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Synchronized Motion of Coupled Stirling Engines

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Abstract—Because of an increasing demand on electric power and limited resources for conventional fuels, highly efficient engine that is capable of converting energy resource to electric power with a small loss is awaited. In this respect, Stirling engine provides a strong potential, because of its efficient thermodynamic cycle close to a theoretical limit of Carnot cycle. The paper presents experimental study of synchronized dynamics in coupled two Stirling engines. It is shown that synchronized operation of the population of engines provides a key technology to extend the system size so as to produce a large-scale electric energy.

1. Introduction

Because of the rapid growth of world economy and wide-spread industrization in developing countries, usage of electric power grows rapidly. Despite such increasing demand, resources for the conventional fuels are getting limited and the danger of nuclear power plants has been realized by earthquake disasters. The global warming induced by the accelerated energy consumption leads to a serious climate change. Under these circumstances, ecological use of natural energy resources such as solar power, wind power, wave power, geothermal heat is expected [1]. Highly efficient engine that is capable of converting energy resource to electric power with a small loss is awaited. Stirling engine is noted for its high efficiency of energy transfer compared to the steam engines or other conventional systems [2]. It has the advantage of quiet operation and is applicable to a wide variety of heat sources including the wasted heat produced in power plants or industrial factories. The heat engine is operated by cyclic compression and expansion of air induced by different temperature levels in a closed system. Internal heat exchanger and thermal store regenerate the energy resource repeatedly. Since the original development by R. Stirling (1816), it has been applied mainly for low-power systems. Development of a large-scale Stirling engine applicable to a huge source of heat has been limited, because it requires a precise and expensive mechanical system that minimizes the energy loss. Small-scale Stirling engines, on the other hand, have the advantage of low-cost devices applicable even to a small temperature deference. Towards production of a large amount of electric energy, a population of small-scale Stirling engines provides a good potential with summation of the population outputs. For efficient summation of the outputs, synchronized operation of the individual engines gives the key technology. The aim of the present paper is to examine synchronization of coupled two Stirling engines.

2. Synchronization

Consider two coupled Stirling engines having oscillation frequencies of ω_1 and ω_2. It has been known that, if the two oscillators have similar natural frequencies ($\tilde{\omega}_1 \approx \tilde{\omega}_2$), even a weak amount of coupling can induce their synchronization, where the two frequencies coincide with each other ($\omega_1 = \omega_2$) and the phase difference is bounded within a constant interval ($|\phi_1(t) - \phi_2(t)| < const$) [3, 4]. For the electric power generation with alternating current, synchronized oscillation of the phase of the engines is of significant importance. For synchronized engines, summation of their output currents simply add their amplitudes. If there is a small frequency mismatch, however, the engines are not synchronized and summation of their outputs shows an amplitude, which slowly varies in time, known as *beating*. The time-varying amplitude crucially reduces the output energy, resulting in a large loss. Thus, it is desired to assure synchronized motion of the two engines. If the mismatch of the natural frequencies is relatively small, the synchronization can be achieved only with a small interaction between the engines. This provides a potential of increasing the system size without developing an expensive and precise engine system.

3. Experimental set-up

Figure 1 shows schematic illustration of the experimental set-up. Two Stirling engines (K's Home Planning Co., Inc.) are located on a separate heater (Sakaguchi E.H VOC Corp.). Each heater enables a temperature control of the metal plate with a precision of 1 °C. As a simple way to introduce interaction between the two Stirling engines, a thin plastic cord of 19 cm length was attached to the piston bar of the engine devices. Movement of the Stirling engines are monitored and recorded by a digital video camera (SONY, HDR-CX180, 30 fps). To quantify the dynamics, the movie data was analyzed as follows. In the video image, the location of the piston moving up and down was manually identified and the time series of the image intensity (RGB, 255 bit resolution) was extracted. The *Fourier* analysis was then applied to compute frequencies (ω_1, ω_2)

of the oscillators. The Hilbert transform was further applied to extract the phases ($\phi_1(t)$, $\phi_2(t)$) of each signal. It has been confirmed that the calculated frequencies and the phases were consistent with the video observation of the piston movement.

As discussed earlier, synchronized dynamics of the two Stirling engines can be controlled by two factors: (1) coupling strength and (2) frequency mismatch. The coupling strength can be controlled by distance between the two engines. As the distance is increased, the tension of the connecting cord increases and, as the result, the two engines are more tightly connected to each other. To weaken the coupling strength, the distance should be shortened. On the other hand, frequency mismatch between the two engines can be controlled by the heater, since difference between the bottom plate of the Stirling engine and the room temperature is the major determinant of the natural oscillation frequency of the engine. By adjusting the heating temperature, the frequency mismatch of two engines can be changed.

Two examine these factors, two experiments were conducted: first experiment studied dependence of the synchronized motion on the coupling strength, whereas second experiment examined its dependence on the frequency mismatch.

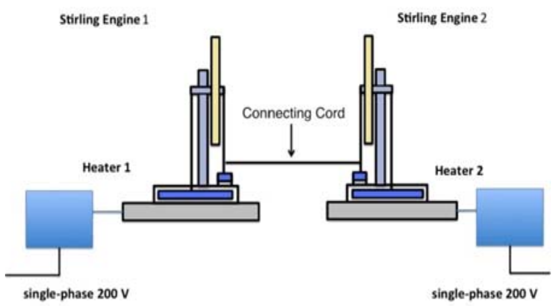

Figure 1: Schematic illustration of the experimental set-up. Two Stirling engines are coupled by a thin plastic cord.

4. Experimental Results

4.1. Measurement of Natural Frequencies

Natural frequency of the Stirling engine is primarily determined by difference between the heater temperature and the room temperature. As a preliminary measurement, we first studied dependence of the natural frequency of the individual Stirling engine on the heater temperature. Fig. 2 shows dependence of their frequencies on the heater temperature increased from 35 °C to 80 °C with an increment of 5 °C. The room temperature was 16 °C. As the heater temperature was increased, frequency of the Stirling engine also increased monotonically.

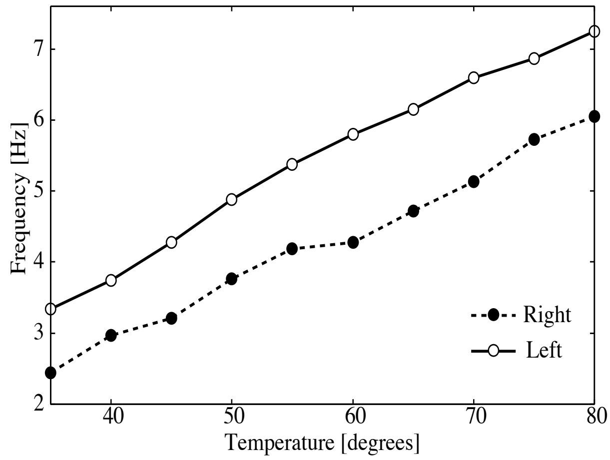

Figure 2: Dependence of natural oscillation frequency of the Stirling engines (solid line for left engine; dotted line for right engine) on the heater temperature increased from 35 °C to 80 °C.

4.2. Coupling-induced synchrony

The heater temperature was fixed to 35 °C and 40 °C for the left and right engines, respectively. The room temperature was 16 °C. Then the distance between the two engines, denoted as d, was changed from 14 cm to 19 cm. Fig. 3 shows dependence of frequencies of the left and right Stirling engines on their distance. The two engines showed different frequencies for small distances ($d < 16$ cm). Phase difference between the two engines, $\phi_1(t) - \phi_2(t)$, which grows linearly in time in Fig. 4a, indicates that the two engines were not synchronized. With the distance of $d = 16$ cm, the two frequencies coincided with each other and the two engines were synchronized. The phase difference between the two engines was indeed bounded within a finite range in this regime (Fig. 4b). Since the line was not strongly stretched and the tension was not too high, the interaction between the two engines were rather weak. However, because the frequency mismatch was not too large, such a weak coupling was enough to induce their synchronized oscillations.

The synchronization observed at $d = 16$ cm was rather fragile and a further increase in the distance destroyed the synchronized motion ($16 < d < 19$). At $d = 19$ cm, the two oscillators were synchronized again. Namely, the left and right engines take the same oscillation frequency and their phase difference was bounded within a finite range (Fig. 4c). Here, the line was stretched with a high tension, which induced a strong coupling between the two engines. Since the interaction decelerated the rotation speed, the entrained frequency was slightly lowered from the nonsynchronized frequencies.

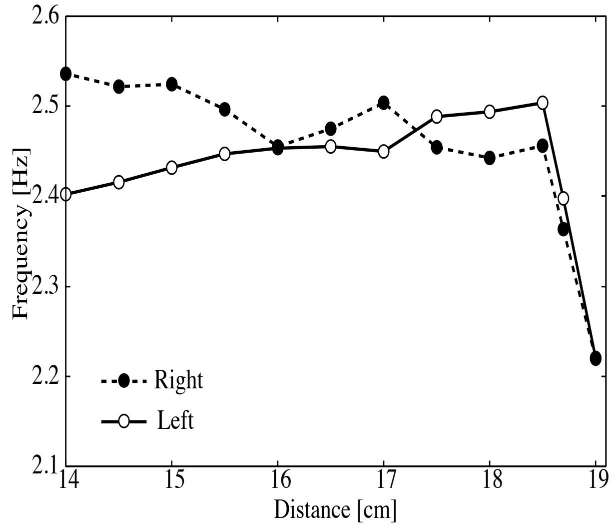

Figure 3: Dependence of oscillation frequency of the Stirling engines (solid line for left engine; dotted line for right engine) on their distance changed from 14 cm to 19 cm.

4.3. Mismatch-induced synchrony

In the next experiment, dependence of the synchronized motion on the frequency mismatch was studied. The room temperature was 15 °C. The distance between the two engines was fixed to $d = 19$ cm, which induces a strong coupling. The heater temperature was fixed to $T_l = 45$ °C for the left engine, whereas the heater temperature was changed from $T_r = 35$ °C to $T_r = 80$ °C for the right engine. The room temperature was 15 °C. Fig. 5 shows dependence of the oscillation frequencies of the left and right engines on the right heater temperature. As the heater temperature of the right engine is increased from $T_r = 35$ °C, oscillation frequency of the right engine was increased monotonically, whereas the two engines showed different frequency values. In the temperature range of 45 °C$\leq T_r \leq$49 °C, frequency of the right engine coincided with that of the left engine, indicating a clear synchronization. As the heater temperature is further increased ($T_r > 49$ °C, frequency of the right engine went apart from that of the left engine and it grew monotonically. Since the distance between the two engines was fixed, the line tension that determines the coupling strength remains the same in this experimental setting. Instead, the heater temperature changed natural oscillation frequency of the right engine. This implies that the synchronization observed in 45 °C$\leq T_r \leq$49 °C was due to a reduced frequency mismatch between the right and left engines.

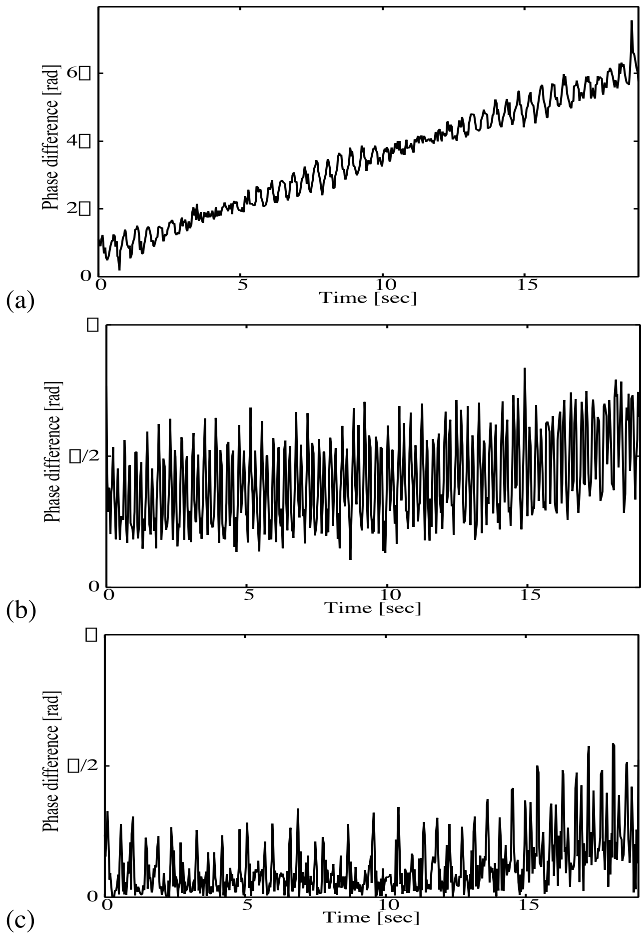

Figure 4: Time course of phase difference between the two engines, $\phi_1(t) - \phi_2(t)$. (a) distance of $d = 14$ cm, (b) distance of $d = 16$ cm, and (c) distance of $d = 19$ cm.

5. Evaluation of power generation

Experimental observation of the coupled Stirling engines was evaluated in terms of its electric power generation. As a standard electrical generator, we considered an alternator that converts mechanical energy to electrical energy in the form of alternating current. In a stationary magnetic field having a magnetic flux density of B, an armature (section area of S) is rotated mechanically with a frequency of ω. The electromotive force (EMF) is then given by

$$\phi = BS\omega \sin(\omega t).$$

Considering an electric current I that is generated through a resistor R (*i.e.*, $\phi = IR$), the output electric power is evaluated as

$$P = I\phi = \frac{\phi^2}{R} = \frac{(BS)^2}{R}\omega^2 \sin^2(\omega t).$$

In our framework, the left and right Stirling engines produce individual electromotive forces as $\phi_l = BS\omega_l \sin(\omega_l t)$

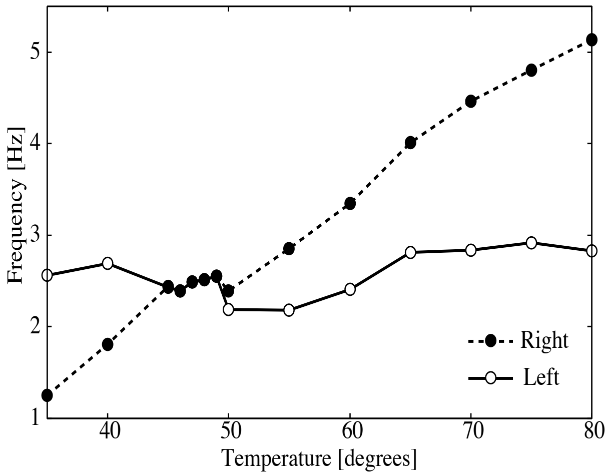

Figure 5: Dependence of oscillation frequency of the Stirling engines (solid line for left engine; dotted line for right engine) on the heater temperature for the right engine ($T_r \in [35\,°C, 80\,°C]$). Heater temperature for the left engine was fixed to $T_l = 45\,°C$, whereas the room temperature was 15 °C.

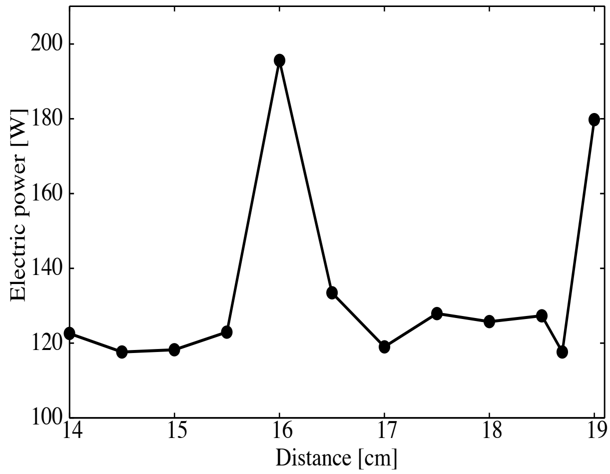

Figure 6: Dependence of total electric power generated from coupled two Stirling engines on their distance.

and $\phi_r = BS\,\omega_r \sin(\omega_r t)$, which are serially connected to form a total force. The total force ϕ_{total} is therefore given by a summation of the left and right forces as $\phi_{total} = \phi_l + \phi_r$. This yields the total output power of

$$P_{total} = \frac{\phi_{total}^2}{R} = \frac{(BS)^2}{R}\{\omega_r \sin(\omega_r t) + \omega_l \sin(\omega_l t)\}^2.$$

For the experimental observation of two Stirling engines in Fig. 3, the output power was computed in accordance with the above formula. For simplicity, the constant product $\frac{(BS)^2}{R}$ was set to be unity. Fig. 6 shows dependence of the total electric power on the distance between the two engines. In the case of synchronized motion realized at the distance of $d = 16$ cm, the electric power was 195.61 W. In the case of synchronized motion realized at the distance of $d = 19$ cm, the electric power was 179.82 W. Compared to power of 122.61 W generated by nonsynchronized engines at $d = 14$ cm, the output energy was significantly increased in the synchronized state.

6. Discussions

To summarize, we have introduced coupled two Stirling engines and studied the synchronization property. As the interaction, the engines are connected through a thin cord attached to the piston bar of each engine. By adjusting the coupling strength (tension of the connecting cord) and the frequency mismatch (heating temperature), synchronized dynamics of the two engines was successfully observed. Evaluation of the electric power generation showed that the synchronized engines produced much higher energy compared to nonsynchronized engines. Thus, synchronized operation provides a crucial technology to utilize a population of Stirling engines to extend the system size.

The present research is still preliminary and further investigations are needed. The present form of coupling using the thin cord has an effect of slowing down the engine cycles. Alternative coupling such as noise-induced synchrony [6] shall be considered. Experiment on synchronization between more than three Stirling engines will be performed in our future study.

Acknowledgments

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References

[1] *Renewables Global Status Report (GSR)* (Renewable Energy Policy Network for the 21st Century, 2012).

[2] G. Walker, *Stirling Engines* (Oxford University Press, 1980).

[3] Y. Kuramoto, *Chemical Oscillations, Waves and Turbulence* (Springer, Berlin, 1984).

[4] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization - A Universal Concept in Nonlinear Sciences*, (Cambridge University Press, Cambridge, 2001).

[5] S.P. Thompson, *Dynamo-Electric Machinery, A Manual for Students of Electrotechnics, Part 1* (Collier and Sons, New York, 1902).

[6] J. Teramae and D. Tanaka, "Robustness of the noise-induced phase synchronization in a general class of limit cycle oscillators," *Phys. Rev. Lett.*, vol. 93, p. 204103, 2004.