Generalized Debye Model for PCB Dielectrics and Conductors

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***Abstract*—Electromagnetic (EM) simulators are commonly used for electromagnetic compatibility and signal integrity (SI) analysis of printed circuit boards (PCB). The accuracy of such analysis depends on the models used for PCB dielectrics and conductors. These models should be broadband and preserve the physical properties of the materials, such as causality and passivity. One such model for dielectrics is the Debye model. Conductors have typically been modeled using a simple surface impedance formula, which is accurate for smooth conductors and small skin depth. With the evolution of high-speed designs, surface roughness loss has also become increasingly important for signal integrity of PCB interconnects. In this paper, we propose a Debye-like model for conductors possibly having rough surfaces. The advantages of a Debye model are its flexibility for arbitrary variation of surface roughness; guaranteed passivity in SI analysis; and availability of an equivalent circuit representation.**

***Keywords*—Debye model, skin effect, surface roughness.**

I. Introduction

The complex permittivity of lossy substrates can be approximated using a Debye model as

$$\varepsilon = \varepsilon'_\infty + \sum_{i=1}^{N} \frac{a_i}{s+b_i}, \tag{1}$$

where a_i and b_i are physically related to the strength and time constants of various relaxation processes, and s is the Laplace variable. The order of the approximation N can be chosen as high as possible as long as the extracted a_i and b_i are all positive coefficients. This ensures that the obtained model represents a passive network over all frequencies and therefore satisfies Kramers-Kronig relations for dielectrics. The Debye model is quite useful in time-domain simulations, as it allows the consideration of the frequency-dependent material properties using an RC type of an equivalent-circuit model.

There is no corresponding model for lossy conductors. In this paper, a generalization of Debye models will be presented to represent conductors as well. This will ensure a physics-based RL type of model for conductors with surface-roughness loss. We propose the following Debye-like model to represent the surface impedance of conductors:

$$Z_s = R_{dc} + \sum_{i=1}^{N} \frac{s a_i}{s+b_i}, \tag{2}$$

where Z_s is the surface impedance and R_{dc} is the dc resistance of the conductor. This Debye model represents the variation of surface-impedance from dc up to the frequency range defined by the order of the model.

We present both closed-form solutions and curve-fitting approaches to calculate the coefficients of the Debye model. Closed-form solutions are available for dielectrics with a constant loss tangent and for conductors with smooth surfaces.

II. Debye Model for Dielectrics

A. Closed-Form Solution

Commonly used PCB substrates such as FR-4 tend to have an approximately constant loss tangent in the frequency range of interest [1], [2]. Hence, an average loss tangent value can be used to obtain a simple broadband model for such dielectrics. A constant loss tangent however implies that the dielectric constant is frequency-dependent according to Kramers-Kronig relation. Since the complex permittivity is a minimum-phase function, the dielectric constant can be exactly defined (up to a constant) for a given frequency-independent loss tangent using the equation

$$\varepsilon = a s^{-2\delta/\pi}, \tag{3}$$

where a is an arbitrary positive constant, δ is the argument of $\tan\delta$, and s is the Laplace variable [3], [4].

A good overview of methods for network realizations of nonrational functions such as (3) can be found in [5], [6], [7]. Following the approach in [8], [9], a closed-form Debye model can be obtained, which results in an RC model for interconnect capacitance as shown in Fig. 1. The parameters of this model is available in closed-form as shown in [8].

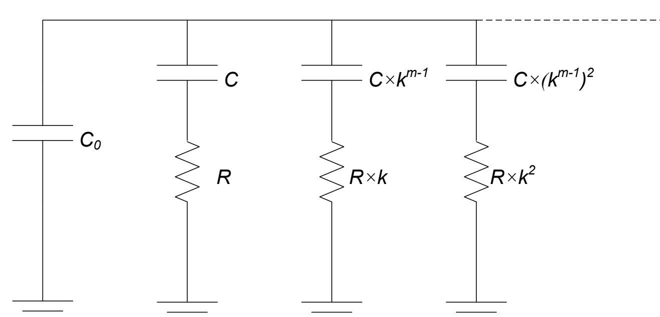
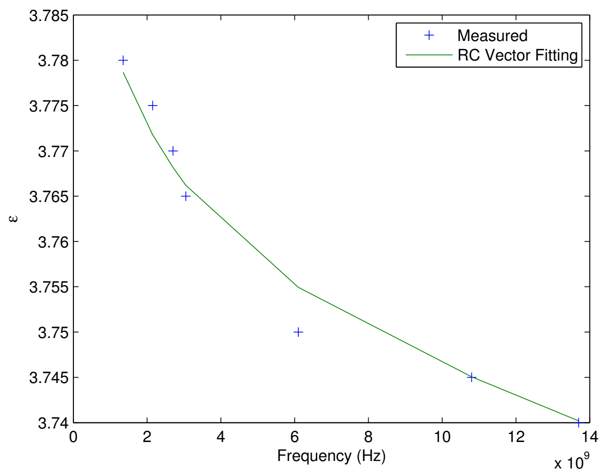


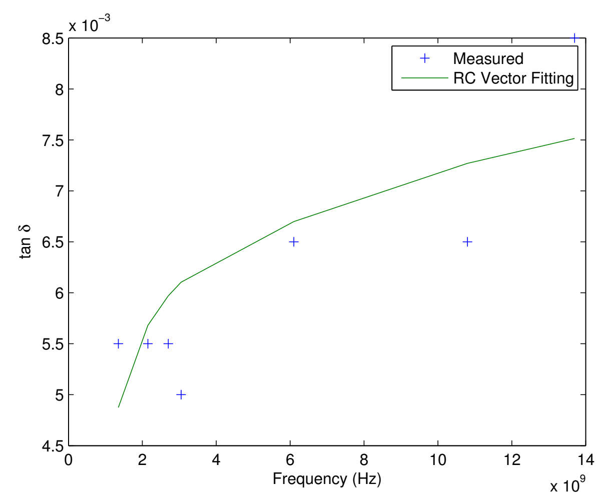
Fig. 1. Debye model representing a lossy dielectric.

B. RC Vector Fitting

For arbitrary variation of the loss tangent, each individual element in the RC equivalent circuit model in Fig. 1 needs to be adjusted. This is equivalent to fitting a rational function in the form of a Debye model in (1). This can be achieved by the vector-fitting algorithm [10], which however does not guarantee positive RC elements. One approach for enforcing the constraint of positive elements is using heuristic optimization methods such as genetic algorithms [11]. Recently, an RC vector fitting approach has been developed, which easily enforces this constraint [12]. In [12], the RC vector fitting approach has been used to fit measured data as shown in Fig. 2. The frequency-dependent behavior of the complex permittivity is well captured. Note that the measured data may contain measurement errors, hence an excellent match between the measurement and the Debye model is not expected. Rather, the Debye model may provide additional confidence in the extracted data, as the Debye model represents a physically plausible permittivity variation with frequency.



(a)



(b)

Fig. 2. (a) Dielectric constant; (b) loss tangent fitted to measured data using RC vector fitting

III. Debye Model for Conductors

A. Closed-Form Solutions

Efficient RL realizations for skin effect have been reported in [4], [13], [14], [15], [16], [17]. These approaches are adequate when the skin-effect resistance varies with $\sqrt{f}$, which is accurate for conductors with a smooth surface. In this case, the surface impedance is given as

$$Z_s = \sqrt{\mu/\sigma}\, s^{1/2} \tag{4}$$

This impedance function has exactly the same form as the lossy dielectric admittance in (3). We can therefore create an equivalent circuit model using a similar approach as in [8]. The main difference is that the exponent of the complex frequency s is always fixed as $1/2$ for the surface impedance.

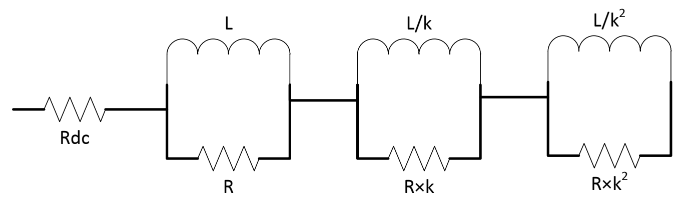


Fig. 3. Debye model representing a lossy conductor. R_{dc}: dc surface resistance; k: spacing factor; ω_0: on-set frequency for skin effect; $R = R_{dc}(k-1)$; $L = R/\omega_0$.

The resulting RL equivalent circuit model is shown in Fig. 3. This RL model requires the following four parameters:

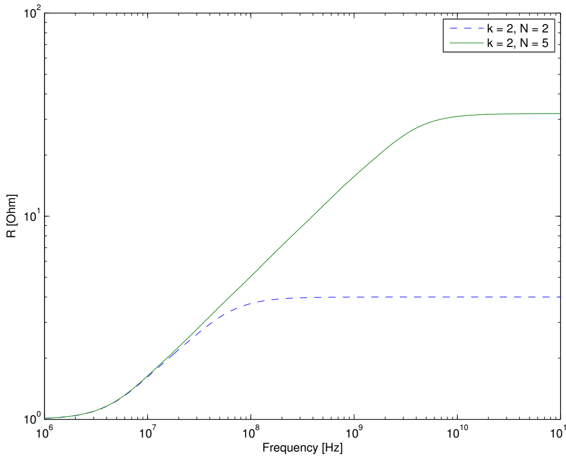
- R_{dc}: dc surface resistance
- ω_0: on-set frequency for skin effect
- k: Spacing factor to be chosen based on the required accuracy vs. bandwidth of the model
- N: Number of RL branches

Based on this input, the values of the circuit elements in Figure 3 can be obtained analytically following [9] as

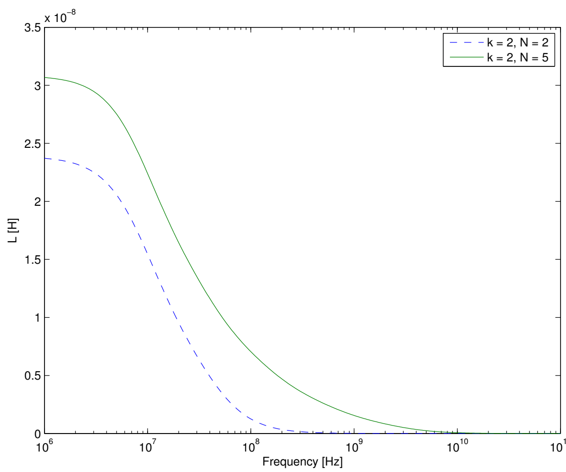
$$R = R_{dc}(k-1) \tag{5}$$
$$L = R/\omega_0 \tag{6}$$

As the number of RL branches N in this network is increased, the bandwidth of the model is increased as shown in Figure 4. Note that the dc internal inductance may also change with the number of branches. This is usually not critical, since the external inductance is much larger than the internal inductance. As more and more branches are added to the model, the dc internal inductance would approach to an asymptotical value. The on-set frequency of the model can also easily be adjusted as shown in Figure 5.

Finally, the bandwidth can be adjusted with the spacing parameter k. Using a larger k increases the bandwidth, but introduces oscillations as shown in Figure 6. The on-set frequency was also adjusted to align the two plots in this figure.



(a)



(b)

Fig. 4. RL model for $\omega_0 = 2\pi \times 10^7$ rad/s, $R_{dc} = 1\Omega$.

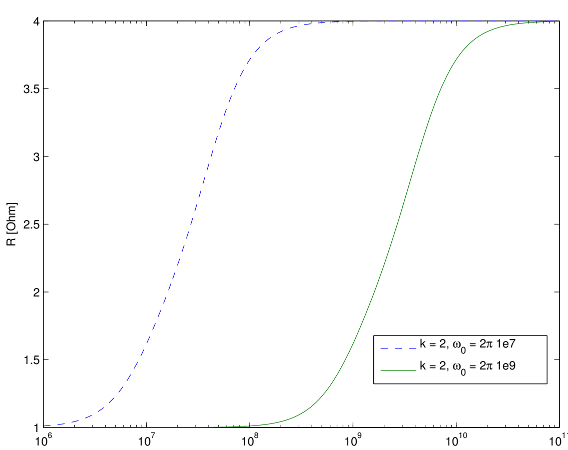


Fig. 5. Variation of the on-set frequency. RL model for $N = 2, R_{dc} = 1\Omega$.

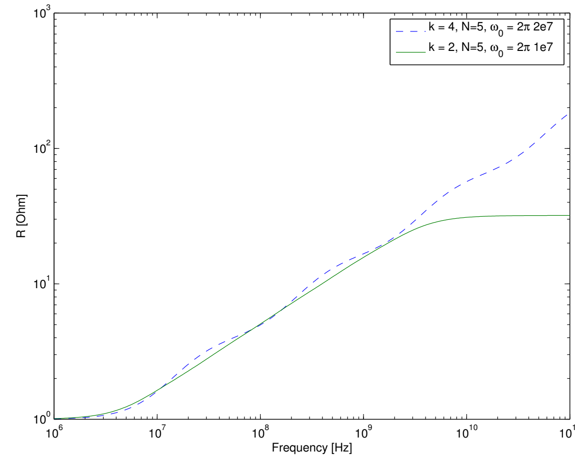


Fig. 6. Variation of the spacing parameter k. RL model for $N = 5, R_{dc} = 1\Omega$.

B. RL Vector Fitting

For practical PCB interconnects, the surface impedance may deviate from the analytical model in (4). For example, surface roughness loss results in impedance increasing faster than $\sqrt{f}$. For such arbitrary variations, an RL macromodel for impedance is necessary. This can be achieved by suitably modifying the RC vector fitting algorithm in [12].

In the presence of surface roughness loss, the real part R_s of the surface impedance can be modified as

$$R_s = \kappa\sqrt{\pi f \mu/\sigma} \tag{7}$$

where κ is the surface roughness loss correction factor. A commonly used approximation for κ is the Hammerstadt formula

$$\kappa = 1 + \frac{2}{\pi}tan^{-1}\left[1.4\left(\frac{\Delta}{\delta}\right)^2\right] \tag{8}$$

where Δ is the rms value of surface roughness and δ is the skin depth. The knowledge of the real part R_s is sufficient to create a macromodel in the form of (2) for the complex surface impedance Z_s on the basis of the causality principle.

As an example, a 35μm thick conductor made of copper ($\sigma = 5.8 \times 10^7$S/m), and a surface roughness rms value of 1.61 μm is considered. Figure 7 demonstrates good fitting results of the RL macromodeling routine against the real part of Z_s, for an order of $N = 14$. The fitting demonstrated here is adequate to replicate the behavior of the real part of the surface impedance, including surface roughness effects, while achieving a passive result.

An RL macromodel is also desirable to represent the per unit length resistance of transmission lines. As an example, Ansys Maxwell 2D simulator is used to model a 200μm wide, 35μm thick, copper microstrip transmission line on FR4 substrate of 100μm thickness, across frequencies ranging from DC to 9 GHz. The final objective is to obtain a Foster RL impedance expression as in (2). The vector fitting algorithm is implemented to obtain a converging set of stable, real poles, which are sent into a residue extraction routine. The

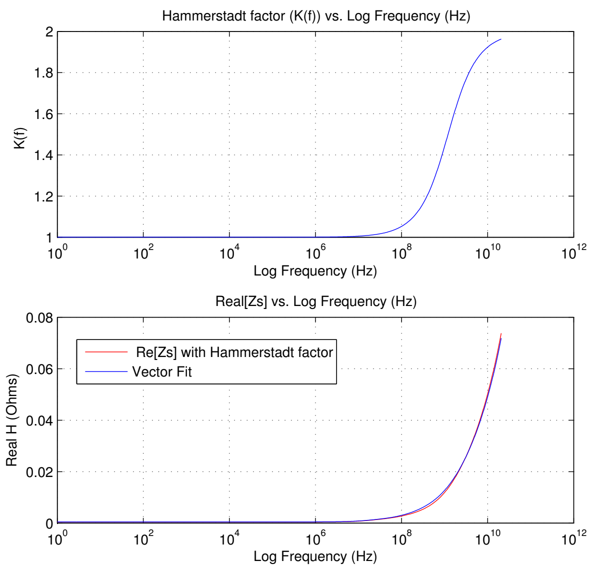


Fig. 7. Top: Hammerstadt surface correction factor. Bottom: corresponding surface resistance R_s with accurate fitting of an RL macromodel.

residues, in the Foster RL impedance form, have the same constraints, as discussed in the previous section, and must be necessarily positive to achieve a minimum-phase, realizable function. The residue extraction routine implements this by using non-negative least squares fitting.

The vector fitting provides accurate results in the entire frequency band for a final order of $N = 9$ as shown in Figure 8. This methodology ensures that the resulting macromodel is a passive RL network.

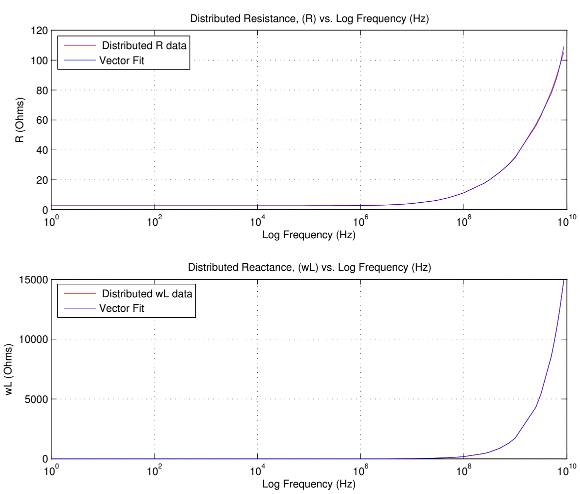


Fig. 8. Distributed resistance and reactance of a microstrip line.

IV. Conclusion

This paper presented for the first time a Debye model for conductors. This allows a generalized loss model for both dielectrics and conductors. Extraction of the Debye model was demonstrated using an analytical approach and a modified RL/RC vector fitting algorithm. The generated Debye model for conductors corresponds to an RL network. This approach was applied to create a model for conductors in the presence of surface-roughness loss. The proposed RL macromodeling approach was also shown to be suitable for generating skin-effect models for transmission lines. The presented models can be easily used in circuit and electromagnetic simulation of SI and EMI behavior in PCBs.

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