

# Analysis of Electromagnetic Scattering Responses in Time Domain for Conducting Cylinders with Apertures

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## 1. Introduction

Analysis of electromagnetic scattering is important for identification of various targets [1]. In this paper, we propose a novel computational method for electromagnetic scattering problems, which is based on the combination of the method of moments (MoM) [2] and the fast inversion of the Laplace transform (FILT) [3]. Applying this technique, transient electromagnetic scattering from a conducting cylinder with an aperture is investigated.

## 2. Formulation

We assume that the scatterer is uniform along the  $z$ -axis as shown in Fig.1. The incident  $E$ -wave in the rectangular coordinate system can be expressed by

$$E_z^{(i)} = \hat{E}_0 \exp[S(X \cos \phi_m + Y \sin \phi_m)], \quad (1)$$

where  $S = s\sqrt{\epsilon_0\mu_0}$ ,  $X = x/a$ ,  $Y = y/a$ , and  $s$  is the complex frequency of the Laplace transform,  $a$  is the radius of a circle which encloses the cylinder,  $\phi_m$  is the angle of incidence, and  $\hat{E}_0$  is the image function of the incident pulse at  $X = Y = 0$ .

In MoM, elements of impedance matrix  $l_{mn}$  can be expressed by

$$l_{mn} = \begin{cases} \frac{\eta}{2\pi} S \Delta C K_0(SR_{mn}), & (m \neq n) \\ \frac{\eta}{2\pi} S \Delta C \log\left(\frac{4e}{\gamma S \Delta C}\right), & (m = n) \end{cases} \quad (2)$$

where  $R_{mn} = \sqrt{(X_m - X_n)^2 + (Y_m - Y_n)^2}$ ,  $\eta$  is the intrinsic impedance,  $K_0(\cdot)$  is the zero order of the modified Bessel function,  $\Delta C$  is the size of segments, and  $\gamma = 1.783$ .

Using the surface current density  $j_z$ , the scattered wave in the far field is defined by

$$\lim_{R \rightarrow \infty} \hat{E}_z^{(S)} = \sqrt{\frac{\pi}{2R}} \exp(-SR) F(S), \quad (3)$$

where

$$F(S) = \frac{\eta}{2\pi\sqrt{S}} \sum_{i=1}^N j_z(X_i, Y_i) \exp[S(X_i \cos \theta + Y_i \sin \theta)] S \Delta C, \quad (4)$$

and  $\theta$  is the observation angle.

Here,  $F(S)$  is transformed into the time domain applying FILT [3]. This technique is based on the approximation of the exponential function in the Bromwich integral. The scattered wave in the time domain  $F(T)$  can be evaluated as follows:

$$F(T) = \frac{e^\alpha}{T} \left( \sum_{k=1}^{K-1} F_k + 2^{-p} \sum_{q=0}^{p-1} A_{pq} F_{K+q} \right), \quad (5)$$

where

$$F_k = (-1)^k \text{Im} \left[ F \left\{ \frac{\alpha + j(k-0.5)\pi}{T} \right\} \right], \quad (6)$$

$$A_{pq} = 1, A_{p0} = 2^p, A_{pq-1} = A_{pq} + \frac{(p+1)!}{q!(p+1-q)!}, \quad (7)$$

$$T = \frac{ct}{a}, c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}, \quad (8)$$

and  $\alpha$  is the approximate parameter,  $K$  is the truncation number of FILT, and  $p$  is the truncation number of the Euler transformation.

### 3. Numerical Results

We consider an half-sine pulse as the incident wave. The waveform in the time domain can be expressed by

$$E_0(T) = \sin \left( \frac{\pi T}{T_w} \right) [u(T) - u(T - T_w)], \quad (9)$$

where  $T_w = \pi / \Omega_0$  is the pulse width and  $u(T)$  is the unit step function. The function can be transformed into the complex frequency domain such as

$$\hat{E}_0(S) = \left( \frac{\Omega_0}{S^2 + \Omega_0^2} \right) [1 + \exp(-T_w S)]. \quad (10)$$

Figure 2 shows the backscattered responses for the parallel plate waveguide cavity when  $\phi_{in} = 180^\circ$  and  $T_w = \pi / 3$ . The time origin is defined when the earliest scattered wave is arrived at the observation point. In this case, the initial response which is the reflection from the bottom plate can be observed at  $T = T_D$ . The waveform at  $T = T_p$  corresponds to the end of the incident pulse. The computational result indicated by dots is obtained by using a highly reliable mode matching technique, the point matching method (PMM) [4] [5]. Both results are in excellent agreement.

Figure 3 shows the backscattered response for the parallel plate waveguide cavity when  $\phi_{in} = 0^\circ$  and  $T_w = \pi / 3$ . The initial responses  $T_D \leq T \leq T_R$  is the reflection from the edges. The response at  $T = T_R$  is the reflected wave from the bottom plate. The MoM result is in excellent agreement with the PMM one.

Figure 4 shows the backscattered response for the L-shaped cylinder when  $\phi_{in} = 0^\circ$ . The initial response  $T_D \leq T \leq T_{R1}$  is the reflection from at the edge. The amplitude is almost half of that for the parallel plate waveguide cavity. In the late time, the response is stable without oscillation.

## 4. Conclusions

We propose a novel computational method which is based on the combination of the method of moments and the fast inversion of the Laplace transform for transient electromagnetic analysis. This method can be applied to solving the electromagnetic scattering of conducting cylinders with apertures. We clarify the scattering mechanism of transient responses for the two types of geometries. Computational results are in excellent agreement with those obtained by a highly reliable mode matching technique.

## Acknowledgments

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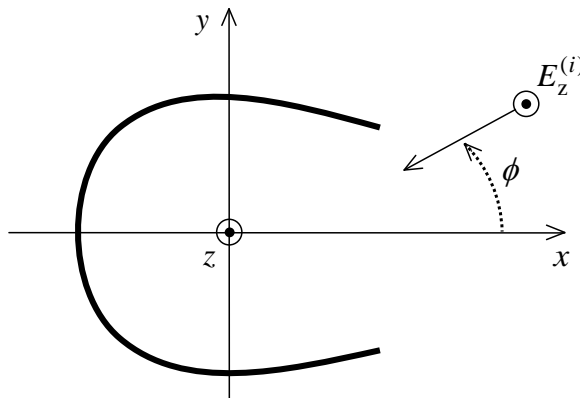


Figure 1: Coordinate system of a conducting cylinder with an aperture

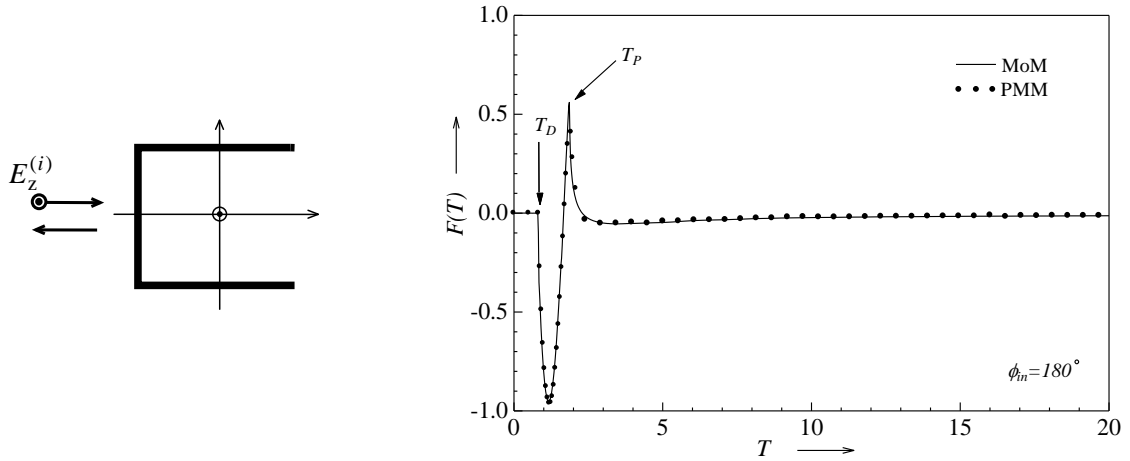


Figure 2: Backscattered responses of the parallel plate waveguide cavity for the pulse incidence from the closed side

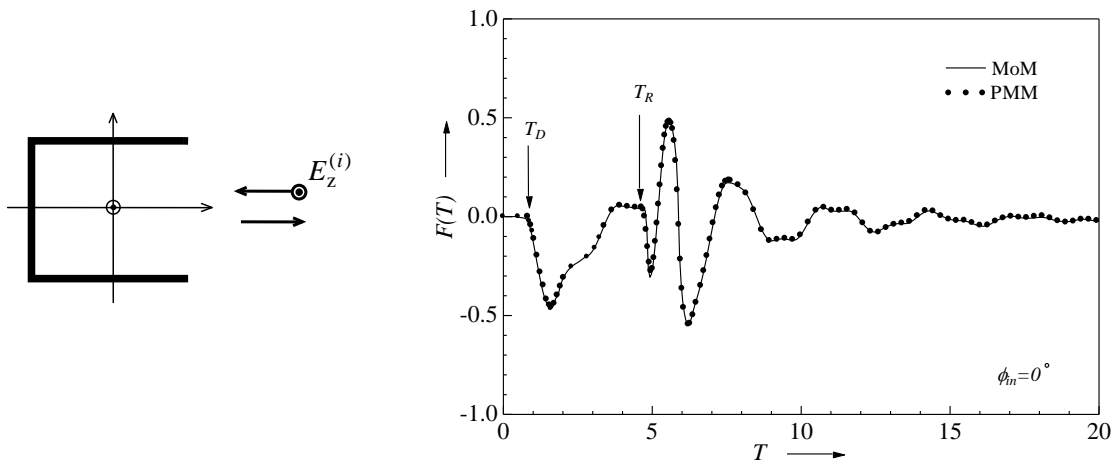


Figure 3: Backscattered responses of the parallel plate waveguide cavity for the pulse incidence from the open side

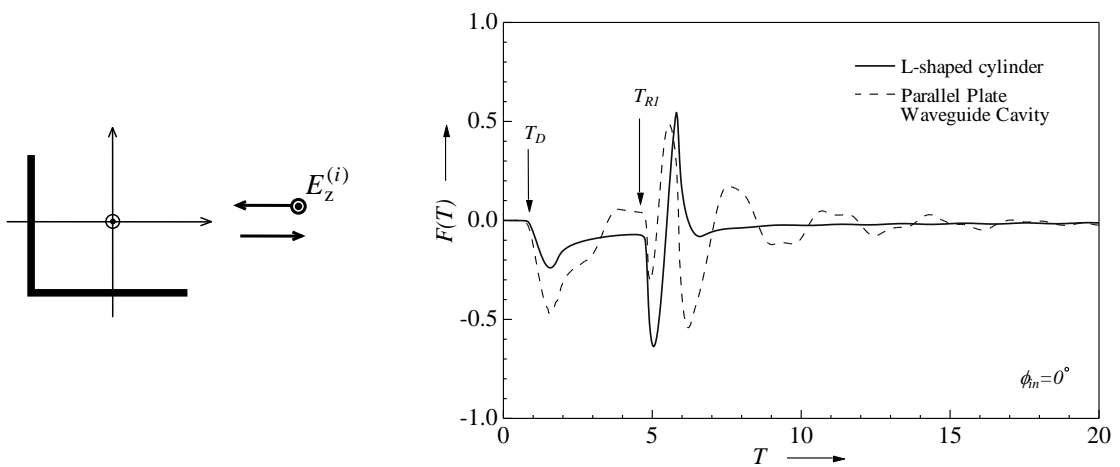


Figure 4: Backscattered responses of the L-shaped cylinder for the pulse incidence