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SPICE Simulation of the Propagating Wave and the Switching Solutions in a Ring of Coupled Hard-Type Oscillator Systems

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Abstract—We investigated generation mechanism of various wave pattern such as the propagating wave and the switching solutions in a ring of several number of coupled hard-type oscillator systems in our previous papers. We clarified for a ring of six coupled oscillators by using bifurcation theory that birth and death of the propagating wave and the switching solutions were due to pitchfork and heteroclinic bifurcations. The propagating wave has very unique characteristic such as non-decaying propagation. The switching solution shows interesting property such as pitchfork bifurcation of the quasi-periodic solution. As a first step toward hardware implementation, we perform realistic computer simulation by using LTspice software. As a result, we have succeeded to obtain these solutions in a realistic ring of six coupled hard-type oscillator systems.

1. Introduction

Recently, researches on coupled oscillator systems become very popular in relation to self-organization systems such as neural networks, central pattern generator, etc. [1] [2] [3] [4]. We investigated various oscillation modes in a ring of coupled oscillators via averaging method in 70s [5] and found various modes of oscillation. They include the same and the reverse phase oscillations, the double mode oscillations, and the rotating wave oscillations, etc. However, the results based on averaging method are limited to weakly nonlinear case, and therefore, comparatively simple oscillation patterns can appear. Recently, we investigate the ring of six coupled hard-type (bistable) oscillators with medium strength nonlinearity and found more complex oscillation patterns such as the propagating quasi-periodic wave and the switching quasi-periodic solutions in addition to those already found for the weakly nonlinear case [6] [7] [8]. The switching solution is very important as an example of bifurcation of the quasi-periodic solutions which has not been investigated before. Moreover, the propagating wave solution, in which the oscillating part propagates the transmission line with no decay, seems to have an engineering application as an information carrier. From these points of view, we aim at hardware implementation of the ring of coupled hard-type oscillator systems to realize the above interesting oscillation patterns. In this paper, as a first step of hardware implementation, we perform SPICE simulation of the system. As a result, we are succeeded in obtaining various oscillation patterns including the propagating wave and the switching solutions. In this paper, we focus our attention to realize the propagating wave and the switching solutions in a ring of six coupled oscillators by LTspice simulator.

2. LTspice Simulation

At first, we will design the circuit model of a ring of six coupled hard-type oscillators shown in Fig. 1 by using LTspice simulator. Here each oscillator consists of a

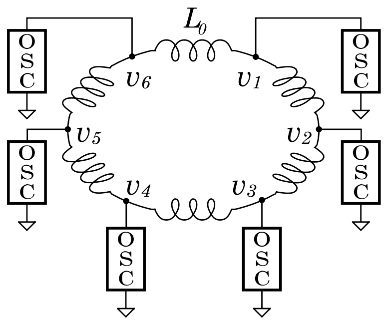

Figure 1: A ring of six coupled hard-type oscillators. L_0 is a coupling inductor.

parallel connection of an inductor L, a capacitor C and a nonlinear conductor NC as shown in Fig. 2. The important point is how to implement NC by using practical circuit elements. Fig. 3 is our circuit realization of NC whose V-I characteristic is given in Fig. 4. The positive slope around the origin is adjusted by using Rg. The negative slope in the middle voltage region is fixed by R2. The positive slope in the large voltage region is fixed by Rr. A pair of diodes surrounded by dotted square Da determines the voltage Va. Also that surrounded by dotted square Db determines the voltage Vb in Fig. 4. We apply 9th-power polynomial approximation to this V-I curve such as $i_{NC} = g_1 v - g_3 v^3 + g_5 v^5 - g_7 v^7 + g_9 v^9$, where $g_1 = 0.000907, g_3 = 0.00422, g_5 = 0.00528, g_7 = 0.00257$, and $g_9 = 0.000441$. In our theoretical model we assume NC by fifth-power polynomial: $i_{NC} = g_1 v - g_3 v^3 + g_5 v^5$,

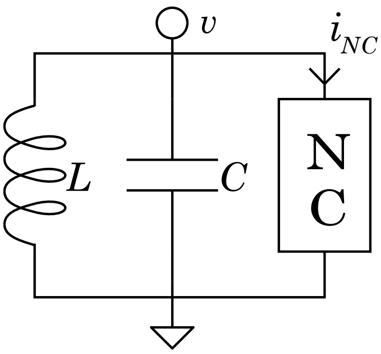

Figure 2: Circuit model of a hard-type oscillator. L = 1mH, and C = 500pF. NC is a nonlinear conductor shown after.

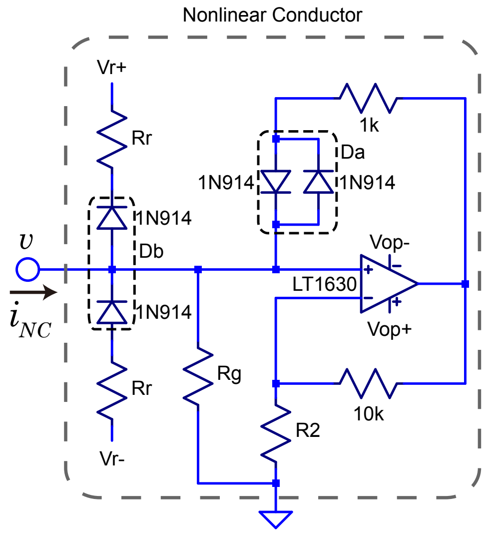

Figure 3: Circuit of a nonlinear conductor. Parameters are as follows. Vr = 35V, Vop=10V, Rr = 500Ω, Rg = 1kΩ, and R2 = 5kΩ.

but in actual circuit we need up to 9th power. In spite of the difference of NC between our theoretical and practical models, we can obtain almost the same phenomena. This means that most important terms to realize the propagating wave and the switching solutions are only g_1, g_3 and g_5, and higher order terms do not affect the qualitative nature of the circuit. Namely, we believe that most important nature of the V-I characteristic of NC to realize oscillation patterns obtained from the theoretical model is the following three points: 1) positive slope around the origin; 2) negative slope in the medium voltage region; 3) positive slope in the large voltage region.

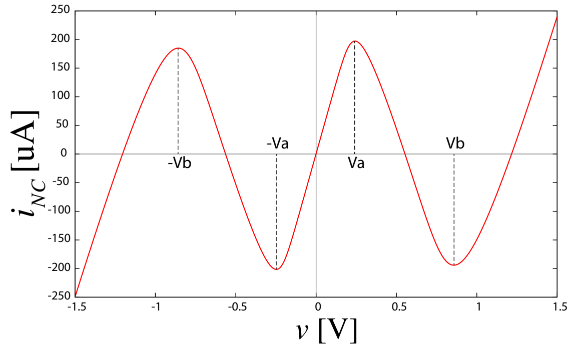

Figure 4: V-I characteristic of a nonlinear conductor.

3. Initial Conditions to Realize the Propagating Wave and the Switching Solutions

To give initial condition to each oscillator, we add a current source i_{0k} for $k = 1, 2, 3, 4, 5, 6$ in parallel to each oscillation circuit as shown in Fig. 5. By adjusting the initial

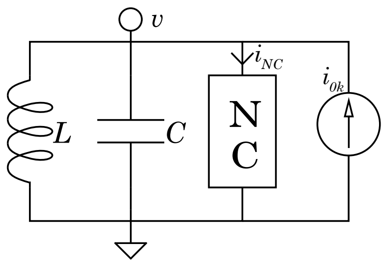

Figure 5: The current source connected in parallel to each oscillator for giving the initial condition.

current sources we can obtain various patterns of oscillations. Since the period of each oscillator $T = 2\pi\sqrt{LC} = 4.44\mu$sec where $L = 1$ mH and $C = 500$ pF, we fix the length of initial pulse is about 1μsec $\simeq T/4$ from our experience. The coupling inductor L_0 is fixed as 2.2 mH, which results in the coupling factor $\alpha = \frac{L}{L+L_0} = 0.312$ in our theoritical model [1] [7].

At first, we will demonstrate the propagating quasi-periodic wave solution in the following manner. Fig. 5 shows a pattern of initial current sources for oscillator 1 ($k = 1$) and oscillator 2 ($k = 2$) for obtaining the counter-clockwise rotating propagating wave shown in Fig. 7. The magnitude of current pulse is slightly beyond the threshold value of each isolated oscillator. In this case the initial current sources for oscillators 3 to 6 ($k = 3$ to 6) are zero. By interchanging the initial current pulses for $k = 1$ and 2 in Fig. 6, we can obtain the clockwise rotating propagating wave as shown in Fig. 8. By observing the pulse

[1] As stated before, the NC used in this LTspice simulation is not the fifth-power nonlinearity used in the theoritical model. Therefore, this value of α is only for reference.

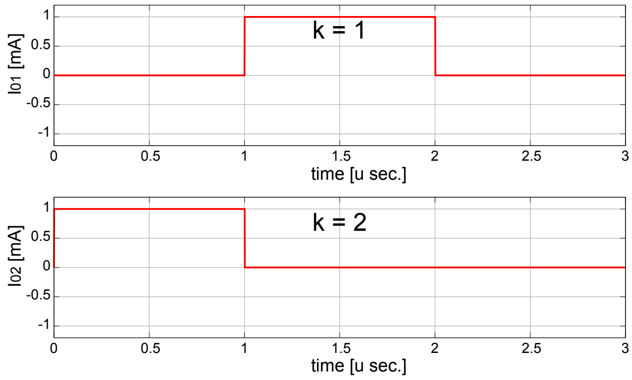

Figure 6: The initial pulse waveforms for oscillator 1 (upper trace) and 2 (lower trace) for the counterclockwise rotating propagating wave. Those for oscillators 3 to 6 are zero.

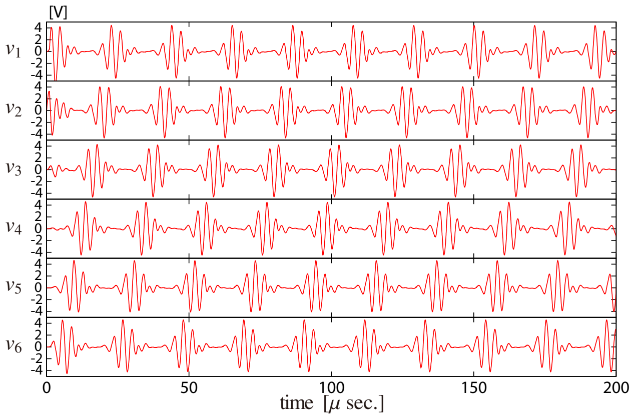

Figure 7: The counterclockwise rotating propagating wave in a ring of six coupled system.

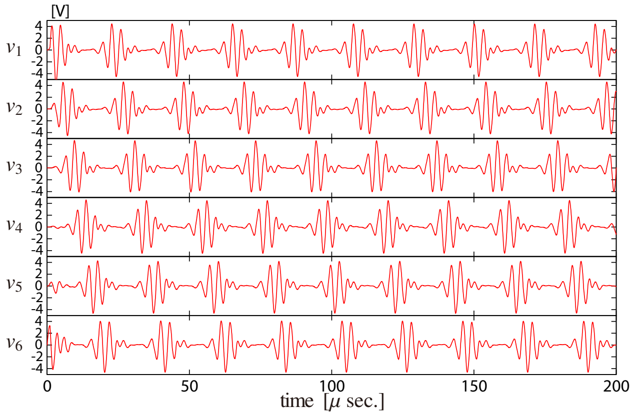

Figure 8: The clockwise rotating propagating wave in a ring of six coupled system.

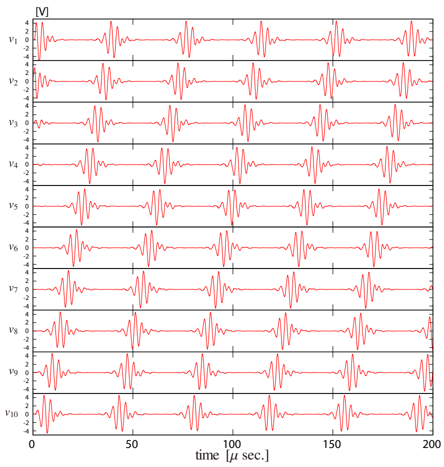

Figure 9: The counterclockwise rotating propagating wave in a ring of ten coupled system.

waveforms in Figs. 7 and 8 we notice that pulse waves can rotate either counterclockwise or clockwise. By giving the same initial current sources as those in Fig. 6, we may obtain the rotating propagating wave solutions for the ring of arbitrary number of coupled hard-type oscillators. Fig. 9 demonstrates an example of the counterclockwise rotating propagating wave for a ring of ten coupled oscillators.

Next we will demonstrate the switching quasi-periodic solution. Fig. 10 presents the initial current pattern for oscillator 1 and oscillator 2. Namely, to realize the switching solution, we give both positive and negative current pulses with 1 μsec length simultaneously for oscillator 1 and 2, respectively whose magnitude is slightly beyond the threshold value (1 mA). The initial current for the rest oscillators are zero. Fig. 11 presents the switching solution in which two pairs of ($v1, v2$) and ($v4, v5$), synchronized with reverse phase, repeat oscillation and no oscillation periods alternatively.

At last, we will show the isolated oscillation pattern observed for large L_0 = 5 mH (α = 0.167) in Fig. 12. Other parameters are the same as before. In this case, we give an initial current pulse whose magnitude and length are 1 mA and 1 μsec, respectively for oscillator 1 only. After all, by giving appropriate initial current pulse for each oscillator we can realize all kinds of oscillation patterns predicted theoretically [5] [7].

4. Conclusion

We perform LTspice simulation of a ring of six coupled hard-type oscillators to confirm actual realization of various oscillation patterns predicted by our theoretical analysis [7]. We show by giving an appropriate initial current source to each oscillator, we can realize various modes of oscillations. The important point is the realization of the nonlinear conductance. Namely, we do not intend to realize the fifth-power nonlinear conductance used in our theoret-

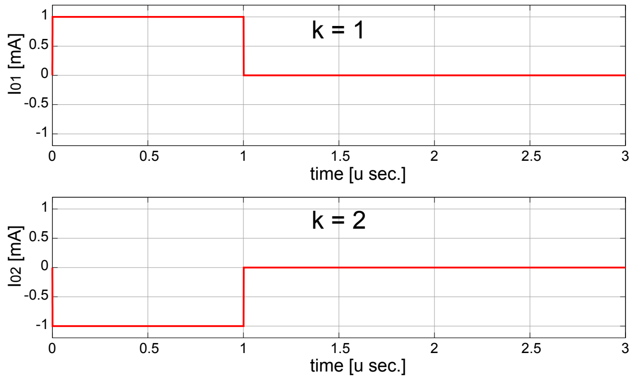

Figure 10: The initial pulse patterns for the switching solution. Those for oscillators 3 to 6 are zero.

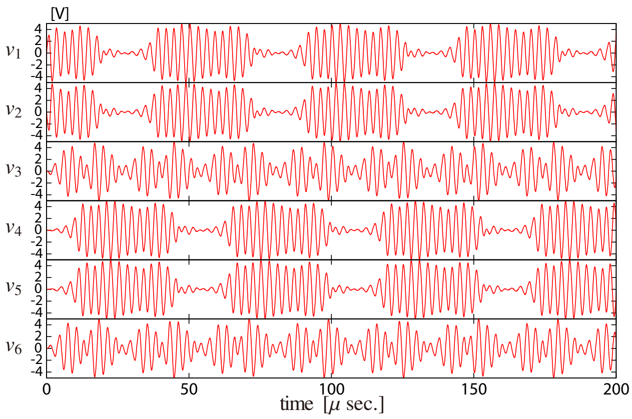

Figure 11: The switching quasi-periodic solution.

ical analysis but we try to implement more "natural" nonlinear conductance as a hardware. In spite of the difference of nonlinearity we confirm most important oscillation patterns; namely, the propagating wave and the switching solutions. From this result, we conjecture that most of oscillation patterns predicted by theory assuming the fifth-power nonlinearity can be observed for more "general" nonlinearity cases showing bistable oscillation.

Acknowledgment

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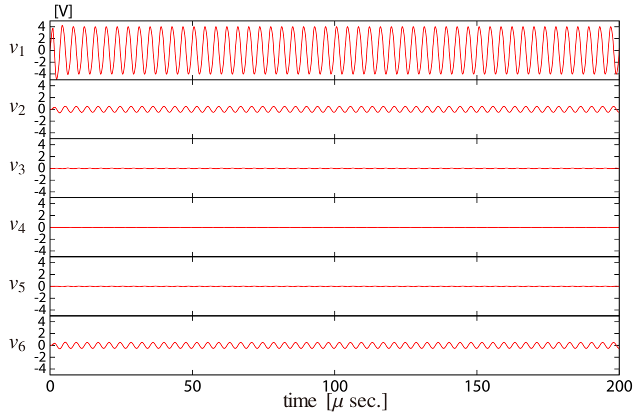

Figure 12: The standing wave solution.

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