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Detection of learning in neural networks only from spike sequences

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Abstract—In this paper, we proposed a new method for estimating evolution of neural network structures only from multi-spike sequences. In the proposed method, we used a spike time metric which quantifies distance between two spike sequences and applied partialization analysis to the spike time metric. To check the validity of the proposed method, we conducted numerical experiments by using an evolving neural network model with spike-timing-dependent plasticity learning. As a result, we could detect existence of learning in the neural network and estimate how the neural network structure evolves.

1. Introduction

One of the important issues in neural science is to capture the neural network structure as well as their nonlinear dynamics. To solve this issue, one of the natural and essential ways is to observe multi-spike sequences simultaneously, to analyze them and to estimate connectivities between neurons, because these multi-spike sequences reflect essential information about the neural network structure.

In recent works [1, 2], methods for estimating connectivities between neurons by transforming spike sequences into continuous time series have been proposed. Although these methods work well, one should be careful to apply these methods, because it is possible to lose essential information of spike sequences by transforming spike sequences into continuous time series.

From this point of view, the methods for estimating connectivity between neurons only from spike sequences without transforming spike sequences into continuous time series have been proposed [3, 4]. We have also proposed an estimation method of a neural network structure and direction of couplings only from observed multi-spike sequences [5, 6] using a spike time metric [8]. In Refs. [3, 4, 5, 6], static neural network structures are estimated.

However, it is also important to estimate dynamic structures, or to detect how the neural network structure changes, because one of the intrinsic properties in neural networks is learning. When the neural networks accept external stimulation, neural networks change their structure by learning.

In this paper, we proposed a method for estimating evolution of neural network structures by applying the methods of estimating static neural network structures [5, 6] to the case that neural networks dynamically evolve. To check the validity of the proposed method, we conducted numerical experiments by using a neural network model with a learning rule of spike-timing-dependent plasticity [7]. In the experiments, we first observed the multi-spike sequences from the neural network with the STDP learning [7]. Next, we divided the observed multi-spike sequences into small temporal epochs. Then, we applied the methods in Refs. [5, 6] to the temporally divided multi-spike sequences and estimated their connectivities. As a result, we could estimate the evolving neural network structure with high estimation accuracy.

2. Method

2.1. Spike time metric

We used a spike time metric [8] to quantify distance between two spike sequences. The spike time metric consists of two operations. The first one is deletion or insertion of a single spike. The cost of this operation is unity. The second operation is movement of a single spike. The cost of this operation is $q\Delta t$ where q is a parameter and Δt is an interval which a single spike is moved.

Let us assume that the ith spike sequence is described as

$$X_i(t) = \sum_{k=1}^{m_i} \delta(t - t_i^k), \tag{1}$$

where t_i^k is the kth spike timing and $t_i^1 < t_i^2 < \ldots < t_i^{m_i}$. Then, the distance between two spike sequences $X_i(t)$ and $X_j(t)$ is defined as

$$D(X_i(t), X_j(t)) = \min\{\sum_{k=1}^{N-1} c(V_k, V_{k+1})\}, \tag{2}$$

where $V_1 = X_i(t)$, $V_N = X_j(t)$, $V_1, V_2, \ldots, V_N$ are elementary steps that transform $X_i(t)$ into $X_j(t)$, and $c(V_k, V_{k+1})$ is the cost of an elementary step that transforms V_k into V_{k+1}.

Then, the distance between the two spike sequences is defined as the minimum total cost of the elementary steps to transform $X_i(t)$ into $X_j(t)$. Figure 1 shows an example of transforming $X_1(t)$ into $X_2(t)$ by the spike time metric.

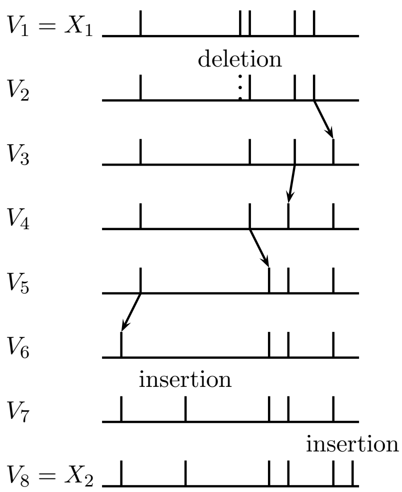


Figure 1: An example of transforming $X_1(t)$ into $X_2(t)$ by the spike time metric.

2.2. Spike time metric coefficient and partialization analysis

To estimate connectivity of neurons only from spike sequences, we have already proposed a spike time metric coefficient (STMC) and a partial spike time metric coefficient (PSTMC) [5]. The STMC is defined as

$$S_{ij} = 1 - \frac{D(X_i(t), X_j(t))}{\max_{k,l} D(X_k(t), X_l(t))}. \tag{3}$$

If two spike sequences are similar, the STMC is close to unity. Otherwise, the STMC is close to zero. Then, the STMC is a similar measure to the correlation coefficient. However, the STMC could be spuriously biased if two neurons are driven by common inputs from other neurons.

To remove such spurious correlation, we applied partialization analysis to the STMC. Then, the PSTMC is defined as

$$P_{ij} = \left| \frac{\alpha(i,j)}{\sqrt{\alpha(i,i)\alpha(j,j)}} \right|, \tag{4}$$

where $\alpha(i, j)$ is the (i, j)th element in an inverse matrix of $\boldsymbol{S} = (S_{ij})$. The PSTMC can measure the degree of correlation between the spike sequences without any spurious correlation.

2.3. Directional spike time metric

To estimate the direction of couplings between neurons, we defined a directional spike time metric [6]. We calculated the spike time metric between two spike sequences $X_i(t)$ and $X_j(t+\tau)$ defined as

$$D_{ij}(\tau) = D(X_i(t), X_j(t+\tau)), \tag{5}$$

where τ is a temporal difference between two spike sequences X_i and X_j. We distinguish the direction of couplings by whether the difference τ at the minimum value of $D_{ij}(\tau)$ is positive or not. If the difference τ at the minimum value of $D_{ij}(\tau)$ is positive, we judged that the direction of coupling is from the ith neuron to the jth neuron.

2.4. How to decide parameter q in spike time metric

To apply the proposed measures to the spike sequences, it is important to decide the parameter q in the spike time metric appropriately, because it determines a relative weight between the two operations in the spike time metric: deletion and insertion or the movement. We experimentally decided q only with the observed asynchronous spike sequences in the following manner. First, let us assume that we have two spike sequences V and V' that are identical except for a single spike that occurs at t_V^i in V and $t_{V'}^j$ in V' under the condition that $t_V^i < t_{V'}^j$. To transform V into V', we have two possible operations. The first operation is the insertion and deletion. Its cost is two (each cost is unity). The second operation is the movement. Its cost is $q(t_{V'}^j - t_V^i)$. Then, if we solve the equation $2 = q(t_{V'}^j - t_V^i)$ (in the case that both costs are same), we obtain a critical value of q (if $2 > q(t_{V'}^j - t_V^i)$, the movement is selected, otherwise the insertion and deletion are selected). To decide q appropriately, we have to define a possible range for the movement of a single spike, $t_{V'}^j - t_V^i$, because it decides the critical values of q. Then, we evaluated an average minimum time difference by

$$\overline{\Delta T} = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1, i\neq j}^{n} \overline{\Delta T}_{ij}, \tag{6}$$

where

$$\overline{\Delta T}_{ij} = \frac{1}{N_i} \sum_{k=1}^{N_i} \min_l (t_i^k - t_j^l), (0 < t_i^k - t_j^l < \frac{\overline{ISI}}{2}), \tag{7}$$

N_i is the number of spikes in the ith sequence, n is the number of spike sequences, t_i^k is the kth spike timing in the ith spike sequence, and $\overline{ISI}$ is the mean interspike interval for all the multi-spike sequences. To exclude long time difference, we applied the condition that $t_i^k - t_j^l < \frac{\overline{ISI}}{2}$.

To obtain the critical value of q, we solve the equation $2 = q\overline{\Delta T}$. Although this determination procedure is heuristic, we have confirmed that it works well in other cases.

3. Simulations and Results

To evaluate the validity of our method, we used a neural network constructed from a mathematical model of Izhikevich's simple neuron model [9] and generated multi-spike sequences. The dynamics of the ith neuron is described by the following equations:

$$
\begin{aligned}
\frac{dv_i(t)}{dt} &= 0.04v_i^2(t) + 5v_i(t) + 140 - u_i(t) + I_i(t), \\
\frac{du_i(t)}{dt} &= a(bv_i(t) - u_i(t)), \\
\text{if } v_i(t) &\geq 30 \text{ [mV], then} \begin{cases} v_i(t) \leftarrow c, \\ u_i(t) \leftarrow u_i(t) + d, \end{cases}
\end{aligned} \tag{8}
$$

where $v_i(t)$ is the membrane potential, $u_i(t)$ is the membrane recovery variable; and a, b, c, and d are dimensionless parameters. The parameters are set to $a = 0.02$, $b = 0.2$, $c = -65$, and $d = 8$. If $v_i(t) \geq 30$[mV], v_i and u_i are reset according to Eq. (8). The variable $I_i(t)$ is the sum of the external and synaptic inputs from coupled neurons which is defined as follows:

$$
I_i(t) = \sum_{j=1}^{n} \sum_{k=1}^{m_j} g_{ij} w_{ij} \delta(t - t_j^k - \tau_{ij}) + 5\eta_i(t) \tag{9}
$$

where g_{ij} is a coupling strength from j to i, τ_{ij} is a delay time between i and j which is randomly set to a value between 1 [ms] to 4 [ms], and w_{ij} is the (i, j)th element of the connection matrix of the network structure. If the neurons are coupled from j to i, w_{ij} takes unity. Otherwise, w_{ij} takes zero. The amplitude of the external inputs is set to 5 times $\eta_i(t)$, where $\eta_i(t)$ is a Gaussian random number with a mean value and standard deviation of zero and unity, respectively. The number of neurons is 100. The neural network is composed of only excitatory neurons which are regular spiking neurons. Each neuron connects to 10 postsynaptic neurons.

We use an STDP function proposed by Song et al. [7] which is defined by

$$
\Delta g = \begin{cases} A_p e^{-|\Delta t|/\tau_p} & \text{if } \Delta t > 0, \\ -A_d e^{-|\Delta t|/\tau_d} & \text{otherwise,} \end{cases} \tag{10}
$$

where A_p and A_d are the learning rates of the long-term potentiation (LTP) and depression (LTD), and τ_p and τ_d are the time constants that determine the exponential decays of the LTP and the LTD. We set the parameters $A_p = 0.01$, $A_d = 0.012$, and $\tau_p = \tau_d = 20$ [ms]. In Eq. (10), the variable Δt represents a relative spike timing between a presynaptic and a postsynaptic neuron. The coupling strength is updated as $g \leftarrow g + \Delta g$ at every second. The coupling strength is limited between 0 to 10. At an initial condition, we set that the coupling strength is 7 and an initial network structure is a random network.

We conducted numerical experiments in the following way. First, we generated multi-spike sequences by constructing a neural network using Izhikevich's simple neuron model and the STDP rule of Eq. (10). Next, we calculated the PSTMC between spike sequences for 1,000 [s]. Then, we divided this total temporal length of 1,000 [s] into small temporal windows. The length of the small temporal window is 100 [s]. Using multi-spike sequences in this 100 [s] small windows, we applied the methods of Refs [5, 6]. We classified coupled pairs and uncoupled pairs by the Otsu thresholding [10]. Then, we estimated the direction of couplings by calculating $D_{ij}(\tau)$ for the estimated coupled pairs. Finally, we evaluated the estimation accuracy.

To confirm the estimation accuracy, we compared the estimated network with the original network. We used the following two characteristics defined by

$$
\text{C}-\tilde{\text{C}} = \frac{\sum_{i,j}(w_{ij}\tilde{w}_{ij})}{\sum_{i,j} w_{ij}}, \tag{11}
$$

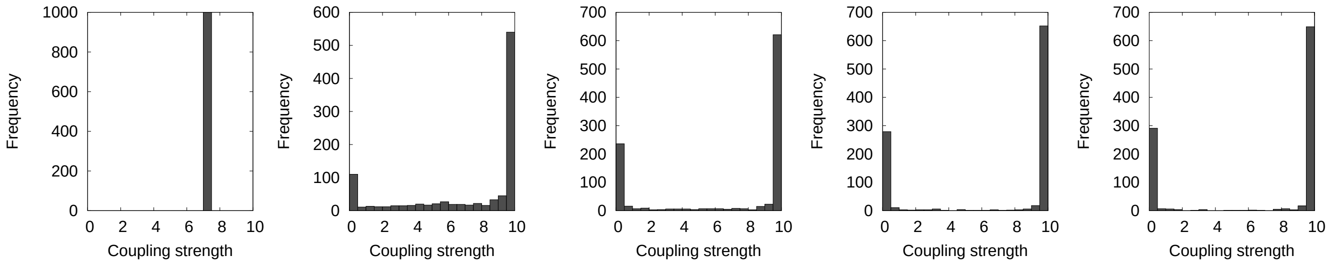
$$
\text{U}-\tilde{\text{U}} = \frac{\sum_{i,j}(1 - w_{ij})(1 - \tilde{w}_{ij})}{\sum_{i,j}(1 - w_{ij})}, \tag{12}
$$

where w_{ij} ($\tilde{w}_{ij}$) is the (i, j)th element of the adjacency matrix of the original (estimated) network structure. If the neurons are coupled from j to i, w_{ij} and $\tilde{w}_{ij}$ take unity. Otherwise, w_{ij} and $\tilde{w}_{ij}$ take zero. C$-\tilde{\text{C}}$ indicates a ratio that coupled pairs are estimated as the coupled correctly, and U$-\tilde{\text{U}}$ indicates a ratio that uncoupled pairs are estimated as the uncoupled correctly. If both of C$-\tilde{\text{C}}$ and U$-\tilde{\text{U}}$ are close to unity, it means that the estimation accuracy is high. In the experiments, we defined the original network structure in the following way: if the coupling strength is larger than 7, we regarded the pairs of neurons as coupled pairs. Otherwise, we regard the pairs of neurons as uncoupled pairs.

Figure 2 shows results of histograms of coupling strength. From these results, we can see that the distribution of the coupling strength changes, namely, the network structure evolves. We also show results of estimation accuracy of the evolving neural network structures in Fig. 3. From the results, the estimation accuracy C$-\tilde{\text{C}}$ takes relatively a low value at 0 [s]. This reason is that the coupling strength changes rapidly at the initial stage with the STDP. Namely, it is relatively hard to estimate the structure because the evolution of the network structure has fast dynamics. However, the value of both of U$-\tilde{\text{U}}$ takes relatively a high value. The temporal epoch proceeds, the estimation accuracy converges to higher values. It means that the proposed method can detect the evolution of STDP neural network structures.

4. Conclusions

In this paper, we estimated the evolving neural network structure through the spike-timing-dependent plasticity (STDP) by extending the estimation method for network structures. In the proposed method, we used the spike



(a) 0 [s] (b) 250 [s] (c) 500 [s] (d) 750 [s] (e) 1000 [s]

Figure 2: Histograms of coupling strength at each time.

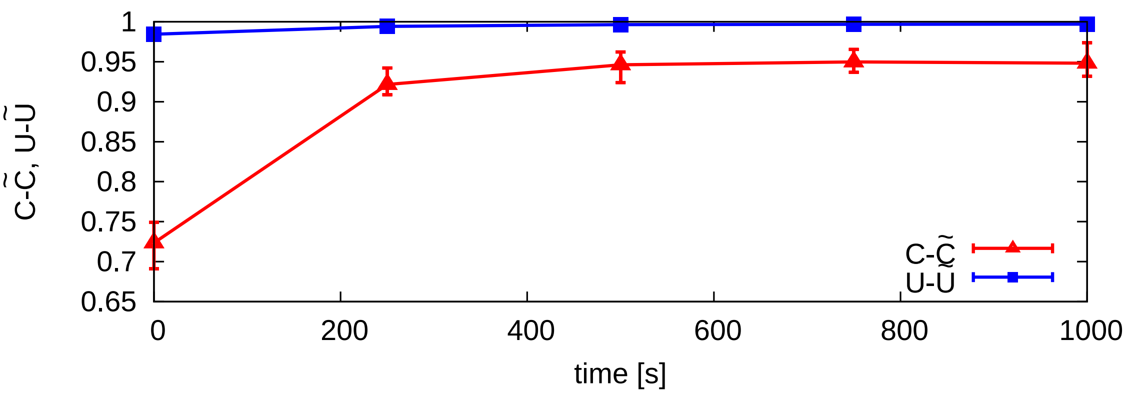


Figure 3: Estimation accuracy of the evolving neural network structures. Red line shows C−C̃ and blue line shows U−Ũ. Error bars indicate minimum and maximum values with 20 trials are also provided.

time metric to quantify the distance between two spike sequences and applied the partialization analysis to the spike time metric to detect couplings between neurons from spike sequences. As a result, the proposed method can estimate the evolving neural network structures and the direction of couplings with high estimation accuracy.

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