Inductance Extraction of a Meander Line on a Coplanar Plane using Partial Element Method

Bo Pu, Kwang Ho Kim, Wansoo Nah
Electromagnetic Compatibility Laboratory, Department of Electrical and Electronic Engineering
Sungkyunkwan University
Suwon, Republic of Korea

***Abstract*—This paper proposes an inductance extraction method for a compact meander line structure. The Green House algorithm is introduced to solve the complicated inductive coupling effects and make it possible to calculate a meander line with any shape. Three different cases with a meander line or a straight line as a return path are discussed. Self and mutual inductances are demonstrated based on partial element theory, and then the loop inductance for the three structures are derived. The effect of the gap between adjacent signal and return lines on the mutual inductance is also addressed.**

Keywords—meander line; inductive coupling;partial element method;gap effect

I. Introduction

During the never-ending development of the semiconductor technology, there is a typical trend that the complexity of integration and speed of operation in modern electronic systems is continuing to increase. As electrical devices become more and more mobile, the integration density in package or Printed Circuit Board (PCB) is higher, and the size of the PCB in electrical devices gets smaller [1]. As a method for reducing the timing error in the PCB design, delay lines have been adopted in critical nets to provide predetermined timing delay of the clock traces or the signal traces. A space and cost effective delay line structure should have a regular and delay-predictable shape with a compact design [2]. The most popular delay line scheme in PCB design is the meander-type structure, which consists of a group of unit delay lines with equal lengths.

Meander line technology also allows designing antennas with a small size and provides wideband performance. Meander line antenna is a type of printed antenna that achieves miniaturization in size by embedding the wire structure on a dielectric substrate. In basic form meander line antenna is a combination of conventional wire and planer strip line. Benefits include configuration simplicity, easy integration to a wireless device, inexpensive and potential for low SAR features [3].

Futuremore, meander line structure is possible to be applied as a busbar in hybrid vehicle system or a filter to suppress the wideband noise since its inductive characteristic. Publications of inductance extraction have discussed the similar inductive structure − spiral line [4] but the inductance calculation of a meander line is rarely mentioned, especially at three dimensional physics based view.

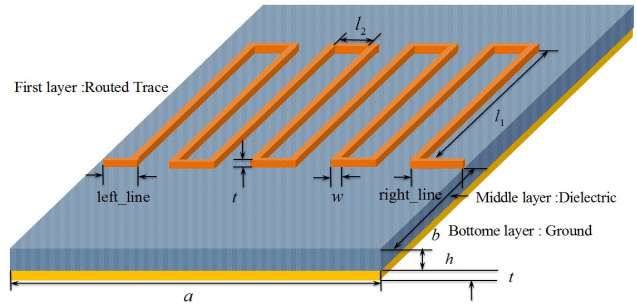

Fig. 1. Physical configuration of a meander line structure.

This paper proposes an analytical method for the inductance calculation of symmetrical meander lines. Section II discusses the basic theory of partial element method and the parameter optimization procedure in general. Partial self and mutual inductance extractions for three different signal and return pairs are explained, and loop inductance are also derived in section III, respectively. Here, the analysis of the complicated mutual coupling situation is addressed. Gap effect between the symmetrically located meander lines is also demonstrated in section IV and compared with three dimensional simulator results.

II. Fundamental of Partial Element Method and Design Optimization

The advantage of the partial elements modeling is the avoidable to the complicate return path. In this method, whatever the properties for the traces, signal or power, propagating or return, they are all regarded as the parts of the composition for the current loop. The Green House algorithm is used to combined the results generated by partial element method, and derive a loop element value. In the coming section, inductance extraction for the meander lines is explained in detail.

A typical N-turn meander line structure with physical description is shown in Fig. 1. Fundamental elements of a meander line are longer conductor with length l_1 and shorter conductor with length l_2, and they are placed orthogonally. Other parameters are line width w, thickness t and substrate thickness h, etc. An appropriate inductance value is obtained by an optimal arrangement of the size parameters. These parameters determine the self and mutual inductances and affect the filter efficiency. Here, we designed a flow chart to describe the optimization procedure and corresponding variable effects in it as depicted in Fig. 2.

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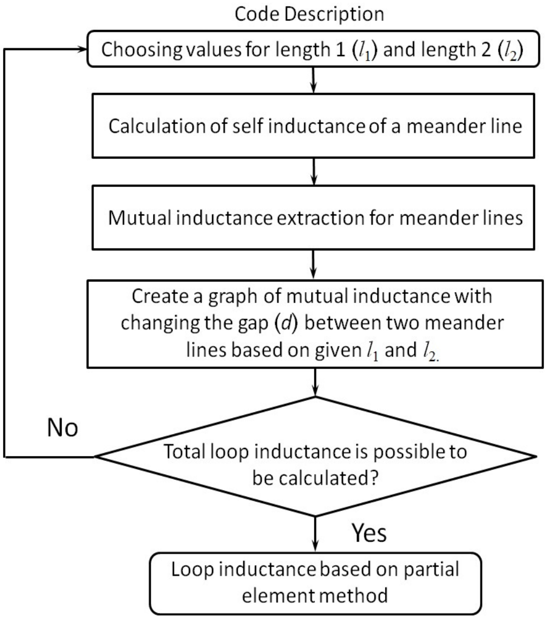

Fig. 2. Optimization flow chart for filter design based on arranging the parameters of a meander line.

III. Inductance Extraction for Meander lines

A. Fundamental theory for inductance calculation of meander lines

In a meander line, the most common and basic element is the cuboid cell. Equations for calculating self and mutual partial inductances of filaments were introduced in [5]−[7]. A rectangular solid element could be regarded as the assemblage of finite filaments. Thus, the self partial inductance of a cuboid is described as the combination of mutual inductances of all sub filaments. Equation to derive a self-inductance for a rectangular solid element with length l, width w and thickness t is in (1). It is represented as a integral of mutual inductances among constituent filaments inside the cuboid.

$$L_{\text{self}} = \frac{\mu_0}{4\pi}\frac{1}{w^2 t^2}\int_{x_2=0}^{w}\int_{x_1=0}^{w}\int_{y_2=h}^{t}\int_{y_1=0}^{t} M_{f1} dy_1 dy_2 dx_1 dx_2$$
$$= \frac{\mu_0 l}{2\pi}\left[\ln(\frac{2l}{w+t}+\frac{1}{2}+0.2235\frac{w+t}{l}-\mu_r\left(0.25-X\right)\right] \quad (1)$$

Where M_{fl} is the mutual inductance of filaments, and w and t are the width and thickness of the rectangular land,respectively. A fitted parameter X, which is defined in [8], is applied to describe the skin effect term in partial inductance.

Mutual inductance between two near conductors is generated by the mutual inductive coupling. Relative position of those two conductors is the most important factor to the mutual inductance. In reality, the relative location between the nearby routed races could be on the same horizontal surface or with vertical difference. As described in Fig. 3, the near trace lands with cross-sectional areas A and A□ are seated in different layers with a geometric mean distance (GMD) d. The horizontal pitch for them is p and vertical difference is h. Characteristics of the conductors are w_1, w_2 for width and t_1, t_2 for length, respectively.

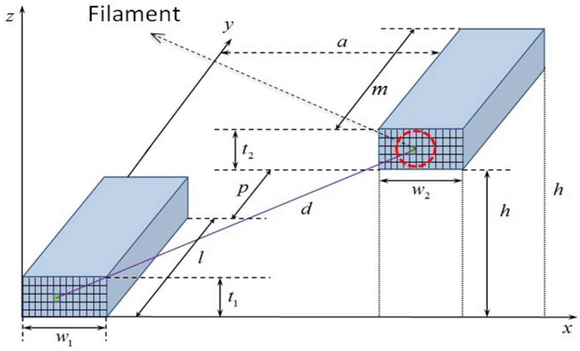

Fig. 3. Determination of partial mutual inductance for two nearby rectangular solid elements.

Equation (2) represents the mutual inductance M_p for any two cuboids with some distance. The M_{f2} here is calculated in [6] as the mutual inductance for any two filaments in different cuboids and shown in (3).

$$M_p = \frac{\mu_0}{4\pi}\frac{1}{VV'}\int_V\int_{V'} M_{f2} dV dV' = \frac{\mu_0}{4\pi}\frac{1}{VV'}[M] \quad (2)$$

$$M_{f2} = \left[f(z)\right]\Big|_{(p+m),(l+p)}^{(l+p+m),p} = \left[f(l+p+m)-f(p)+f(p+m)-f(l+p)\right]$$
$$\left[f(z) = z\ln\left(z+\sqrt{z^2+d^2}\right)-\sqrt{z^2+d^2}\right] \quad (3)$$

To extend the equation from filaments to cuboid, the thickness t_1, and t_2, width w_1 and w_2 is taken into account and the mutual partial inductance between two trace lands becomes the equation as in (4).

$$M = \left[\left[\left[f(x,y,z)\right]\Big|_{(a+w_2-w_1),(l+s)}^{(a-w_1),(a+w_2)}(x)\right]_{(s+m),(l+s)}^{(l+s+m),s}(y)\right]_{(b+t_2-t_1),b}^{(b-t_1),(b+t_2)}(z) \quad (4)$$

Until now, self and mutual partial inductances for a rectangular cross-section structure have been extracted and can be applied to an analysis of a symmetrical meander lines structure in B.

B. Mutual inductance for a symmetrical meander lines structure

The return path for a signal/power trace in another plane is always hard to determine and figure out in miniature or complex components such as multilayer PCB and packages, therefore, the return path designed in the same plane or as nearby bonding wire is an effective way and could mention the backward signal. Here, two different kinds of return paths, which are a meander line and a straight line for a meander or straight type signal line, are proposed for characteristics comparison as shown in Fig. 4. There are total three different structures for a signal and return pair. They are symmetrical meander lines pair, a meander, a straight line pair, and two straight lines pair, respectively. Meander lines in the three different case have the same physical configuration with width 1.47 mm, longer length 20 mm, shorter length 5 mm and a same total length M in x axis. Gaps for different line pairs also keep the same with 4 mm.

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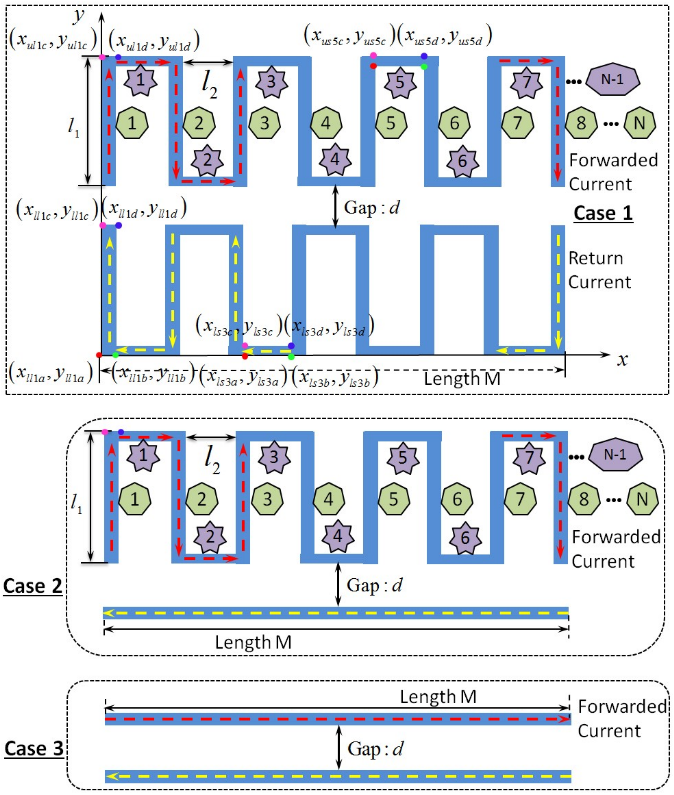

Fig. 4. A description for three different kinds of forwarded and return line pairs in the cartesian coordinate system.

Currents propagate in opposite directions at high and low lines for the two different return path cases. The symmetrical meander lines consist of longer and shorter rectangular solid elements and the current direction of any two elements are not the same. Thus the total mutual inductance is influenced by the positive and negative results of each mutual inductance between any two elements.

Mutual inductances are divided into two different kinds, depending on the relative positions between each two elements. They are mutual inductances of elements in the same meander line and in different meander lines.

There are mutual inductances for two longer conductors located in parallel and that for two aligned or offset short conductors. The sequence is assigned to those conductor elements for a further determination of the positive or negative mutual couplings in accordance with current directions. Current of longer conductors in the same meander line displaces in a uniform direction for two elements with same parity order while in contra-direction between the odd and even order elements. Short conductor elements transfer current with opposite directions in upper and lower meander lines separately.

Initially, we discuss the mutual inductance extraction for the longer conductors in the same meander line. It is regarded as the fundamental and most important part of all mutual inductance calculation. Mutual couplings here are divided into the two aspects as described and determined as positive for odd (even) □ odd (even) pair and negative for odd-even pair. As a result, total mutual inductance for N longer elements in the same meander line is calculated in (5)

$$\sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{i=1}^{N} M_{i,j} = \sum_{i=1}^{N}\left(\sum_{\substack{j=1\\ j\neq i\\ \mathrm{mod}(i,2)=\mathrm{mod}(j,2)}}^{N} M_{i,j} - \sum_{\substack{j=1\\ j\neq i\\ \mathrm{mod}(i,2)\neq\mathrm{mod}(j,2)}}^{N} M_{i,j} \right), \quad (5)$$

where the symbol mod means modulus after the division of the order to 2, and it distinguishes the positive and negative sub-mutual inductances. Opposite currents direction of nearby longer elements of the meander line is an advantage compared to the spiral line. The radiated filed declines by the reduction of the currents, and therefore avoids a severe EMI problem to other integrated circuits.

The extraction of mutual couplings for shorter elements in a meander line seems to be easier than the calculation for longer parts since current transfers through the short ones in the same direction. Particularly, equations for calculating mutual inductances are different because there are two relative positions which are aligned and offset. The mutual inductance for short elements is described in (6).

$$\sum_{\substack{j=1\\ j\neq i}}^{N-1}\sum_{i=1}^{N-1} M_{i,j} = \sum_{i=1}^{N-1} \sum_{\substack{j=1\\ j\neq i\\ \mathrm{mod}(i,2)=\mathrm{mod}(j,2)}}^{N-1} M_{i,j\_\mathrm{aligned}} + \sum_{i=1}^{N-1} \sum_{\substack{j=1\\ j\neq i\\ \mathrm{mod}(i,2)\neq\mathrm{mod}(j,2)}}^{N-1} M_{i,j\_\mathrm{offset}} \quad (6)$$

A more complicate structure is the symmetrical meander lines which is also depicted in Fig. 4. Mutual inductance of this configuration includes not only the couplings in a meander line, but also the mutual effects between the upper and lower meander lines. Considering the current directions in the two meander lines, the positive mutual couplings belong to the odd-odd and even-even pairs while the negative mutual inductance are generated between the odd and even order elements.

The extraction of mutual inductance only between the two meander lines is addressed here to discuss the gap effect for the symmetrical meander lines. Equation to describe the mutual inductance for longer elements between two lines is in (7).

$$\sum_{j=1}^{N}\sum_{i=1}^{N} M_{i,j} = \sum_{i=1}^{N}\left(\sum_{\substack{j=1\\ \mathrm{mod}(i,2)==\mathrm{mod}(j,2)}}^{N} M_{i,j} - \sum_{\substack{j=1\\ \mathrm{mod}(i,2)\sim=\mathrm{mod}(j,2)}}^{N} M_{i,j} \right) \quad (7)$$

The difference in the mutual couplings calculation between (5) and (7) are two different longer elements in one meander line of the former equation and mutual effects extracted for any two longer elements in two distinct meander lines for the latter.

$$\sum_{j=1}^{N-1}\sum_{i=1}^{N-1} M_{i,j} = -\sum_{i=1}^{N-1}\sum_{j=1}^{N-1} M_{i,j} \quad (8)$$

Currents in the upper line and lower line move in different directions for shorter conductors, thus description of mutual inductance for the short parts in diverse meander lines is not hard to obtain and shown in (8).

The total loop inductance for the symmetrical meander lines structure is possible to obtain based on (1) □ (8). Given a gap with 4 mm, the total loop inductance comparison for symmetrical meander lines and a meander line with a straight line as the return path is shown in Fig. 5.

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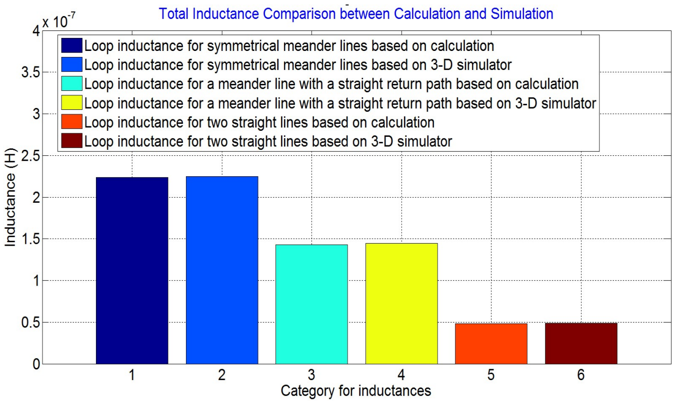

Fig. 5. Loop inductances comparison from calculation and simulation for three different kinds of structures.

The obvious divergence of the loop inductance for these two cases is caused by a reduction of the self inductance from a meander line to a straight line. We also developed a code for a convenient calculation of inductance to any size of meander lines based on MATLAB. The extracted value, which has a skin effect term but ignores the proximity effect, are valid up to 3GHz comparing with the simulation result since the inductance difference caused by proximity effect is only 0.76% at that frequency for our miniature structure based on the derivation at high frequency [5].

IV. Gap Effect Discussion and Validation

Gap effect is the most common phenomena for any two nearby conductors in a discussion of mutual couplings. In a normal case, it is thought the mutual inductance decreases with a rising of gap distance since the corresponding coupling magnitude declines. However, the mutual coupling effect in the symmetrical meander lines structure generates an increase in total mutual inductance. An explanation for this result is the negative mutual effects which take a large proportion of all mutual inductances are reduced largely by increasing the gap between two meander lines.

Mutual inductance for the mentioned symmetrical meander lines configuration is (9) and defined as the sum of inductances in (7) and (8) with consideration of couplings between the two lines.

$$M_{\mathrm{mutual}} = \sum_{i=1}^{N}\left(\sum_{\substack{j=1 \\ \mathrm{mod}(i,2)==\mathrm{mod}(j,2)}}^{N} M_{i,j} - \sum_{\substack{j=1 \\ \mathrm{mod}(i,2)\sim=\mathrm{mod}(j,2)}}^{N} M_{i,j} \right) - \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} M_{i,j} \quad (9)$$

Parameter to describe the gap distance is determined as a part of the y variable in (4). To demonstrate the gap effect to the symmetrical meander lines structure, a chart depicting the relationship between mutual inductance and gap for two different return path cases is shown in Fig. 6. A validation based on a 3-D simulator named Q3d of ANSYS is added into the chart to verify our proposed numerical analysis results.

A descendent trend for the mutual inductance with an ascendant gap distance can be observed in this figure. Mutual inductance starts with negative 4 nH for a gap around 4 mm and increases up to minus 2.5 nH with a distance about 20 mm. Results achived from the proposed extraction method and those

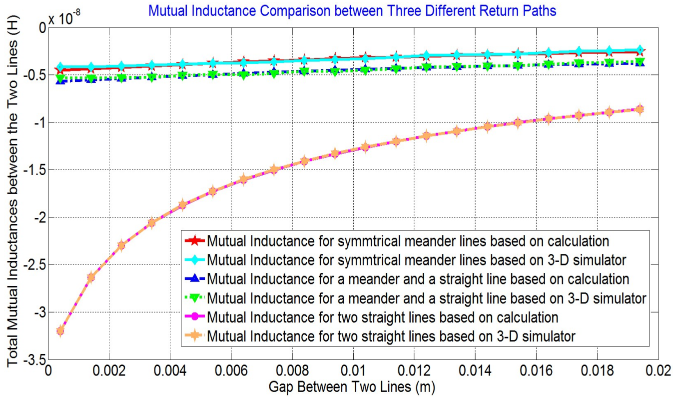

Fig. 6. Mutual inductances extracted from calculation and simulation for two different kinds of return paths.

obtained with the 3-D simulator are in a good agreement. Mutual inductance for a straight line return path is larger than the meander line type return trace since the average gap for former one gets smaller than the latter.

Proper inductance values are available for establishing LC resonators in a filter, and then the procedure in Fig. 2 for designing an appropriate filter used to enhance the immunity of integrated circuits is completed with the numerical analysis.

V. Conclusion

A physics-based closed loop inductance expression based on the partial element menthod for three dimensional meander line structures is proposed in this paper. Three different inductive coupling situations consisted of both meander lines and straight lines are discussed and verified by a 3-D simulator. The gap effect on the mutual coupling behavior is also addressed for the divergence cases. Implementing our proposed approach, the loop inductance is easily and convenient to be achieved for meander lines with any sizes and shapes.

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