). Similar to the conventional PSO, the rotation angle parameter is set for each iteration t. The conventional PSO can discover good solutions when the particles exhibit damped oscillations. For this reason, we set the parameters of the deterministic PSO to generate such damped oscillations. And we discuss a method for the deterministic PSO to search the solution whose ability is equivalent to the conventional PSO. The rotation angle θ for every dimension d of each particle j is calculated as follows. D is a dimension of an evaluation function.

$$\theta_{nt} = \{jD + (d + 1) + (t + 1)D\}\phi \tag{8}$$

$$\theta = \frac{\theta_{nt} \pmod{2\pi}}{2\pi}\phi \tag{9}$$

In this article, using such time-variant rotation angle, the parameter setting procedure of the deterministic PSO is proposed. w and c derived from the rotation angle by using Eqs. (6) and (7).

4. Numerical Simulations

In order to confirm the performance of the procedure of PSO which uses the golden angle, we compares with the conventional PSO. We carry out the numerical simulation for two cases. One is the deterministic PSO. The other one is the golden angle is used for every iteration.

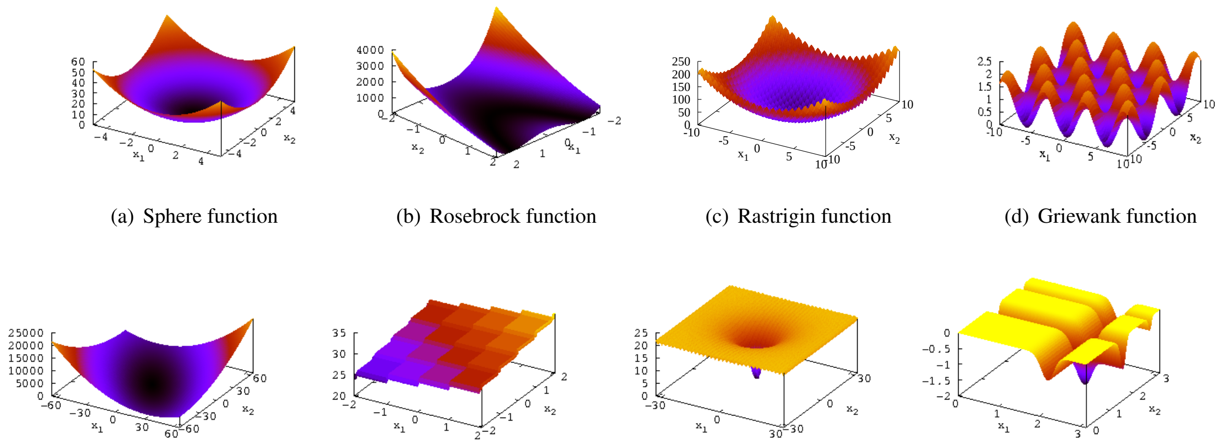
The numerical simulations are carried out by using eight standard benchmark functions as shown in TABLE 1. f_1, f_2, f_5, and f_6 functions are unimodal functions. f_3, f_4, f_7, and f_8 functions are multimodal functions. Excepting f_2, f_6, and f_8 function, the optimum value of each function is 0. The optimum value of f_2 function is 0 and the corresponding optimum solution is $x_d = 1$. The surface of each function of $N = 2$ is shown in Fig. 3. The parameters; searching range and initializing range are set as shown in TABLE 1 for each benchmark function. $v_{\max}$ is a divergent control parameter. The upper bound is given at each particle velocity calculated in Eq. (1). Initializing range is determined as an asymmetric range in the searching range. This operation provides the biased initial values.

5. Results

The simulation results of $N = 2$ are illustrated in Fig. 4. The horizontal axis denotes an iteration, and the vertical

Table 1: Benchmark Function

Function		Search Range	Initial Range
Sphere function	$f_1(\boldsymbol{x}) = \sum_{d=1}^{N} x_d^2$	$(-100,\ 100)$	$(50,\ 100)^N$
Rosenbrock function	$f_2(\boldsymbol{x}) = \sum_{d=1}^{N-1}(100(x_{d+1} - x_d^2)^2 + (x_d - 1)^2)$	$(-100,\ 100)$	$(50,\ 100)^N$
Rastrigin function	$f_3(\boldsymbol{x}) = 10N + \sum_{d=1}^{N}((x_d)^2 - 10\cos(2\pi x_d))$	$(-10,\ 10)$	$(2.56,\ 5.12)^N$
Griewank function	$f_4(\boldsymbol{x}) = 1 + \frac{1}{4000}\sum_{d=1}^{N} x_d^2 - \prod_{d=1}^{N}\cos\left(\frac{x_d}{\sqrt{d}}\right)$	$(-600,\ 600)$	$(300,\ 600)^N$
Rotated hyper-ellipsoid function	$f_5(\boldsymbol{x}) = \sum_{d=1}^{N}\left(\sum_{k=1}^{d} x_k\right)^2$	$(-65.536,\ 65.536)$	$(32.768,\ 65.546)^N$
Modified 3rd De Jong's function (step)	$f_6(\boldsymbol{x}) = 30 + \sum_{d=1}^{N}\lfloor x_d \rfloor$	$(-2.048,\ 2.048)$	$(1.024,\ 2.048)^N$
Ackley's function	$f_7(\boldsymbol{x}) = -20\exp\left(-\frac{1}{5}\sqrt{\frac{1}{N}\sum_{d=1}^{N} x_d^2}\right) - \exp\left(\frac{1}{N}\sum_{d=1}^{N}\cos(2\pi x_d)\right) + 20 + \mathrm{e}$	$(-32.768,\ 32.768)$	$(16.384,\ 32.768)^N$
Michaelwicz's function	$f_8(\boldsymbol{x}) = -\sum_{d=1}^{N}\sin(x_d)\cdot\left(\sin\left(\frac{dx_d^2}{\pi}\right)\right)^{2m}$	$(0,\ \pi)$	$(3\pi/4,\ \pi)^N$



(a) Sphere function (b) Rosebrock function (c) Rastrigin function (d) Griewank function

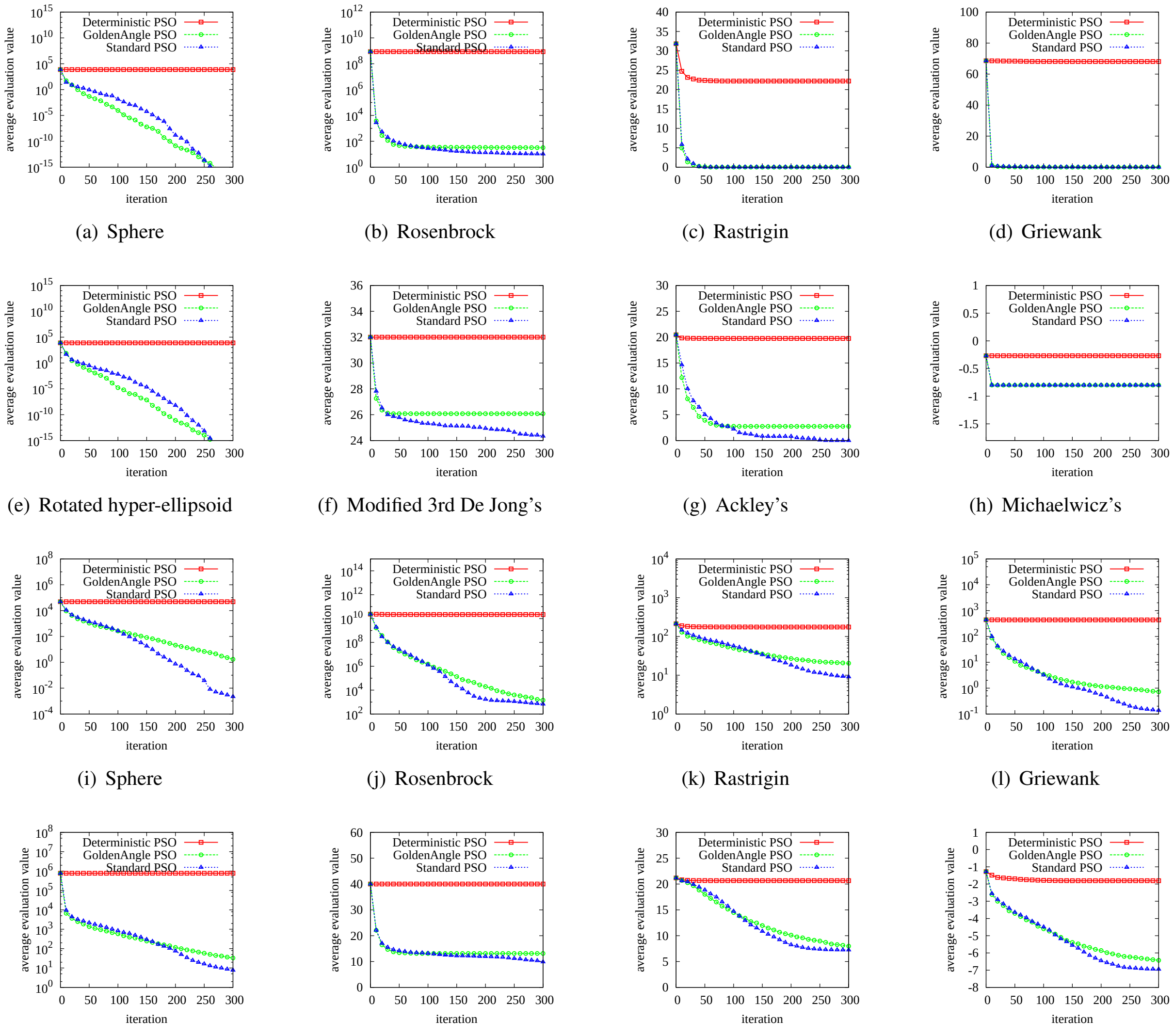
(e) Rotated hyper-ellipsoid (f) Modified 3rd De Jong's (g) Ackley's function (h) Michaelwicz's function

Figure 3: The surface of benchmark function $N = 2$

axis denotes the average evaluation value. We carry out 50 times trials. The parameters of PSO are set to $w = 0.729$ and $c_1 = c_2 = 1.494$. However, w use IWA which linearly decreases with iteration. The number of particles is 10. Moreover, N of benchmark function was set as 2 and 10. Figure 4 shows that the solution search performance in which golden angle PSO is equivalent to the conventional PSO is obtained, when the dimension of a evaluation function is low. Especially, in some benchmark functions, when there is little iteration, the solution better than the conventional PSO is obtained. Moreover, in all the benchmark functions, the performance of golden angle PSO is improving rather than deterministic PSO. The simulation results of $N = 10$ are illustrated in Fig. 4. As compared with deterministic PSO, performance improves golden angle PSO with all the benchmark functions like the case of $N = 2$. On the other hand, as compared with the conventional PSO, golden angle PSO becomes the searching performance in which iteration is almost the same to about 100 times. However, in subsequent iterations, an evaluation value has a difference. This is considered to be influence of inertia weight approach (IWA).

6. Conclusions

In this article, we proposed a novel parameter setting procedure which deterministically reproduce the diversity of the conventional stochastic PSO. The proposed procedure is based on the effect of the stochastic factor of the conventional PSO. We confirmed that the deterministic PSO using the proposed procedure exhibits the similar performance of the conventional PSO by using the golden angle.



(a) Sphere (b) Rosenbrock (c) Rastrigin (d) Griewank

(e) Rotated hyper-ellipsoid (f) Modified 3rd De Jong's (g) Ackley's (h) Michaelwicz's

(i) Sphere (j) Rosenbrock (k) Rastrigin (l) Griewank

(m) Rotated hyper-ellipsoid (n) Modified 3rd De Jong's (o) Ackley's (p) Michaelwicz's

Figure 4: Standard PSO vs Deterministic PSO with Golden Angle ($N = 2$ (a)–(h), $N = 10$ (i)–(p))

Acknowledgments

This work was supported by JSPS KAKENHI Grant-in-Aid for Scientific Research (C) 22560389.

References

[1] J. Kennedy and R. Eberhart, "Particle Swarm Optimization", in *Proc. IEEE Int. Conf. Neural Networks,* pp. 1942-1948, 1995.

[2] J. Kennedy, "The particle swarm: Social adaptation of knowledge," in *Proc. IEEE Int. Conf. Evolutionary Computation,* pp. 303-308, 1997.

[3] Y. Shi and R. Eberhart, "Empirical study of particle swarm optimization," in *Proc. ICEC 1999*, pp. 1945-1950, 1999.

[4] M. Clerc and J. Kennedy, "The particle swarm - explosion, stability, and convergence in a multidimensional complex space," *IEEE Trans. Evol. Comput.,* vol. 6, no. 1, pp. 58-73, 2002.

[5] F. van den Bergh and A. P. Engelbrecht, "A cooperative approach to particle swarm optimization," *IEEE Trans. Evol. Comput.,* vol. 8, no. 3, pp. 225-239, 2004.

[6] V. Kadirkamanathan, K. Selvarajah, and P. J. Fleming, "Stability analysis of the particle dynamics in particle swarm optimizer," *IEEE Trans. Evol. Comput.,* vol. 10, no. 3, pp. 245-255, 2006.

[7] E. F. Campana, G. Fasano, D. Peri and A. Pinto "Particle swarm optimization: efficient globally convergent modifications," in *Proc. III European Conference on Computational Mechanics Solids*, Structures and Coupled Problems in Engineering, 2006.

[8] T. Kanda, T. Kurihara, T. Shindo, and K. Jin'no, "A-2-21 investigation of circuit implementation of particle swarm optimizer," *Proceedings of the Society Conference of IEICE*, vol. 2011, p. 58, 2011.