# Scattering Analysis of a Formation of Ships Using Parallel Higher Order Method of Moments

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## 1. Introduction

As is well known, the bottleneck with the method of moments (MoM) for the solution of integral equations relates to the $O(N^3)$ solution time and $O(N^2)$ memory storage requirements of direct solution, where $N$ is the number of unknowns. To reduce the overall time and memory requirements, various fast algorithms such as the conjugate gradient fast Fourier transform (CG-FFT), adaptive integral method (AIM), and fast multipole method (FMM), have been developed. These algorithms reduce the CPU time for carrying out the matrix-vector product in the iterative loop from $O(N^2)$ down to $O(N^{1.5})$ and $O(N \log N)$. However, when dealing with targets with thin structures and complex materials, these algorithms may have a very slow rate of iterative convergence or a trend of divergence even if preconditioners are employed.

On the other hand, with the development in computer hardware capabilities, massively parallel computing on computer clusters and multi-core processors has been the method of choice for solving modern engineering and science problems arising from extremely complicated real-life applications [1]. In this paper, we analyze the scattering properties of a formation of ships by using the pure MoM. The application of MoM is greatly extended by taking advantage of the higher order basis functions (HOBs) and the parallel out-of-core technique [2-6]. Unlike fast algorithms, the proposed method does not lose any accuracy of MoM for reducing memory requirement and improving simulation efficiency.

## 2. Parallel Higher Order Method of Moments

The parallelization of the MoM solution procedure involves two steps. The first step is the matrix filling and the second step is the solution of the matrix equation. Both of these must be handled efficiently. Furthermore, efficient parallel matrix filling for MoM with HOBs introduces new challenges and is quite different from the procedure used in a MoM formulation using the traditional subdomain basis functions, e.g., Rao-Wilton-Glisson basis functions (RWGs) [7].

To parallelize the solution of the large dense matrix in a MoM problem, typically one needs to divide the matrix between processes in such a way that two important conditions are fulfilled: each process should store approximately the same amount of data, and the computational load should be equally distributed among the processes that run on different nodes. An efficient way to parallelize MoM is to partition the large dense matrix among all participating processes. Various block partitioning methodologies have been developed to ensure good load balance according to different matrix equation solvers, employing either direct methods (e.g., Gaussian elimination, LU decomposition) or iterative methods (e.g., conjugate gradient method) [1]. A block-cyclic distribution of a matrix as performed by ScaLAPACK math library is demonstrated in Figure 1. Assume that the matrix is divided into 6×6 blocks, which are distributed to 6 processes in a 2×3

process grid, as illustrated in Figure 1(a). Figure 1(b) shows to which process the blocks of the matrix are distributed using the ScaLAPACK's distribution methodology.

| | 0 | 1 | 2 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| 0 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | 21 | 22 | 23 | 24 | 25 | 26 |
| 0 | 31 | 32 | 33 | 34 | 35 | 36 |
| 1 | 41 | 42 | 43 | 44 | 45 | 46 |
| 0 | 51 | 52 | 53 | 54 | 55 | 56 |
| 1 | 61 | 62 | 63 | 64 | 65 | 66 |

| 0 (0,0) | 2 (0,1) | 4 (0,2) | 0 (0,0) | 2 (0,1) | 4 (0,2) |
|---|---|---|---|---|---|
| 1 (1,0) | 3 (1,1) | 5 (1,2) | 1 (1,0) | 3 (1,1) | 5 (1,2) |
| 0 (0,0) | 2 (0,1) | 4 (0,2) | 0 (0,0) | 2 (0,1) | 4 (0,2) |
| 1 (1,0) | 3 (1,1) | 5 (1,2) | 1 (1,0) | 3 (1,1) | 5 (1,2) |
| 0 (0,0) | 2 (0,1) | 4 (0,2) | 0 (0,0) | 2 (0,1) | 4 (0,2) |
| 1 (1,0) | 3 (1,1) | 5 (1,2) | 1 (1,0) | 3 (1,1) | 5 (1,2) |

(a)                                    (b)

Figure 1: Block-Cyclic Distribution of a Matrix as Performed by ScaLAPACK: (a) a Matrix Consisting of 6×6 Blocks and (b) Rank and Coordinates of Each Process Owning the Corresponding Blocks in (a).

The solution of the matrix equation is essentially the same regardless of the type of basis functions used for the MoM. The parallel LU decomposition solver based on the ScaLAPACK library package is described in Chapter 2 in [2]. Also, one can refer to Chapter 6 in [2] for a detail discussion about the iterative solvers based on conjugate gradient (CG) method. The computational complexity of LU decomposition, scales with $O(N^3)$, is much higher than that of CG type methods with $O(N^2)$, where $N$ is the number of unknowns. However, in this paper, the parallel LU decomposition based on the ScaLAPACK is utilized as the parallel equation solver rather than the parallel iterative CG method due to the fact that the iterative CG method may encounter a divergence problem when dealing with complicated targets composed of thin structures and various materials.

## 3. Description of the Cluster

The computational cluster used in this paper is a high-performance computing (HPC) cluster with one head node and 24 computing nodes. Each computing node has two quad-core Intel Xeon E5310 1.6 GHz EM64T processors (2×4 MB L2 Cache and 1066 MHz FSB), 4 GB RAM, and two 72 GB 15K rpm SAS hard disks. The nodes are connected with two Infiniband switches. The parallel code is developed using the FORTRAN language based on message passing interface (MPI) and can be applied to both shared memory and distributed memory systems.

## 4. Numerical Results

In this section, the bistatic RCS from a single ship and a formation of five ships are calculated. The ship model is 153.0 m in length and 16.5 m in width, as shown in Figure 2. It is modelled as a PEC surface. The plane wave is incident from the –x direction and is polarized along the y direction, as depicted by the arrows in Figure 2.

The bistatic RCS of the full scale ship is simulated at 100 MHz. The formation of five ships is given in Figure 3. The distance between any two neighbouring ships is 60.0 m. The structure is considered in free space (not over an ocean surface) for demonstration purposes.
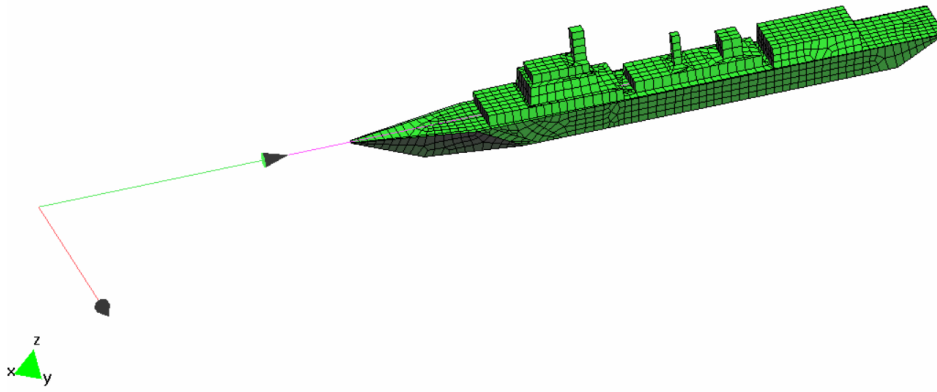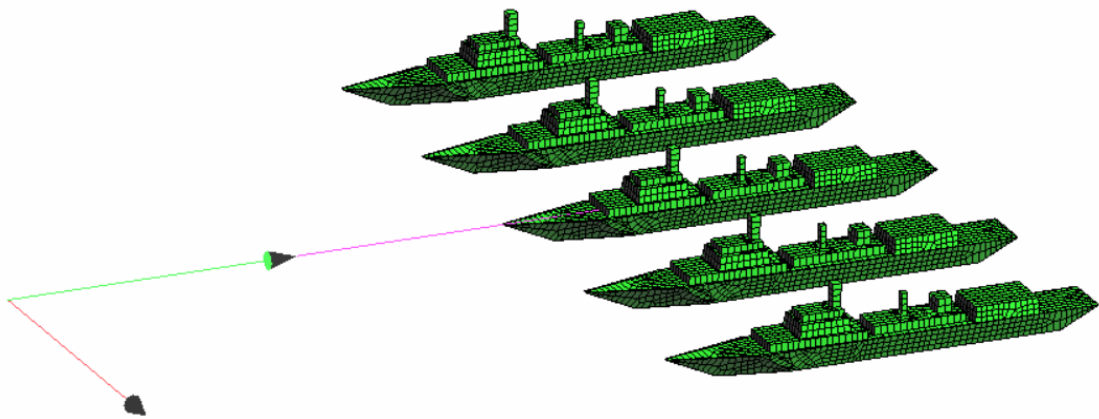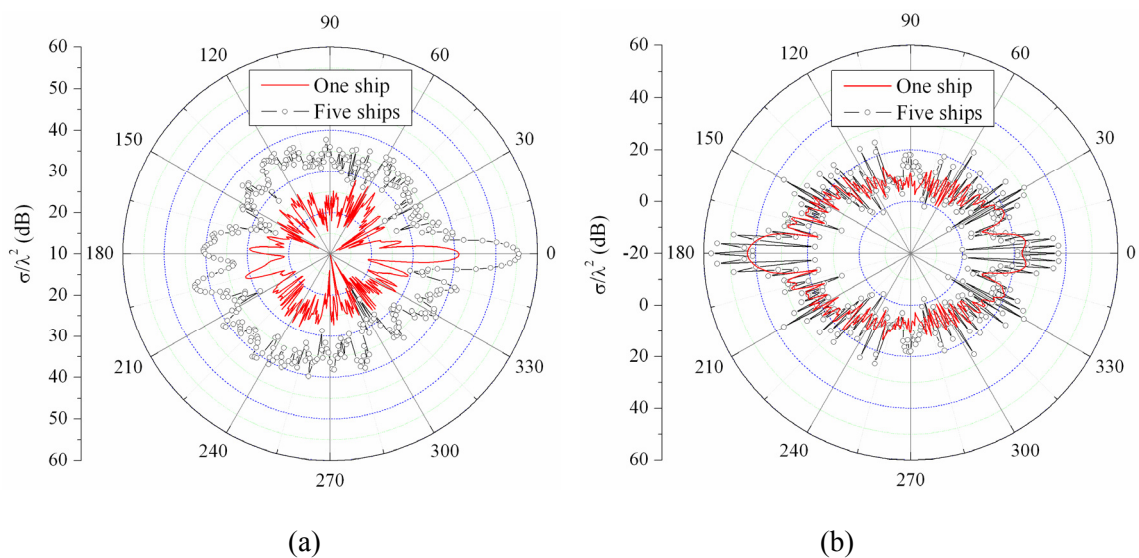
Figure 2: A Single Meshed Ship Model



Figure 3: A Formation of Five Ships

The bistatic RCS of a single ship and the ship formation in the XOZ plane and the XOY plane are plotted in Figure 4. From comparison, it is clearly seen that the RCS of the ship formation is much larger than that of a single ship in many angles. In detail, the difference between the two is about 10–20 dB.



(a)             (b)

Figure 4: Bistatic RCS in (a) XOZ Plane and (b) XOY plane
(0° starts from $x$ axis in both the XOZ and XOY planes)

The number of unknowns, number of processes and wall clock time are listed in Table 1. The single ship is simulated by using a parallel in-core MoM, while the ship formation is simulated by using a parallel out-of-core MoM due to the limitation of memory. Note that the parallel LU decomposition based on the ScaLAPACK math library is utilized as the matrix equation solver in both parallel solvers. From this example, it is concluded that large objects can be dealt with by pure MoM in reasonable time.

Table 1: Comparison of Simulation Parameters

| Model | Number of Unknowns | Number of Processes | Matrix Filling Time (s) | Matrix Solving Time (s) |
|---|---|---|---|---|
| Single ship | 37,536 | 192 | 123 | 648 |
| Ship formation | 187,680 | 192 | 7920 | 29,051 |

## 5. Conclusion

The scattering characteristics of a formation of five ships are accurately and efficiently analyzed by the massively parallel higher order method of moments. Through comparison, the RCS of the ship formation is larger than that of a single ship by about 10–20 dB in many angles. Numerical results show that the method of moments is able to solve electrically large and complex problems on computer clusters nowadays.

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