

# Analysis of Emission From a Slot Nearby a Microstrip Line on a Printed Circuit Board

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**Abstract**—By advance of miniaturization of electronic devices, a ground plane of a printed circuit board is tend to be small and complicated and the ground plane may have some defects, such as connectors or clearance halls of vias. In this paper, we analyzed the coupling between a microstrip line and a slot in a ground on a PCB using the MTL method to examine an effect of the signal integrity for the slot in the ground. Especially, in order to increase the accuracy the per-unit-length self conductance of the slot line was considered. And also we analyzed emission from a PCB with slot using coupling slot voltages which was calculated by the MTL method. To calculate by using the MTL method, some self and mutual parameters of a microstrip line and a slot were calculated using the FDTD method. Finally, we showed the validity of the method in this paper, comparing results using the MTL method with numerical results using the FDTD method. It is shown that the proposed method is validated by comparing results by the MTL method with results by the FDTD.

## I. INTRODUCTION

Recently with the progress of information processing technology an operating speed of electronic devices becomes very fast. Consequently the problem of undesired emission from such fast electronic devices arises. Since electronic devices are becoming small and lightweight, demands of miniaturizing PCBs in these devices increase. Miniaturization of the PCBs makes those ground to be small, and some defects of the ground, such as for connectors or clearance halls of vias, may appear. Particularly, when many clearance halls are arranged along a line intensively, some slot lines are formed in the ground. If these ground slot lines are placed near a microstrip line, the microstrip line's characteristics, such as impedance or wave numbers, may be affected by the slot lines[1],[2]. And also if these ground slot lines may be coupled with microstrip lines or other electromagnetic sources, they may behave like antennas and become the cause of undesired emission.

The authors showed emission from a printed circuit board with a ground slot line placed nearby a microstrip line[3]. In order to estimate emission, the coupling between the microstrip line and the ground slot line have to be analyzed using the multi-transmission line method (MTL)[4]–[6]. Each self per-unit-length parameters for the microstrip line and the slot line and mutual per-unit-length parameters between them are calculated by using the FDTD method[7]. In this paper, in order to increase the accuracy the authors add the per-unit-length self conductance of the slot line in this MTL method. This conductance value is calculated using emission from the slot line. Finally we estimate emission from the PCB with the slot using the voltage distribution in the slot obtained by the

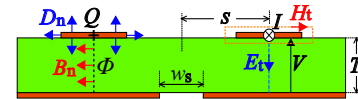


Fig. 1. Cross section of two microstrip lines nearby a slot line.

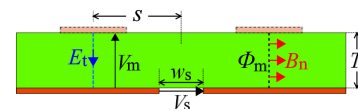


Fig. 2. Cross section of a slot line.

MTL method. And its validity is discussed by comparing with results for exact PCB model using the FDTD method.

## II. THEORY

### A. Per-unit-parameters of a microstrip line nearby a ground slot line

Per-unit-parameters of a microstrip line are calculated using FDTD method when a ground slot line is placed nearby a microstrip line. We will consider the configuration of two symmetrical microstrip lines on both sides of the slot line, as shown in Fig.1. In this case, when two microstrip lines are excited by even mode, the slot line can not be excited. In even mode, the per-unit-length self capacitance and self inductance of the microstrip lines with the slot line are respectively defined by  $C_{\text{even}}$  and  $L_{\text{even}}$ . In the case of odd mode, the effect of the parameters of the microstrip lines by the slot line is very small, then two microstrip lines are excited by odd mode without a slot line. The per-unit-length inductance of the microstrip lines without the slot line is defined by  $L_{\text{odd}}$  in odd mode. Therefore the per-unit-length capacitance  $C_m$  and inductance  $L_m$  of one microstrip line nearby a slot line are respectively given as

$$C_m = C_{\text{even}} \quad (1)$$

$$L_m = \frac{1}{2} (L_{\text{even}} + L_{\text{odd}}). \quad (2)$$

### B. Mutual parameters between a microstrip line and a slot line

There is the MTL method to analyze coupled transmission lines which consists of any number of parallel conductors[4]. In this method, per-unit-parameters of a slot line and mutual parameters between a microstrip line and a slot line can not be calculated easily since a signal in a slot line is

not propagated in quasi-TEM modes. Then we also use the approximate equations and the calculated value using FDTD method. The characteristic impedance  $Z_s$  and the effective relative permittivity  $\epsilon_{es}$  of a slot line can be calculated using the approximate equation[8]. Using  $Z_s$ ,  $\epsilon_{es}$  and the speed of the light in vacuum  $c_0$ , per-unit-length self capacitance  $C_s$  and self inductance  $L_s$  of a slot line are respectively given as

$$C_s = \frac{\sqrt{\epsilon_{es}}}{c_0 Z_s}, \quad L_s = \frac{Z_s \sqrt{\epsilon_{es}}}{c_0}. \quad (3)$$

The current  $I_s$  which is flowing in a slot line is formally calculated by using  $Z_s$  and voltage  $V_s$  on the slot line.

In following, per-unit-length capacitance matrix  $\mathbf{C}$  and inductance matrix  $\mathbf{L}$  for a microstrip line and a slot line are calculated. It is considered that a slot line with width  $w_s$  is placed in the ground plane without a microstrip line as shown in Fig.2. The per-unit-length capacitance matrix  $\mathbf{C}$  is calculated using the coefficients of potential  $\mathbf{P}$  indicated as  $\mathbf{P}=\mathbf{C}^{-1}$ [6]. Diagonal elements,  $p_{11}$  and  $p_{22}$ , of  $\mathbf{P}$  are each reciprocal of self capacitance when the other charge is zero and are given by

$$p_{11} = \frac{1}{C_m}, \quad p_{22} = \frac{1}{C_s}. \quad (4)$$

Off-diagonal elements,  $p_{12}$  and  $p_{21}$ , are given as

$$p_{12} = p_{21} = \frac{V_m}{Q_s} \Big|_{Q_m=0} = \frac{V_m}{C_s V_s} \Big|_{Q_m=0} \quad (5)$$

where  $Q_m$  is the charge of the microstrip line and  $Q_s$  is the charge of the slot line,  $V_m$  is the voltage at the point, where the microstrip line is going to be placed, away a distance  $s$  from the center of the slot line when the line conductor does not exist. Therefore the per-unit-length capacitance matrix  $\mathbf{C}$  is given by

$$\mathbf{C} = \mathbf{P}^{-1} = \begin{bmatrix} \frac{1}{C_m} & \frac{V_m}{C_s V_s} \\ \frac{V_m}{C_s V_s} & \frac{1}{C_s} \end{bmatrix}^{-1}. \quad (6)$$

The mutual inductance  $L_{ms}$  between the microstrip line and the slot line is given by

$$L_{ms} = \frac{\Phi_m}{I_s}$$

where  $\Phi_m$  is magnetic flux penetrated between the point, where the microstrip line is going to be placed, and the ground when the line conductor does not exist. Therefore the per-unit-length inductance matrix  $\mathbf{L}$  is given by

$$\mathbf{L} = \begin{bmatrix} L_m & L_{ms} \\ L_{ms} & L_s \end{bmatrix}. \quad (7)$$

$V_m$  and  $\Phi_m$  are derived by integrating  $E$  and  $H$  calculated in the FDTD method, respectively.

### C. Electric field radiated by a slot line

Using voltages in a slot line, we calculate the electric field radiated by the slot line. When a slot line with width  $w_s$  and infinitesimal length  $\Delta l$  is placed at the origin along the  $z$  axis on the  $xz$  plane as shown in Fig. 3, the  $z$  components of vector

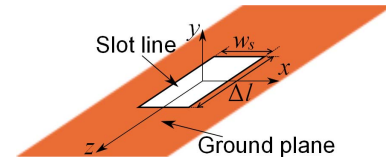


Fig. 3. Configuration of a slot line with infinitesimal length.

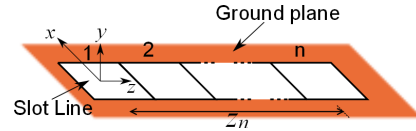


Fig. 4. Slot line configuration for calculation.

potential  $A_z$ , which is placed at a distance  $r$  from the origin, is given by

$$A_z(r, \theta, \phi) = \frac{e^{-jk_0 r}}{4\pi r} E_x \int_{-w_s/2}^{w_s/2} e^{-jk_0 x' \sin \theta \cos \phi} dx' \times \int_{-\Delta l/2}^{\Delta l/2} e^{-jk_0 z' \sin \theta \cos \phi} dz' \quad (8)$$

where  $E_x$  is the the  $x$  component of electric field on the slot and  $k_0$  is wave number of the free space [9]. If the slot width  $w_s$  and length  $\Delta l$  is small enough compared to the wavelength of free space, Eqn.(8) is approximately given as

$$A_z(r, \theta, \phi) = \frac{e^{-jk_0 r}}{4\pi r} E_x(z_0) w_s \Delta l = \frac{e^{-jk_0 r}}{4\pi r} V_s \Delta l \quad (9)$$

where  $V_s$  is the voltage in the slot given as  $V_s = E_x w_s$ . Therefore the  $\phi$  component of electric field  $E_\phi$ , which is placed at the distance  $r$  from the origin, is given by

$$E_\phi(r, \theta, \phi) = jk_0 \sin \theta A_z(r, \theta, \phi). \quad (10)$$

When length  $l$  of a slot is not small enough compared with the wavelength the total electric field from the slot is calculated by dividing the slot into many infinitesimal length slots. Finally, the total electric field is calculated by adding all the radiated electric field from each small slot taking into account the position of them.

### D. Per-unit-length self conductance of slot line by radiation power from slot

In this subsection, we show the calculation method of per-unit-length self conductance of the slot line using the total radiated power from the slot. The slot line with length  $l$  is divided by  $n$  pieces infinitesimal length slot with  $\Delta l$  as shown in Fig. 4.

Electric field  $E_\phi$  from whole the slot line is the sum of the electric field from divided infinitesimal length slot and it is shown by

$$E_\phi^{\text{line}}(r, \theta, \phi) = \sum_0^n E_\phi(r, \theta, \phi) e^{jk_0 z_n \cos \theta} = \sum_0^n \frac{jV_s(z_n) \Delta l}{2\lambda_0 r} e^{-jk_0(r+z_n \cos \theta)} \sin \theta. \quad (11)$$

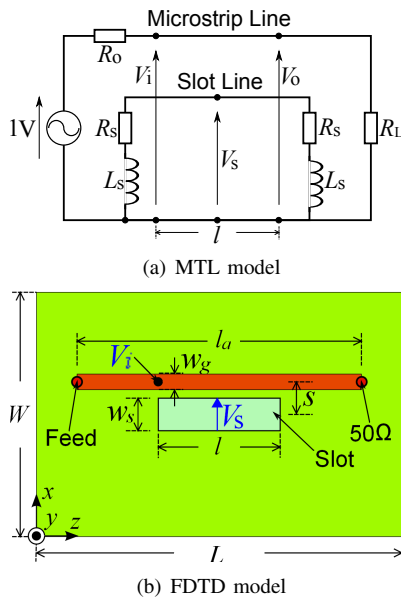


Fig. 5. Configuration of analysis model.

Where, it is assumed that a one end of the slot line is at origin and  $r$  is large enough. The total radiated power is calculated with integrating the radiated power density, which is the square of the electric field  $E_\phi$  divided by the characteristic impedance  $\eta_0$  of free space, over a sphere of radius  $r$  and shown by

$$P^{\text{rad}} = \frac{1}{2\eta_0} \oint |E_\phi^{\text{line}}|^2 r^2 \sin\theta d\theta d\phi. \quad (12)$$

When it is assumed that the per-unit-length self conductance of the slot line is  $G_s$  and the whole conducted loss  $P^{\text{loss}}$  of the slot line is given by

$$P^{\text{loss}} = \frac{1}{2} G_s |V_s^{\text{ave}}|^2, \quad (13)$$

the total radiated power  $P^{\text{rad}}$  from the slot is equal to the dissipated power  $P^{\text{loss}}$  in the slot. Therefore the  $G_s$  is given by

$$G_s = \frac{1}{\eta_0 |V_s^{\text{ave}}|^2} \oint |E_\phi^{\text{line}}|^2 r^2 \sin\theta d\theta d\phi, \quad (14)$$

where we use the average voltage  $V_s^{\text{ave}}$  in the slot to make  $G_s$  constant over the slot line.

### III. ANALYSIS

#### A. Analysis Model

In order to validate the derived the radiated power from the slot line, we calculate coupling between a microstrip line and a slot and emission from the PCB with a slot using MTL method. And we also compare the results using the MTL with results using the FDTD method.

In the case of the MTL method, the configuration of the microstrip line and the slot line on the PCB is used as shown in Fig.5(a). The microstrip line has width  $w_g=2.8\text{mm}$  and length  $l$ , the characteristic impedance of about  $50\Omega$  if the slot does not exist. A signal circuit which consists of an open-circuit voltage 1V and output impedance  $50\Omega$  is connected to one

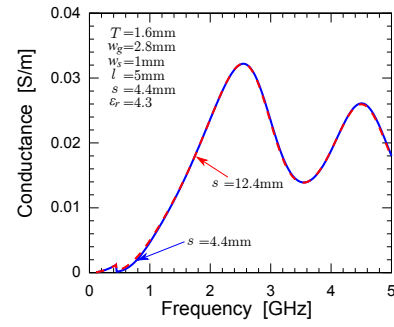
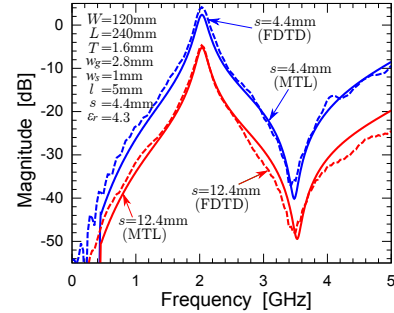


Fig. 6. Per-unit-length conductance derived by radiated power.

Fig. 7. Normalized voltages  $V_s/V_i$  in slot line for various space  $s$ .

end of the microstrip line and an  $50\Omega$  resistor is connected to the other end. The slot line with width  $w_s$  and length  $l$  in the ground is placed parallel in the position  $s$  away from the microstrip line. The both ends of the slot line are terminated by a series connection of a resistance of  $R_s$  and a inductance of  $L_s$ . The values of  $R_s$  and  $L_s$  are derived using the FDTD method[3]. The PCB consists of a dielectric substrate which has thickness  $T=1.6\text{mm}$  and infinite length, infinite width and is made by FR4 of relative permittivity  $\epsilon_r=4.3$ . The ground plane is placed on the other side of the microstrip line. The voltages  $V_s$  at the center of the slot is calculated. Since  $V_i$  is indicated as the incident voltage to the line,  $V_i = 0.5\text{V}$  is used because of the  $50\Omega$  output impedance of signal and the  $50\Omega$  characteristic impedance of the line.

In the case of the FDTD method, the configuration of the microstrip line and the slot line on the PCB is used as shown in Fig.5(b). The length of the microstrip line is longer than the slot line length because its length is long enough to ignore the higher mode wave created in the termination point and the feed point. The length of the microstrip line is  $l_a=90\text{mm}$ . The PCB has length  $L=110\text{mm}$  and width  $W=100\text{mm}$ . The voltages  $V_s$  at the center of the slot is calculated. The voltage  $V_i$  at the point in the microstrip near the end of slot is also calculated. However, since  $V_i$  is indicated as the incident voltage to the line, the calculation of  $V_i$  is finished after the propagation wave passes through the observation point. Other conditions are the same as the MTL method.

#### B. Coupling between microstrip line and slot line

First, in order to derive the per-unit-length conductance of the slot line, the total radiated power is calculated without the per-unit-length conductance of the slot line. The results of the

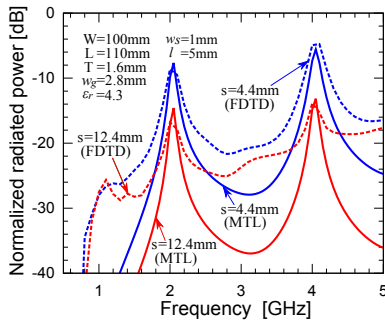


Fig. 8. Normalized radiated power from slot line for various space  $s$ .

per-unit-length conductance of the slot line for the slot width  $w_s=1\text{mm}$ , length  $l=5\text{mm}$  and space  $s=4.4\text{mm}$  and  $s=12.4\text{mm}$  are shown in Fig.6. Both conductance values are almost same. It is considered that both voltage distribution over the slot are almost same though the coupling voltage for  $s=12.4\text{mm}$  is smaller than that for  $s=4.4\text{mm}$ . These calculated per-unit-length conductance of the slot line is used in following results.

The voltages  $V_s$  at the center of the slot line excited by the coupling with the microstrip line is calculated using the MTL method and the FDTD method with conductance of the slot line. The calculated results is normalized by the incident voltage  $V_i$ .

The results of  $V_s/V_i$  for the slot width  $w_s=1\text{mm}$ , length  $l=5\text{mm}$  and space  $s=4.4\text{mm}$  and  $s=12.4\text{mm}$  are shown in Fig.7. In this figures, the solid line indicates the results using the MTL method and the dashed line indicates that using the FDTD method. As seen in Fig.7, the large voltages are observed at 2.0GHz regardless of  $s$ . At 2.0GHz where the both voltages have peak value, these peak voltages are caused by the half-wavelength resonance of the slot line with  $l=5\text{mm}$ . And also at 4.0GHz where the voltage  $V_s/V_i$  has the smallest value because these smallest voltages are caused by the one-wavelength resonance, named as anti-resonance, of the slot line. The resonant frequency and anti-resonance frequency calculated by the MTL method and that by the FDTD method are almost same. And also good agreement between the MTL results and the FDTD results is obtained in the frequency range of up to 5GHz, especially around peak value.

### C. Total radiation power from PCB

In order to evaluate emission from the PCB with the slot line, the total radiated power from the PCB is calculated using the MTL method and the FDTD method with conductance of the slot line. In the case of the MTL method, the total radiated power is calculated with integrating the radiation power density, which is the square of the electric field  $E_\phi$  divided by the characteristic impedance of free space, over a sphere of 1m radius including the PCB. In the case of the FDTD method, the total radiated power is calculated with integrating the Poynting vector over a surface enclosing the PCB. The calculated results is normalized by the incident power  $P_{\text{inc}}$  of the microstrip line given by

$$P_{\text{inc}} = \frac{V_i^2}{Z_0}. \quad (15)$$

The results of normalized total power for  $w_s=1\text{mm}$ ,  $l=5\text{mm}$  and various  $s$  are shown in Fig.8. As seen in Fig.8 there are the peaks of radiation power at 2.0GHz and 4.0GHz. These frequencies are same as to the resonance frequencies and anti-resonance frequencies shown in previous subsection, and emission is obviously dominant at the resonance of the slot line. These peak values in the MTL method and those in the FDTD method are almost same. But the local minimum values of radiation power in the MTL method are appreciably smaller than those in the FDTD method because the results in the MTL method are calculated only for the effect of the slot line and ignored the effects of the radiation from microstrip line, common-mode current and so on. Therefore, it can be estimate the maximum total radiation power from the slot line at resonance frequency by using the proposed method, but it is necessary to consider the other effects, such as the microstrip line or common-mode current, to estimate the radiation power precisely.

## IV. CONCLUSION

In this paper, emission from a printed circuit board with a slot line placed nearby a microstrip line was analyzed using the MTL method with the per-unit-length conductance of the slot. The per-unit-parameters of the slot line and the microstrip line were derived by using the FDTD method. And the per-unit-length conductance of the slot was derived by the total radiated power from the slot line calculated by the MTL method without the conductance. To validate the theory, normalized coupling voltages in the slot line and normalized total radiation powers from the PCB were calculated using the MTL method and the FDTD method. Comparing both results, the validity of the proposed method was shown since the results of the case in the MTL method was approximately same as those of the FDTD method. And also, it was shown that considering the other effects, such as a current flowing in the microstrip line or a common-mode current, is necessary in order to estimate the total radiation power precisely.

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