Full-Wave Parallel Dispersive Finite-Difference Time-Domain Modeling of Three-Dimensional Electromagnetic Cloaks

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1. Introduction

Recently, a great deal of attention has been paid to the analysis and design of electromagnetic cloaking structures, since first proposed by Pendry *et al.* [1]. The specially designed cloak is able to guide waves to propagate around its central region, rendering the objects placed inside invisible to external electromagnetic radiations. Besides the coordinate transformation technique originally used to design the cloak [1, 2], a cylindrical wave expansion technique [3], and a method based on the full-wave Mie scattering model [4, 5] have also been applied. In addition, the full-wave finite element method (FEM) based commercial simulation software COMSOL Multiphysics$^{\text{TM}}$ has been extensively used to model different cloaks and validate theoretical predictions [6, 7]. So far, the time domain techniques that have been developed to model the cloaking structures include the time-dependent scattering theory [8], the transmission line method (TLM) [9] and the finite-difference time domain (FDTD) method [10]. However due to the computational complexity, so far the FDTD modeling of three-dimensional (3-D) cloaking structures has not been attempted. In this paper, we extend our previously proposed 2-D FDTD method [10] to the 3-D case and develop a parallel dispersive FDTD method to model 3-D cloaking structures.

2. Parallel dispersive FDTD modeling of 3-D cloaks

A complete set of material parameters of the ideal cloak in spherical coordinate is given by [1]

$$\varepsilon_r = \mu_r = \frac{R_2}{R_2 - R_1}\left(\frac{r - R_1}{r}\right)^2, \quad \varepsilon_\theta = \mu_\theta = \frac{R_2}{R_2 - R_1}, \quad \varepsilon_\phi = \mu_\phi = \frac{R_2}{R_2 - R_1}, \tag{1}$$

where R_1 and R_2 are the inner and outer radii of the cloak, respectively, and r is the distance from a spatial point within the cloak to the center of the cloak. Since the values of ε_r and μ_r are less than one, the cloak cannot be directly modeled using the conventional FDTD method [11]. However, one can map the material parameters using dispersive material models, for example, the Drude model $\varepsilon_r(\omega) = 1 - \omega_p^2/\left(\omega^2 - j\omega\gamma\right)$, where ω_p and γ are the plasma and collision frequencies of the material, respectively. By varying the plasma frequency, the radial dependent material parameters in (1) can be achieved.

The main difference between the modeling of 2-D and 3-D cloaks is the coordinate transformation. For the conventional Cartesian FDTD mesh, since the material parameters given in (1) are in spherical coordinates, the following coordinate transformation is used [12]

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_r & 0 & 0 \\ 0 & \varepsilon_\theta & 0 \\ 0 & 0 & \varepsilon_\phi \end{bmatrix}$$
$$\cdot \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}. \tag{2}$$

The permeability of the 3-D cloak has the same form as Eq. (2). Following the same procedure as in [10] using the inverse Fourier transform, the central finite difference operators in time and the central average operator with respect to time [11], one can obtain the FDTD updating equation set for the 3-D cloaks. Due to the large amount of computer memory required to simulation the 3-D cloaks, a parallel FDTD method based on the domain decomposition is also implemented.

3. Numerical results and discussion

The computational domain for the modeling of the 3-D cloak is shown in Fig. 1. The FDTD cell size in all simulations is $\Delta x = \Delta y = \lambda/150$ where λ is the wavelength at the operating frequency $f = 2.0$ GHz. The time step is chosen according to the Courant stability criterion [11] i.e. $\Delta t = \Delta x/\sqrt{3}c$. The radii of the cloak are $R_1 = 0.1$ m and $R_2 = 0.2$ m. In the present paper, only the ideal lossless cloak is considered. The whole simulation domain is divided along y-direction into 100 sub-domains and in total 100 processors and 220 gigabyte (GB) memory were used to run the parallel dispersive FDTD simulations.

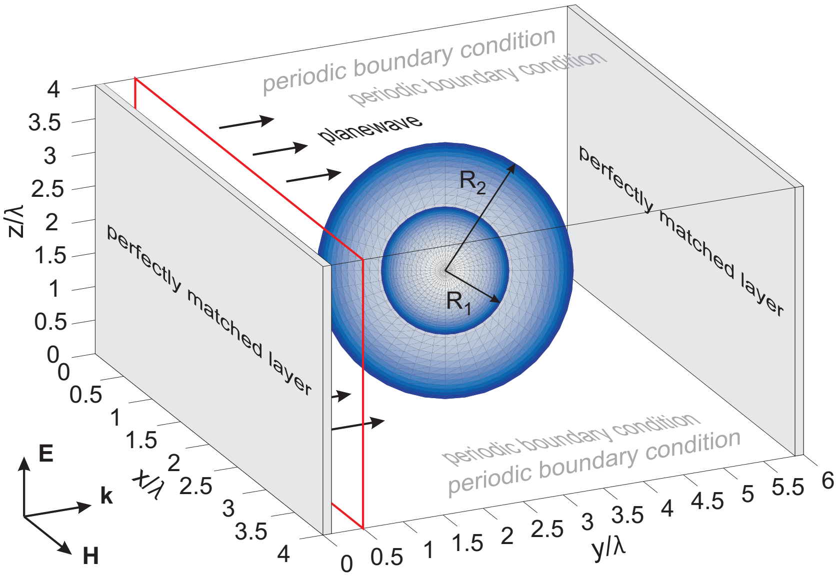

Fig. 1: The 3-D parallel dispersive FDTD simulation domain for the case of plane-wave incidence on the cloak. The red rectangle indicates the location of the source plane.

Figures 2 and 3 show the normalized steady-state field distributions for the E_z and H_x components in y-z and x-y planes, respectively. It can be seen that the plane wave is guided by the cloak to propagate around its central region, and recomposed back after leaving the cloak. There is nearly no reflection (except those tiny numerical ones due to the finite spatial resolution in FDTD simulations), since the material parameters (1) vary continuously in space while keeping the impedance the same as the free space one. It is also interesting to notice that the E_z component in y-z and x-y planes in Fig. 2 and the H_x component in x-y and y-z planes in Fig. 3 have the same distributions (with different amplitude), which is due to the fact that the ideal 3-D cloak is a rotationally symmetric structure with respect to the electric and magnetic fields. The wave behavior near the 3-D cloak can be better illustrated using the power flow diagram, as plotted in Fig. 4. It is shown that the Poynting vectors are diverted around the central area enclosed by the cloak. Therefore objects placed inside the cloak do not introduce any scattering to external radiations and hence become 'invisible'.

4. Conclusion

In conclusion, a parallel dispersive FDTD method has been developed to model the ideal 3-D cloak. The radial dependent permittivity and permeability of the cloak are mapped to the Drude dispersion model and taken into account in simulations using a dispersive FDTD method. FDTD simulation results are validated by those obtained using analytical methods. The developed FDTD method can be also applied to study the effect of losses in cloaks, evaluate the performance of simplified cloaks as well as assist the design and realization of practical cloaks.

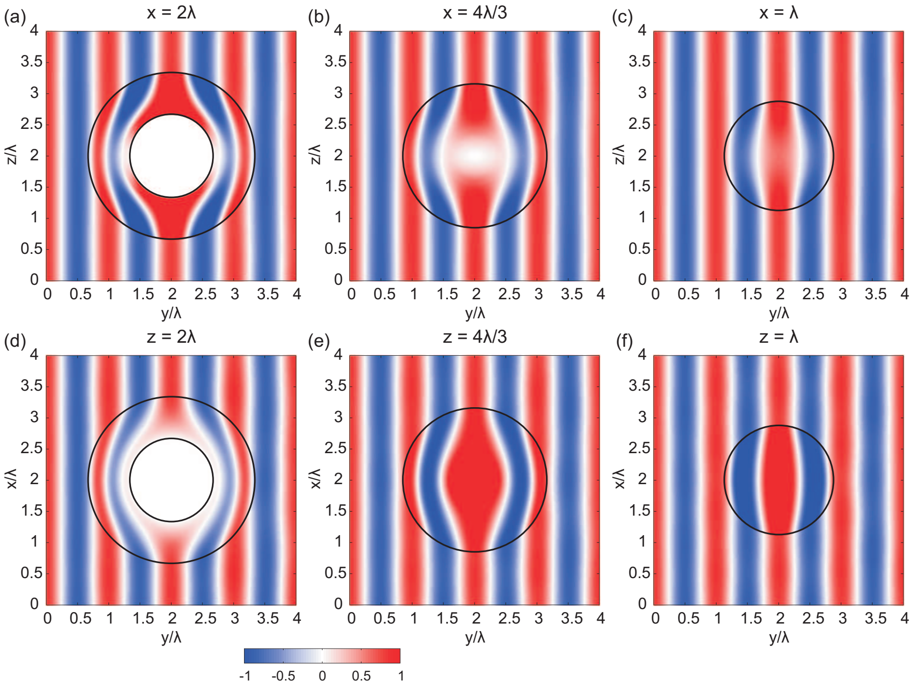

Fig. 2: Normalized field distributions for the E_z component in (a)-(c) y-z plane and (d)-(f) x-y plane in the steady-state of the parallel dispersive FDTD simulations. The cutting planes are (see Fig. 1): (a) $x = 2\lambda$, (b) $x = 4\lambda/3$, (c) $x = \lambda$, (d) $z = 2\lambda$, (e) $z = 4\lambda/3$, (f) $z = \lambda$.

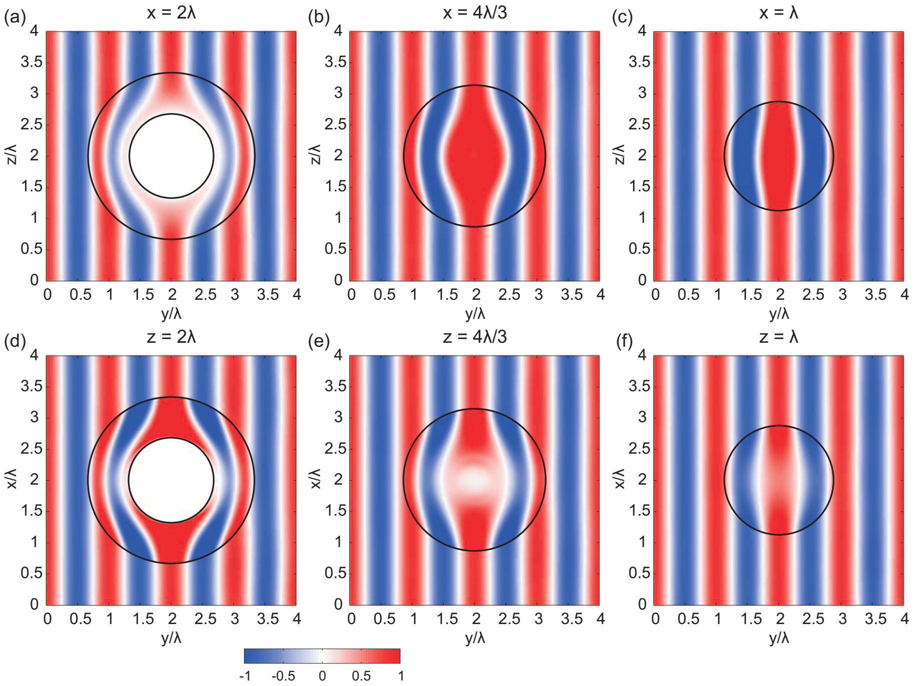

Fig. 3: Normalized field distributions for the H_x component in (a)-(c) y-z plane and (d)-(f) x-y plane in the steady-state of the parallel dispersive FDTD simulations. The cutting planes are (see Fig. 1): (a) $x = 2\lambda$, (b) $x = 4\lambda/3$, (c) $x = \lambda$, (d) $z = 2\lambda$, (e) $z = 4\lambda/3$, (f) $z = \lambda$.

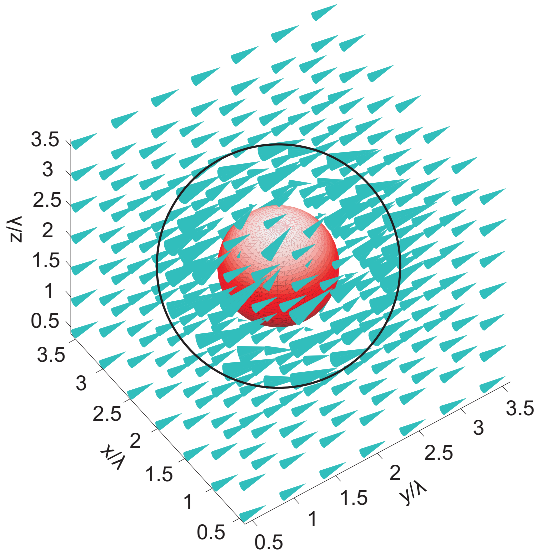

Fig. 4: Power flow diagram of a plane wave incidence on the ideal 3-D cloak calculated from parallel dispersive FDTD simulations.

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