

Investigation of Moment Method Solutions Based on Impulse Expansion Functions

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1. Introduction

The Method of Moments (MM) is a numerical method for solving electromagnetic problems. It is popular because of its simplicity and most important because it can give very accurate results [1]. Inclusion of the edge behaviour of the fields in the expansion functions leads to more accurate and fast converging solution [2]. [3] deals with the characterization of an infinite array microstrip elements, and it is shown that a complete set of trigonometric functions which do not enforce the correct edge behaviour slows significantly the rate of solution convergence. The convergence of the MM is closely related to the choice of expansion functions and, although to a lesser extent to the choice of testing functions. Different expansion and test functions were checked during the last years in order to get more accurate and fast converging solutions. Expansion function consideration for the MM using a lot of different models and techniques. In general, expansion functions can be classified into entire domain expansion functions and sub-domain expansion functions (rectangular, triangular, etc).

Investigation of MM solutions using impulse function defined in infinite domain is illustrated by solving simple electromagnetic problem for which the analytic solution is known. In this paper we check the efficiency of a MM solution when expansion functions contain impulse functions and obey to known physical behaviour of the fields near the edges of the disk and at infinity.

The structure of the paper is as follows: chapter 2 describes the formulation of the problem and includes the analytical solution and the MM solutions. Chapter 3 deals with the selection of the expansion functions and numerical results are presented in chapter 4. The conclusions are discussed in chapter 5.

2. Formulation

2.1 Geometry of the problem and Analytic Solution

The thin, charged disk is shown in fig. 1. The radius of the disk is R and the total charge on it is q . The analytic solution for the charge distribution and $E(\rho)$ are,

$$\sigma(\rho) = \frac{q}{2\pi R} \frac{1}{\sqrt{R^2 - \rho^2}}, \quad 0 \leq \rho < R \quad (1)$$

$$E_\rho(\rho, z=0) = \frac{1}{4\pi\epsilon_0} \frac{1}{\rho\sqrt{\rho^2 - R^2}}, \quad \rho > R \quad (2)$$

and $E_\rho(\rho, z=0) = 0$ for $\rho \leq R$ [4]. This solution will be compared to the MM solution based on impulse based expansion functions defined in the infinite domain.

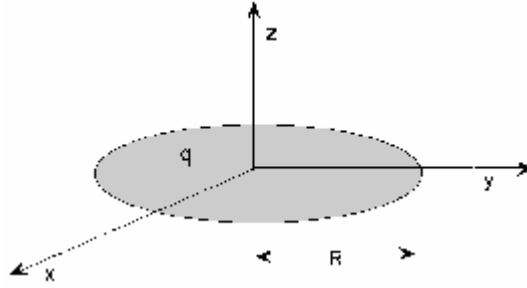


Figure 1: Infinitely thin, flat circular conducting disk with radius R and total charge q

2.2 Electrostatic Theory related to the Problem.

Laplace equation around the disk and the potential function therefore be of the form [4]

$$\nabla^2 \Phi = 0 \quad (3) \quad \Phi(\rho, z) = \int_0^{\infty} dk f(k) e^{-k|z|} J_0(k\rho) \quad (4)$$

where $J_0(\cdot)$ is the Bessel function of the first kind [5] and $f(k)$ must be determined from boundary conditions at $z = 0$ and the input data (total charge q on the disk).

The electric field components are given by,

$$E_\rho = -\frac{\partial \Phi}{\partial \rho}, \quad E_z = -\frac{\partial \Phi}{\partial z} \quad (5)$$

2.3 MM Solution Based on Impulse Expansion Functions Defined in Infinite Domain.

Expanding E_ρ outside the disk, on the disk plane, by the MM set of expansion functions $\{d_n(\rho), n = 1, 2, \dots, N\}$, where N is the number of expansion functions, we obtain

$$E_\rho(\rho, z=0) = \left(-\frac{\partial \Phi}{\partial \rho}\right)_{z=0} = \begin{cases} \sum_{n=1}^N a_n d_n(\rho) & , \quad \rho > R \\ 0 & , \quad \rho \leq R \end{cases} \quad (6)$$

By using [6-7] we can find $f(k)$ in term of expansion functions

$$f(k) = \sum_{n=1}^N a_n \tilde{d}_n(k) \quad (7) \quad \tilde{d}_n(k) = \int_R^{\infty} d\rho d_n(\rho) J_1(k\rho) \rho \quad (8)$$

The MM expansion function coefficients are found from the MM set linear of equations by expressing $f(k)$ in terms of the expansion functions,

$$\int_0^{\infty} dk \sum_{n=1}^N a_n \tilde{d}_n(k) \tilde{b}_m(k) k = 0, \quad m=1, 2, \dots, N \quad (9)$$

where

$$\tilde{b}_m(k) = \int_R^{\infty} d\rho b_m(\rho) J_0(k\rho) \rho \quad (10)$$

and $b_n(\rho)$ are the test functions.

In matrix notation, the (9) can have the form,

$$Aa = 0 \quad (11) \quad , \quad A_{mn} = \int_0^{\infty} dk \tilde{b}_m(k) \tilde{d}_n(k) k \quad (12)$$

and $a = (a_1, a_2, \dots, a_N)$ is the vector containing the un-normalized MM expansion function coefficients. The normalization is determined by the value of the total charge on the disk.

3. Choosing Expansion and Test Functions

The expansion and test functions are chosen such to obey the physical behaviour of $E(\rho)$ near the edge of the disk and at infinity. The behaviour of $E(\rho)$ is given by,

$$E_\rho \sim \alpha_1(\rho - R)^{-1/2} + \alpha_2(\rho - R)^{1/2} + \alpha_3(\rho - R)^{3/2} \dots, \rho \rightarrow R \quad (13)$$

$$E_\rho \sim \frac{\alpha}{\rho^2} + \frac{\beta}{\rho^4} + \frac{\gamma}{\rho^6} + \dots, \rho \rightarrow \infty \quad (14)$$

It can be shown that the set of expansion functions for $E(\rho)$,

$$\{d_n(\rho) = \frac{\delta\left(\frac{\rho^2 - (n\sqrt{5}/2)^2}{\rho^4 - n}\right)}{\rho^{2n-1}\sqrt{\rho^2 - R^2}}, \quad n=1,2,3,\dots \quad \rho > R\} \quad (15)$$

and test functions,

$$\{b_m(\rho) = \frac{\delta\left(\frac{\rho^2 - (m\pi/3)^2}{\rho^4 - m}\right)}{\rho^{2m-1}\sqrt{\rho^2 - R^2}}, \quad m=1,2,3,\dots \quad \rho > R\} \quad (16)$$

are contains all the needed powers near the edge of the disk and at infinity. Note that these functions do not include 'wrong' powers - powers that not belong to the correct edge powers for $\rho \rightarrow R$ and for $\rho \rightarrow \infty$. In additional, these functions have analytical Fourier-Bessel transform.

4. Numerical Results

For simplicity we choose $R = 1$ and $q = 1$. The MM matrix elements where analytically calculated by MATHEMATICA software [8]. The charge density $\sigma(\rho)$ on the surface of the disc is given by:

$$\sigma(\rho) = \sum_{n=1}^N a_n c(n) {}_2F_1\left(\frac{3}{2}, \frac{1}{2}; 1; \frac{4\rho^2}{5n^2}\right) \quad (17)$$

where

$$c(n) = \frac{2^{2n-4} 5^{-1/2-n} n^{-2n} (25n^3 - 16)}{\pi \sqrt{5n^2 - 4}} \quad (18)$$

The surface charge density is shown in fig. 2, and shows a very good rate of convergence for the solution. A zoom on the area near the centre of the disk is shown in fig. 3.

5. Conclusions

The possibility to apply the moment method by using impulse functions as expansion functions describing the fields in an infinite domain (outside the body) has been checked, and it is shown that accuracy and converging rate has been achieved. The key point was the requirement that the expansion functions will exactly obey the physical behavior of the fields near the edge of the body and at infinity.

We can conclude that efficient MM solutions based on impulse functions defined as expansion functions is possible and all results achieved during solution of the problem involves just analytically solved integrals with no numerical techniques. As a result it is possible to use a large number of expansion functions (even hundreds) for achieving much more accurate results. And most important point that all calculations are fast because of the nature of impulse functions. The efficiency of the MM solution based on impulse functions as expansion functions defined in the infinite domain has to be checked in the future for electromagnetic problems.

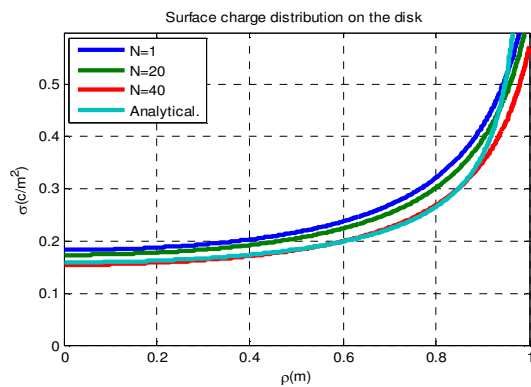


Fig. 2: Surface charge distribution on the disk.

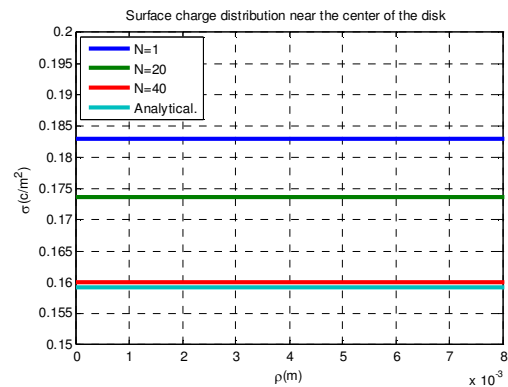


Fig. 3. Zoom on the centre of the disk

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