

# FDTD Analysis of PEC Wire in Contact with Surface Impedance Boundary Condition

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## 1. Introduction

The hole-body resonance phenomenon is observed when a human is standing on the flat earth far field exposure of VHF waves[1][2]. In the resonance phenomenon, intensive current is induced. On the other hand, the VHF waves are widely used as broadcasting, radio communication and so on. Therefore the analyzing of the induced current a human body is important. The Green function for the problem was obtained[3][4][5], therefore, the Method of Moment(MoM) can be used for solving this problem. However, the sommerfeld integral is included in this calculation, the sommerfeld integral is not convergence easily, so, it is very difficult to analyze the problem by using MoM. The FDTD method is widely used to analyze a human body model, because, the numerical human data including the electric property of tissue is provided as the Voxel model[6], the Voxel data can be included to FDTD analysis easily[7]. In this paper, we use the FDTD method.

Fig.1 shows the scatterer on the flat earth and the FDTD analysis model. In the FDTD analysis, a special boundary is needed to truncate the analysis at the end of computation space. In order to realize flat earth, the PML absorbing boundary which matched with lossy ground is used[8]. However, the PML which matched with lossy ground is needed huge computation resources by comparison with the PML for free space. Furthermore, the PML which matched with lossy ground is very complicated, so, the programming is very difficult. We have proposed efficient modeling method to model flat earth[9]. The method uses surface Impedance boundary condition(SIBC). However, the proposed method was effective for only a case of separate scatteter from ground plane. In this paper, we expand the method for the induced current analysis of the metal wire in contact with the ground plane, and we will propose improving calculation accuracy technique for the problem.

## 2. The modeling method of flat earth by SIBC in the FDTD method

In this section, we will explain a derivation of SIBC for low-loss ground plane and its calculating method briefly.

$$Z_s(\omega) = \sqrt{\frac{j\omega\mu_0}{\sigma_s + j\omega\epsilon_s}} \quad (1)$$

but, it is very difficult to include FDTD method by using eq.(1). On the other hand, the electric property of ground is low-loss and high permittivity at VHF bands (e.g.  $\sigma = 0.0016[S/m]$ ,  $\epsilon_r = 15.0@79.4MHz$ ), therefore, the surface impedance can be approximated as

$$Z_s(\omega) = \sqrt{\frac{\mu/\epsilon_s}{1 + \frac{\sigma_s}{j\omega\epsilon_s}}} \approx Z \left( \frac{1}{1 + \frac{\sigma_s/2}{j\omega\epsilon_s}} \right) \quad (2)$$

where  $Z = \sqrt{\mu_0/\epsilon_s}$ .

The tangential component of electric filed at the ground surface can be expressed by using surface impedance as

$$E_{tan} = Z_s \mathbf{n} \times \mathbf{H} \quad (3)$$

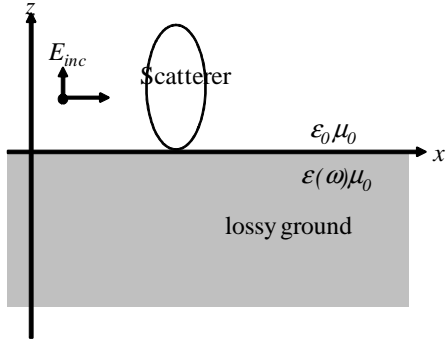


Figure 1: Scatterer on flat earth

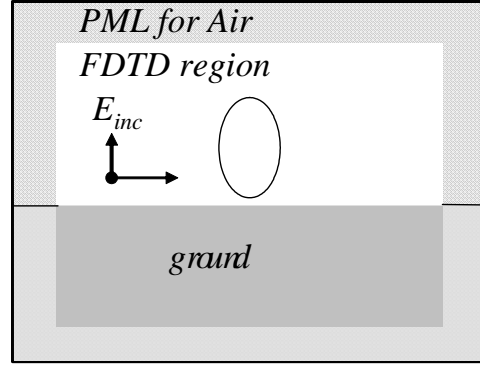


Figure 2: Geometry of the problem for FDTD method

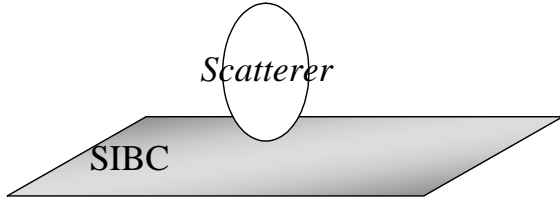


Figure 3: A modeling method of flat earth plane using SIBC

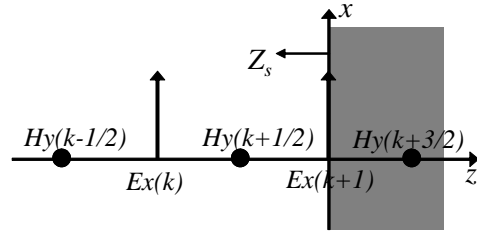


Figure 4: Fields placement in SIBC

The computation resources can be reduced by using the surface impedance, because, the calculation electric field which is inside of ground becomes unnecessary.

Next, we will indicate briefly how to formulate obtained surface impedance to the FDTD method. The FDTD geometry is shown in Fig.4. The electric field on the SIBC is calculated by using surface impedance as

$$E_x(t) = \int_0^t Z_s(\tau) H_y(t - \tau) d\tau \quad (4)$$

Substitution eq.(1) to eq.(4), we can obtain FDTD update equation

$$H_y^{n+\frac{1}{2}} \left( i + \frac{1}{2}, j, k + \frac{1}{2} \right) = \frac{1}{\frac{\mu}{\Delta t} + \frac{Z_s \chi^0}{\Delta z}} \left\{ \frac{\mu}{\Delta t} H_y^{n-\frac{1}{2}} \left( i + \frac{1}{2}, j, k + \frac{1}{2} \right) - \frac{1}{\Delta z} \left( E_x^n \left( i + \frac{1}{2}, j, k \right) - \Phi^n \right) + \frac{1}{\Delta x} \left\{ E_y^n \left( i + 1, j, k + \frac{1}{2} \right) - E_y^n \left( i, j, k + \frac{1}{2} \right) \right\} \right\} \quad (5)$$

where  $\chi^0 = -\frac{\sigma_s}{2\epsilon_s} \int_0^{\Delta t} e^{-\frac{\sigma_s}{2\epsilon_s} t} dt$ . We can derive FDTD equation for  $H_y$  similarly.

### 3. Modeling for wire in contact with ground plane in SIBC-FDTD

In this section, we expand SIBC for PEC wire in contact with flat earth. Fig. 5 shows FDTD cell on the ground plane. In this figure  $H_y$  and  $H_x$  are obtained by using SIBC as eq.(5). In order to model PEC wire touched flat earth,  $E_z$  component which just under the wire should be  $E_z = 0$ . However, the calculated induced current is not good agreement with PML result. In this paper, modeling method of PEC wire in contact with SIBC is proposed. From Fig.5, special treatment is needed to obtain  $H_y$  and  $H_x$  vicinity of the wire. The integral form of Faraday's law

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = \mu \int_s \frac{\partial \mathbf{H}}{\partial t} ds \quad (6)$$

is effective to model this special condition. Applying Faraday 's law to contour C in Fig.5, the update equation of  $H_y$  can be obtained as

$$H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right) = \frac{1}{\frac{\mu}{\Delta t} + \frac{Z+\chi^0}{\Delta z}} \left\{ \frac{\mu}{\Delta t} H_y^{n-\frac{1}{2}}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right) - \frac{2}{\Delta z} \left( E_x^n\left(i+\frac{1}{2}, j, k\right) - \Phi^n \right) \right. \\ \left. + \frac{2}{\Delta x} \left\{ 0.5 E_y^n\left(i+1, j, k+\frac{1}{2}\right) - E_y^n\left(i, j, k+\frac{1}{2}\right) \right\} \right\} \quad (7)$$

#### 4. Improving calculation accuracy

In this section, we will propose the improving calculation accuracy techniques. In order to improve the calculation accuracy, we introduce quasi-static approximation[10]. Fig.6 shows analysis model to obtain static field distribution near the antenna on the ground plane. In this mode, the antenna is uniformly charged. The electro static potential  $\phi(\rho, z)$  is obtained as

$$\phi(\rho, z) = \frac{\sigma}{4\pi\epsilon} \left\{ \frac{\log\left|z + \sqrt{z^2 + \rho^2}\right|}{\log\left|z - l + \sqrt{(z-l)^2 + \rho^2}\right|} + R \frac{\log\left|z + l + \sqrt{(z+l)^2 + \rho^2}\right|}{\log\left|z + \sqrt{(z)^2 + \rho^2}\right|} \right\} \quad (8)$$

where  $R = \frac{1-\epsilon_r}{1+\epsilon_r}$ . The electric field distribution can be obtained by

$$\mathbf{E} = -\nabla\phi \quad (9)$$

The obtained electric field can be included into FDTD method by using eq.(6)[14]. In this paper, the magnetic field is assumed constant in the FDTD cells. This assuming is same as original FDTD method.

#### 5. Numerical results

In this section we calculated PEC wire on flat earth. the radius of the wire is 2.35cm, the length of the wire is 175cm. The wire is set on ground plane. The imputed wave is plane wave and frequency is 79.4MHz. The ground is semi-dry condition, the electric properties are  $\sigma = 0.0016[S/m]$ ,  $\epsilon_r = 15.0$ . The calculated induced current is shown in Fig.7. The accuracy is confirmed by comparison with PML result. The gound plane is carefully modeled by using PML absorbing boundary condition in the PML result.

The proposed method which include quasi-static field is good agreement with PML result. Therore, the proposed method is very effective to analyze PEC wire on flat earth. Fig.8 shows induced current distribution at 70.4MHz. The FDTD cell size and analysis model is same as previous calculation. This result also shows that The proposed method which include quasi-static field is effective to analyze PEC wire on flat earth.

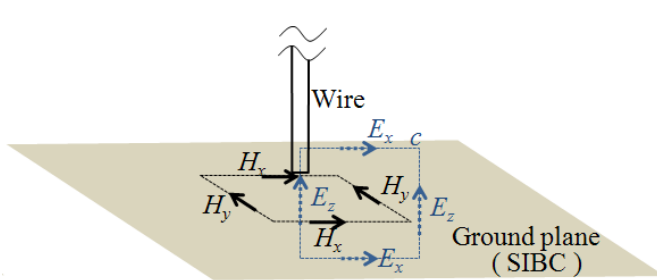


Figure 5: Fields placement near the antenna

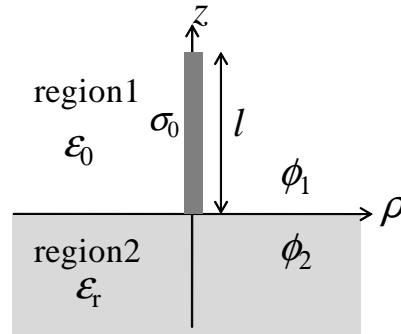


Figure 6: An analysis model

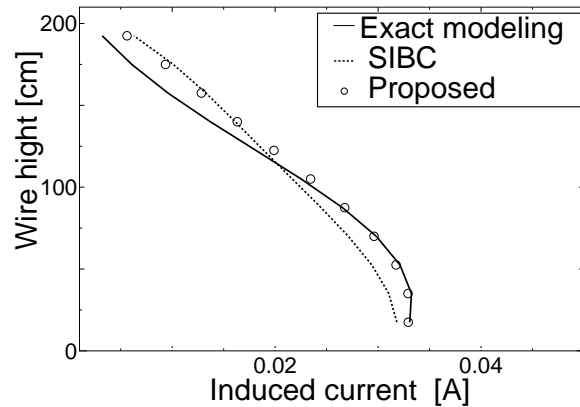
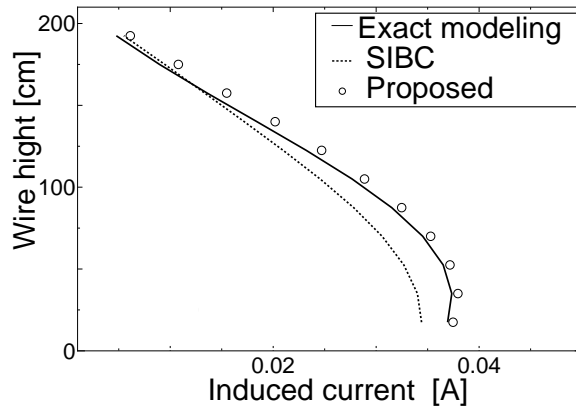


Figure 7: Induced current distribution at 79.4MHz Figure 8: Induced current distribution at 70.4MHz

## 6. Conclusion

In this paper, we proposed which include quasi-static field efficient analysis method for wire in contact with ground plane. From results, the proposed method is very effective by comparison with using PML method. The human model analysis will be our future works.

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