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An Algorithm for Resource Optimization of Consolidating Two Coexisting Networks

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Abstract
We proposed an algorithm to derive the minimum number of fiber links required for resource optimization in consolidating two coexisting networks provided that every two nodes in the two networks are bi-directionally connected.

1 Introduction

With the advancement of optical fiber technologies and the surge demand of internet bandwidth in the last decade, there are many optical networks deployed by different parties. They maybe overlapped extensively in one region. It is also envisaged that more fiber links can be saved when transmission links at high data rate (e.g. 40Gb/s or 100Gb/s link) [1] are employed. In the consolidation of two networks to achieve high utilization of network resources, by traffic grooming and rerouting, some of the links can be suspended. Though for the suspended links, the deployed fibers can not be reallocated, the operation cost of the regenerator site can be saved [2]. For instance, if there are two identical and colocated optical ring networks with d nodes and links each, there are totally $4d$ links considering bi-directional communication. It can be shown that the required number of links can be reduced from $4d$ to d by inserting two short interconnections between all the colocated nodes of the two ring networks, thus substantially reduce the link cost by a factor of 4. Based on this, we propose an algorithm to derive the minimum number of links required for arbitrary connected networks. Only networks that can be viewed as planar graphs are considered. A simple equation for the minimum number of links required is derived for certain networks, whereas for others that do not have an exact solution, an upper bound is given.

2 Problem formulation and the algorithm

We model the existing optical network as a connected planar graph and make the following assumptions:

i. There are two identical optical networks in one region.
ii. The cost of every link between two nodes is identical.
iii. The interconnections between two networks only occur at colocated nodes and cost much less than that of a single link, thus their cost is negligible.

The objective is to derive the minimum number of links required so that there is a path from an arbitrary node to all other nodes in the two networks in which all of the links are directed. Assume that all colocated nodes will be interconnected with interconnections. Thus all traffic to and from the nodes on the second network will go through the colocation interconnection and the original links on the second network can be saved.

In graph theory, bridge is an edge (link) whose removal disconnects a graph [3]. We divide the bridges into two types, namely TP-I bridge and TP-II bridge. A leaf is a vertex of degree 1. TP-I bridge is the link incident to a leaf. The other bridges are TP-II bridges. An articulation point (cut vertex) is a vertex whose removal disconnects the graph. The following states our notations.

L_{min} – Minimum number of links required.
B – Number of bridges in a graph.
A_i – Number of the articulation points with the removal of which the graph will be divided into i subgraphs.

The following states our algorithm. As the two coexisting networks are identical, we concentrate on only one of the networks.

Step 1, remove all the TP-I bridges and the leaves connect to them.

Then some of the TP-II bridges will become TP-I bridges. Remove them as stated in step 1 until no TP-I bridges exist.

Step 2, remove all the TP-II bridges. Denote V as the number of vertices remained after this step.

Step 3, remove all the articulation points.

Thus the graph is divided into several subgraphs.

Step 4, restore the articulation points to all the subgraphs.

Step 5, check whether the resultant subgraphs are Hamiltonian [4] or not. A Hamiltonian cycle is a cycle that visits each node exactly once. If all the resultant subgraphs are Hamiltonian, we can derive that:

$$L_{\min} = 2B + V + \sum_{i=2}^{\infty} A_i (i-1) \tag{1}$$

Proof: It is obvious that every bridge needs two links with opposite directions in order that the two end nodes of the bridge can reach each other. For a single articulation point that will divide a graph into i subgraphs, the total vertices after step 4 should be $V + (i-1)$. As the resultant subgraphs are Hamiltonian, the number of links required is equal to the number of vertices.

If there are some subgraphs that are not Hamiltonian, we concentrate on the non-Hamiltonian graphs and make the following definitions.

D2 node: Nodes of degree two.
Arm: A path consists of entirely D2 nodes and the connecting links plus the two end links connecting to the adjacent non-D2 nodes.
j-D2 arm: Arms with j D2 nodes.

For example, in *Figure 1*, there is one *3-D2 arm* and two *2-D2 arms* in the non-Hamiltonian graph. We denote N_i as node i and $l_{i\text{-}j}$ as the link connecting node i and j. One of the two *2-D2 arms* consists of $l_{1\text{-}2}$, N_2, $l_{2\text{-}3}$, N_3 and $l_{3\text{-}4}$. Then we go to step 6.

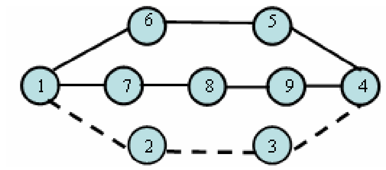

Figure 1. A non-Hamiltonian graph with three arms.

Step 6, find all the arms first, then remove those arms one at a time until all the subgraphs are Hamiltonian.
Denote M_j as the number of j-D2 arms deleted, and V_H as the number of remaining vertices in the resultant Hamiltonian graphs. We come to the following equation:

$$L_{\min} = 2B + \left[V_H + \sum_{j=1}^{\infty} M_j (j+1) \right] + \sum_{i=2}^{\infty} A_i (i-1) \quad (2)$$

Proof: All the j+1 links of the j-D2 arms are required in order that all the D2 nodes in the arm can reach other nodes and be reachable from other nodes. So we need at least $V_H + \sum_{j=1}^{\infty} M_j (j+1)$ links. On the other hand, if an arm with the end nodes, N_{e1} and N_{e2} (they can be same nodes) is added between two nodes, N_m and N_n, of a Hamilton cycle, all the D2 nodes in the arm can reach N_m and thus all other nodes via the link N_{e1} to N_m. And also, all the D2 nodes in the arm can be reached by other nodes via the link N_n to N_{e2}. As for the graph in *Figure 1*, when we view the arm consists of $l_{1\text{-}2}$, N_2, $l_{2\text{-}3}$, N_3 and $l_{3\text{-}4}$ as is added to the Hamiltonian cycle consists of nodes N_1, N_7, N_8, N_9, N_4, N_5 and N_6. N_{e1} and N_{e2}, N_m and N_n, are N_2 and N_3, N_1 and N_4 respectively. Further additions of arms have similar properties as they are not added to the previously added arms. So $V_H + \sum_{j=1}^{\infty} M_j (j+1)$ links are sufficient for all the nodes to be fully connected. Thus, we need exactly $V_H + \sum_{j=1}^{\infty} M_j (j+1)$ links plus the parts denoted by the number of bridges and articulation points, the 1st and the 3rd term in Eq. (2).
For those networks that are still non-Hamiltonian after step 6, it can be proved that the upper bound of L_{min} is:

$$L_{\min} < 2B + 2V + \sum_{i=2}^{\infty} A_i (i-1) \quad (3)$$

3 Example illustration

We illustrate the algorithm using examples of various network topologies.
i) *Ring*. For two coexisting identical ring optical networks, there are no bridges and no articulation points, and both of them are Hamiltonian. So the minimum number of links required L_{min} is equal to the number of nodes V. Thus we can reduce the number of links from $4V$ to V, so 75% of the links are reduced. If there are additional links in the ring networks (For example, two identical mesh networks), a reduction of more than 75% can be achieved.
ii) *Tree.* For two identical tree networks in which all links are bridges, L_{min} is equal to twice the number of the links of one network, namely $2B$. Thus 50% of the links are reduced. It means no link can be saved in network 1, representing the minimum saving that can be achieved.
iii) For two coexisting identical networks that have arbitrary topology as illustrated in *Figure 2*, we will derive L_{min} according to the algorithm discussed in the previous section.

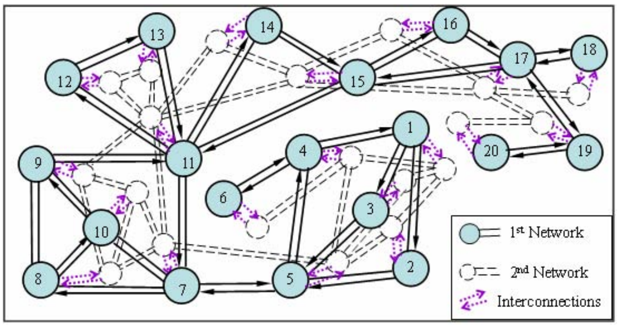

Figure 2. Two coexisting identical networks with interconnections between all the colocated nodes.

For step 1, we delete N_6 and $l_{4\text{-}6}$, N_{20} and $l_{19\text{-}20}$, N_{18} and $l_{17\text{-}18}$ (TP-I bridges). Then delete also N_{19} and $l_{17\text{-}19}$. For step 2, we delete $l_{5\text{-}7}$ (TP-II bridge) and B=5, V=16. After step 4, we derive five subgraphs, namely the subgraphs consist of nodes 1-2-3-4-5, 7-8-9-10-11, 11-12-13, 11-14-15, and 15-16-17 and A_2=1 (N_{15}), A_3=1 (N_{11}). All of the subgraphs are Hamiltonian except the first one. We continue to step 6, delete the arm consists of N_2, $l_{1\text{-}2}$ and $l_{2\text{-}5}$. Thus all of the subgraphs are Hamiltonian and, M_1=1, V_H =15. So the minimum number of links required is:

$$L_{\min} = 2B + \left[V_H + M_j (j+1) \right] + \sum_{i=2}^{\infty} A_i (i-1) = 30$$

Thus 72.2% of the links are reduced considering two networks. One possible result after link reduction is illustrated in *Figure 2* as indicated by the 30 arrows.

4 Summary

We investigated the minimum number of links required in consolidating two duplicated networks to make every two nodes bi-directionally connected. We proposed an algorithm to derive the minimum number of links and found that it is at least the number of the nodes or at least twice the number of the bridges in one of the networks. The algorithm can be extended easily for the consolidation of two non-identical networks.
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5 References

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