

# **Robustness and Stability of a Massively Parallel Out-of-Core Solver for Solving Pure MoM Problems with Million Level Unknowns**

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## **1. Introduction**

With the rapid development of computer technology, especially with the advent of multi-core technology in recent years, parallel computation is playing a more and more important role in computational electromagnetics (CEM). Parallel technology has been used in various CEM methods, such as the method of moments (MoM) [1-3], fast multipole method (FMM) [4] and finite-difference time-domain method (FDTD) [5].

Because of its numerical accuracy, MoM has been a very popular method in radiation and scattering analysis. As is well known, MoM needs to deal with very large, full-density matrices for solving complex problems, and hence it incurs huge memory requirement and computational complexity. Our previous works [1-3] provided a solution to overcome these drawbacks rooted in MoM by taking advantage of the higher order basis functions (HOBs) and the technological advancements in high-performance computing (HPC) hardware, in particular, multi-core CPUs. In detail, we solved a pure MoM problem with million level unknowns by using a parallel out-of-core solver combined with the higher order basis functions [6]. The project with million level unknowns was simulated by using 512 cores and only 1 TB RAM for approximately 16 TB RAM (double precision) problems.

In this paper, the robustness and stability of the parallel out-of-core solver is verified by simulating an aircraft and a formation of aircraft. The solver can deal with general structures composed of metallic and dielectric materials. The ability to solve large, complex problems in reasonable time is the greatest strength of the solver. Unlike fast algorithms (e.g., fast multipole method and adaptive integral method), the solver does not lose any accuracy of MoM for reducing memory requirement.

## **2. Parallel Out-of-Core Schemes of Higher Order Method of Moments**

The parallelization of the MoM solution involves two steps. The first step is the matrix filling and the second step is the solution of the matrix equation. Both of these must be handled efficiently. Furthermore, efficient parallel matrix filling for MoM with higher order basis functions introduces new challenges and is quite different from the procedure used in a MoM formulation using the traditional subdomain basis functions, e.g., RWGs [7].

The reason for developing an out-of-core matrix filling algorithm is to enable one to solve large matrix equations, where the impedance matrix may be too large to be stored in the main memory (RAM) of the system. Compared with the in-core matrix filling algorithm, where the matrix is filled once and kept in the RAM, the main idea of designing an out-of-core algorithm is to fill a portion of the matrix at a time and then write this portion to the hard disk rather than keeping it in the RAM.

By comparing with the in-core matrix filling algorithm [1], it can be found that for each slab, the algorithm is exactly the same. Most of the overhead for filling the out-of-core matrix,

excluding that from the in-core matrix calculation for an individual slab, comes from two parts: (1) calculation of the redundant integrals performed on each process, for different elements of the impedance matrix, which belongs to different slabs; and (2) writing the matrix elements to the hard disk.

After the matrix is filled, the solution of the matrix equation is essentially the same regardless of the type of basis functions used in the MoM formulation. For the parallel out-of-core matrix equation solver, the methodology in this paper is based on the ScaLAPACK math library [1]. As an alternative, the PLAPACK library can be also employed as the parallel out-of-core matrix equation solver [2].

### 3. Description of the Computational Platform

The platform used in this paper has one head node and 64 computing nodes. The head node has two quad-core Intel Xeon E5450 3.0 GHz (2×6 MB L2 Cache/1333 MHz FSB) EM64T processors, 16 GB of RAM, and four 146 GB 10K rpm SAS hard disks (two hard disk drives are configured as RAID1 and the other two as RAID0). Each computing node has two quad-core Intel Xeon E5450 3.0 GHz (2×6 MB L2 Cache/1333 MHz FSB) EM64T processors, 16 GB of RAM, and two 146 GB 10K rpm SAS hard disks (RAID0). The platform is a HP BL460c blade system with three Infiniband (ConnectX IB DDR) switches.

## 4. Numerical Results

### 4.1 An Aircraft with Million Level Unknowns

The bistatic RCS of a single aircraft is calculated at 6.15 GHz, the full size aircraft with mesh is shown in Figure 1. The corresponding electrical size of the aircraft is about  $237.8\lambda \times 143.5\lambda \times 59.86\lambda$ . The incident wave propagates in  $-x$  direction and is  $z$  direction polarization. The numbers of unknowns in this case is 954,618 (approximately one million unknowns) when the structure is meshed at a frequency of 5.95 GHz. The bistatic RCS is given in Figure 2.

To investigate the robustness and stability of the solver, this model was simulated twice. The first run took 637,962 seconds while the second run took 638,365 seconds using 64 nodes (512 cores). The second run took more than about 7 minutes longer than the first run. The relative difference is less than 0.07%, which means that the code is very stable and the results can be repeated even if this project used almost all the hard disk space resources. This simulation demonstrates that the code is stable and can be used for even more unknowns as long as sufficient hard disk space is available.

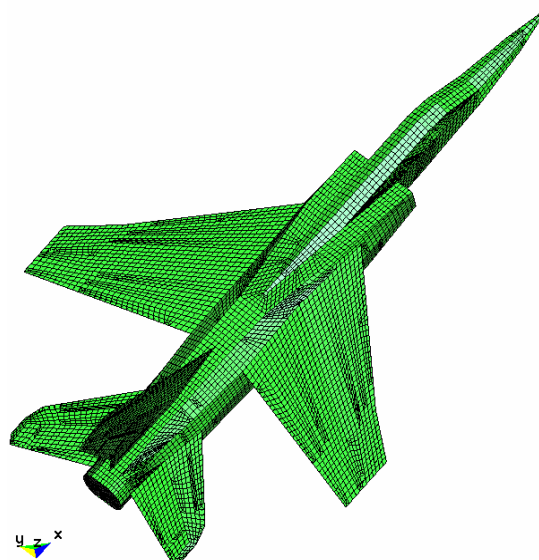


Figure 1: The Meshed Aircraft Model (mesh frequency is 5.95 GHz)

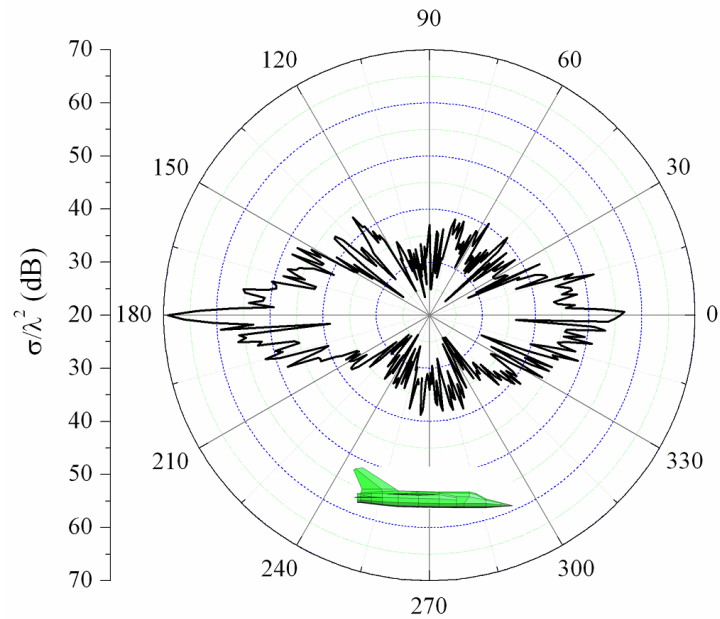


Figure 2: Bistatic RCS at  $\phi = 0^\circ$  Cut Plane ( $0^\circ$  starts from  $x$  axis in the  $xoz$  plane)

#### 4.2 A Formation of Aircraft

The single aircraft model is 17.32 m in length, 11.4 m in width and 3.7 m in height. It is modelled as a PEC surface. The bistatic RCS of a “V” shape aircraft formation is calculated. The plane wave is incident from the  $-y$  axis direction and is polarized along the  $z$  axis. The distances between any two neighbouring aircraft are  $\Delta y = 20.0$  m along the head direction,  $\Delta x = 20.0$  m along the wing direction, and  $\Delta z = 0.0$  m along the height direction. The aircraft formation is shown in Figure 3. The bistatic RCS is calculated at 1.0 GHz. The number of unknowns for the aircraft formation is 897,360. It took 350,176 seconds for filling the matrix and 555,516 seconds for solving the equation (totally about 10.5 days) using 456 cores. The code ran in 10.5 days by using so many cores but without breakdown, which fully demonstrates the robustness and stability of the solver.

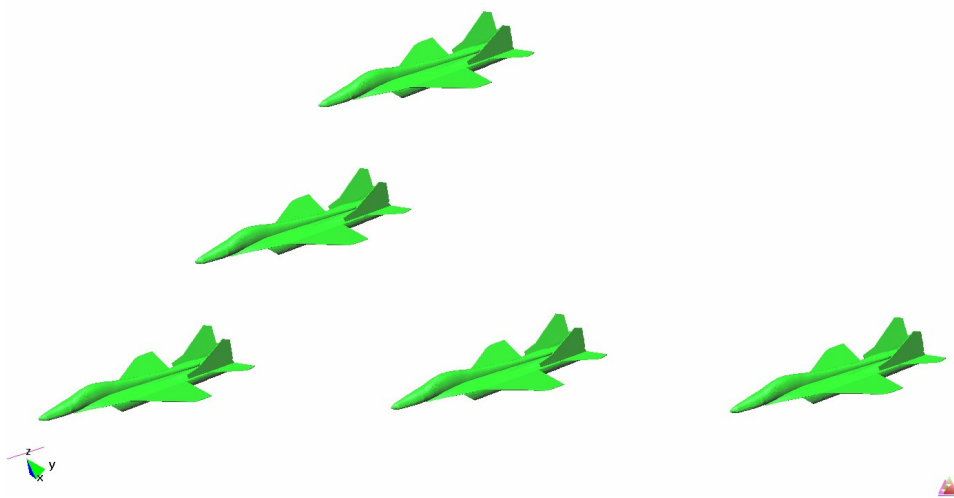


Figure 3: Aircraft Flying in a “V” Formation

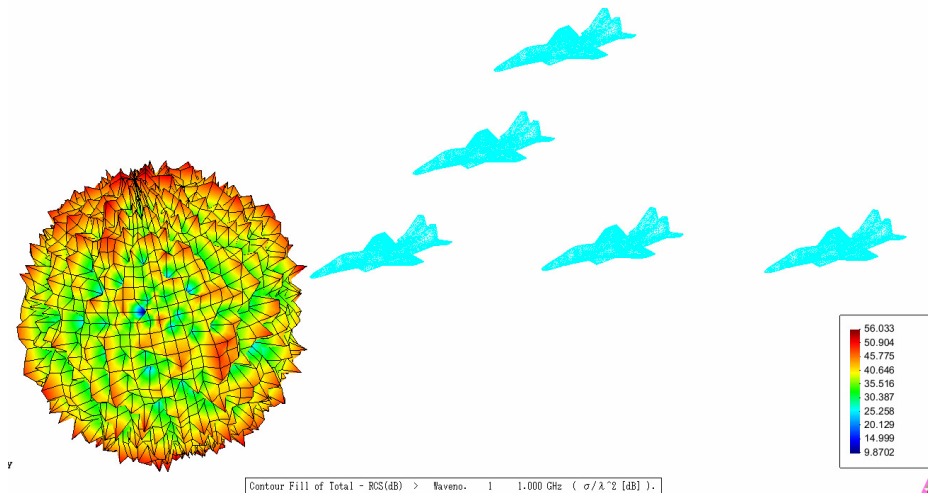


Figure 4: 3D Bistatic RCS of the Aircraft Formation

## 5. Conclusion

The parallel out-of-core schemes for higher order MoM are briefly reviewed in this paper. Two examples of an aircraft and a formation of aircraft are shown to demonstrate the robustness and stability of the proposed parallel out-of-core solver. This research presents a new powerful tool to solve challenging electromagnetic problems including electrically large and complex targets in reasonable time.

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